



17. A trapezium is bounded by four lines, the equations of which are $x = 0$, $x = 4$, $4y = 3x + 8$ and $y = k$, where $k < 2$.

For which value of k is the numerical value of the perimeter of the trapezium equal to the numerical value of the area of the trapezium?

- A $\frac{3}{2}$ B 1 C $\frac{1}{2}$ D $-\frac{1}{2}$ E -1

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17. E The trapezium in question is shown as $ABCD$ in the diagram. The coordinates of its vertices are $A(0, k)$, $B(0, 2)$, $C(4, 5)$, $D(4, k)$. Using Pythagoras' Theorem: $BC = \sqrt{4^2 + 3^2} = 5$.
The perimeter of $ABCD = (2 - k) + 5 + (5 - k) + 4 = 16 - 2k$.
The area of $ABCD = 4(2 - k) + \frac{1}{2} \times 4 \times 3 = 14 - 4k$. So $16 - 2k = 14 - 4k$, that is $k = -1$.

(In the diagram it was assumed that $k > 0$, although it transpires that $k < 0$. However, this does not affect the validity of the solution.)

