24. The factorial of n, written $n!$, is defined by $n! = 1 \times 2 \times 3 \times \ldots \times (n-2) \times (n-1) \times n$. For how many positive integer values of k less than 50 is it impossible to find a value of n such that $n!$ ends in exactly k zeros?

A 0 B 5 C 8 D 9 E 10

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24. D When $n!$ is written in full, the number of zeros at the end of the number is equal to the power of 5 when $n!$ is written as the product of prime factors, because there is at least that high a power of 2 available. For example, $12! = 1 \times 2 \times 3 \times \ \ldots \ \times 12$
$= 2^{10} \times 3^5 \times 5^2 \times 7 \times 11$.
This may be written as $2^8 \times 3^5 \times 7 \times 11 \times 10^2$, so 12! ends in 2 zeros, as $2^8 \times 3^5 \times 7 \times 11$ is not a multiple of 10.
We see that 24! ends in 4 zeros as 5, 10, 15 and 20 all contribute one 5 when 24! is written as the product of prime factors, but 25! ends in 6 zeros because $25 = 5 \times 5$ and hence contributes two 5s. So there is no value of n for which $n!$ ends in 5 zeros.
Similarly, there is no value of n for which $n!$ ends in 11 zeros since 49! ends in 10 zeros and 50! ends in 12 zeros. The full set of values of k less than 50 for which it is impossible to find a value of n such that $n!$ ends in k zeros is 5, 11, 17, 23, 29, 30 (since 124! ends in 28 zeros and 125! ends in 31 zeros), 36, 42, 48.