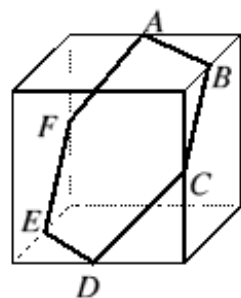




24. A solid red plastic cube, volume  $1 \text{ cm}^3$ , is painted white on its outside. The cube is cut by a plane passing through the mid-points of various edges, as shown.

What, in  $\text{cm}^2$ , is the *total red area* exposed by the cut?

- A  $\frac{3\sqrt{3}}{2}$     B 2    C  $\frac{9\sqrt{2}}{5}$     D 3    E  $\frac{3(\sqrt{3} + \sqrt{2})}{4}$



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24. A Let  $O$  be the centre of the cube. Consider triangle  $ABO$ : from Pythagoras' Theorem,  $OA = AB = BO = \sqrt{\left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^2} \text{ cm} = \frac{1}{\sqrt{2}} \text{ cm}$ . So triangle  $OAB$  is equilateral. A similar argument may be applied to triangles  $OBC$ ,  $OCD$  etc. The area of each of these equilateral triangles is  $\frac{1}{2} \times \frac{1}{\sqrt{2}} \times \frac{1}{\sqrt{2}} \times \sin 60^\circ \text{ cm}^2$ , that is  $\frac{1}{8}\sqrt{3} \text{ cm}^2$ . So the area of hexagon  $ABCDEF$  is  $6 \times \frac{\sqrt{3}}{8} \text{ cm}^2$ . However, the total red area exposed by the cut is twice the area of this hexagon, that is  $\frac{3\sqrt{3}}{2} \text{ cm}^2$ .