10. Which one of the following rational numbers *cannot* be expressed as $\frac{1}{m} + \frac{1}{n}$ where m, n are different positive integers?

A $\frac{3}{4}$ B $\frac{3}{5}$ C $\frac{3}{6}$ D $\frac{3}{7}$ E $\frac{3}{8}$

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10. D By inspection

$$\frac{3}{4} = \frac{1}{2} + \frac{1}{4}; \qquad \frac{3}{5} = \frac{1}{2} + \frac{1}{10}; \qquad \frac{3}{6} = \frac{1}{3} + \frac{1}{6}; \qquad \frac{3}{8} = \frac{1}{4} + \frac{1}{8}.$$

However $\frac{3}{7} \neq \frac{1}{m} + \frac{1}{n}$. [To see why, suppose that $\frac{3}{7} = \frac{1}{m} + \frac{1}{n}$ and note that $\frac{1}{m} > \frac{1}{n}$ or vice versa. We will suppose the former. Then $\frac{1}{m} \geqslant \frac{3}{14} > \frac{3}{15}$ and so $\frac{1}{m} > \frac{1}{5}$ and $m < 5$. Also $\frac{1}{m} < \frac{3}{7}$ and so $3m > 7$. Hence $m \geqslant 3$. So $m = 4$ or $m = 3$. However $\frac{3}{7} - \frac{1}{4} = \frac{5}{28}$ and $\frac{3}{7} - \frac{1}{3} = \frac{2}{21}$ neither of which has the form $\frac{1}{n}$.]