



14. Heather and Rachel each has some pennies. Heather has more than Rachel. In fact, the number of pennies that Heather has is the square of the number that Rachel has. The total number of pennies they have between them makes a whole number of pounds. What is the smallest this total could be?

A £1 B £6 C £57 D £99 E £101

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14. **B** Let Rachel and Heather have x and x^2 pennies respectively. So $x + x^2 = 100n$, where x and n are positive integers. We require, therefore, that $x(x + 1) = 100n = 2^2 \times 5^2 \times n$. Now x and $x + 1$ cannot both be multiples of 5, so their product will be a multiple of 25 if and only if x or $x + 1$ is a multiple of 25. The smallest value of x which satisfies this condition is 24 which is a multiple of 4 so 24×25 is a multiple of 100. Therefore Rachel has 24 pennies, Heather has 576 pennies and, in total, they have £6.