



21. Which of the following numbers does *not* have a square root in the form $x + y\sqrt{2}$, where x and y are positive integers?

- A $17 + 12\sqrt{2}$ B $22 + 12\sqrt{2}$ C $38 + 12\sqrt{2}$ D $54 + 12\sqrt{2}$ E $73 + 12\sqrt{2}$

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21. **D** $(x + y\sqrt{2})^2 = x^2 + 2xy\sqrt{2} + 2y^2$. Note that all of the alternatives given are of the form $a + 12\sqrt{2}$ so we need $xy = 6$. The only ordered pairs (x, y) of positive integers which satisfy this are $(1, 6)$, $(2, 3)$, $(3, 2)$, $(6, 1)$. For these, the values of $x^2 + 2y^2$ are 73, 22, 17, 38 respectively. So the required number is $54 + 12\sqrt{2}$.