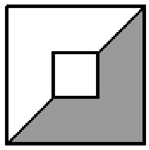
UKMT Fractions Questions
(Answers follow after all the questions)

2005…

4. The diagram shows two squares, with sides of length 1 and 3, which have the same centre and corresponding sides parallel. What fraction of the larger square is shaded?



A $\frac{4}{9}$ B $\frac{4}{11}$ C $\frac{2}{5}$ D $\frac{2}{7}$ E $\frac{6}{11}$

9. What is the value of the expression: $(1+\frac{1}{2})(1+\frac{1}{3})(1+\frac{1}{4})\ldots(1+\frac{1}{2004})(1+\frac{1}{2005})$?

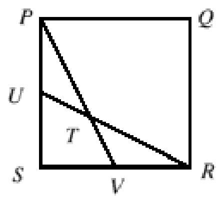
A 1001 B 1002 C 1003 D 1004 E 1005

21. What is the sum of the values of n for which both n and $\frac{n^2-9}{n-1}$ are integers?

A -8 B -4 C 0 D 4 E 8

2006…

15. *PQRS* is a square with *U* and *V* the mid-points of the sides *PS* and *SR* respectively. Line segments *PV* and *UR* meet at *T*.
What fraction of the area of the square *PQRS* is the area of the quadrilateral *PQRT*?



A $\frac{1}{2}$ B $\frac{5}{8}$ C $\frac{2}{3}$ D $\frac{3}{4}$ E $\frac{5}{9}$

20. A positive number $a = [a] + \{a\}$ where $[a]$ is the integer part of a and $\{a\}$ is the fractional part of a.
Given that $x + [y] + \{z\} = 4.2$, $y + [z] + \{x\} = 3.6$, $z + [x] + \{y\} = 2.0$, and $x, y, z > 0$, what is the value of $\{y\}$?

A 0.1 B 0.3 C 0.5 D 0.7 E 0.9

22. Which positive integer n satisfies the equation

$$\frac{3}{n^3}+\frac{4}{n^3}+\frac{5}{n^3}+\ \ldots\ +\frac{n^3-5}{n^3}+\frac{n^3-4}{n^3}+\frac{n^3-3}{n^3} = 60\ ?$$

A 5 B 11 C 31 D 60 E 2006

2007…

1. What is the value of $\frac{2007}{9} + \frac{7002}{9}$?

A 500.5 B 545 C 1001 D 1655 E 2007

2008…

10. Which one of the following rational numbers *cannot* be expressed as $\frac{1}{m} + \frac{1}{n}$ where m, n are different positive integers?

A $\frac{3}{4}$ B $\frac{3}{5}$ C $\frac{3}{6}$ D $\frac{3}{7}$ E $\frac{3}{8}$

21. The fraction $\frac{2008}{1998}$ may be written in the form $a + \cfrac{1}{b + \cfrac{1}{c + \cfrac{1}{d}}}$ where a, b, c and d are positive integers. What is the value of d?

A 2 B 4 C 5 D 199 E 1998

2009…

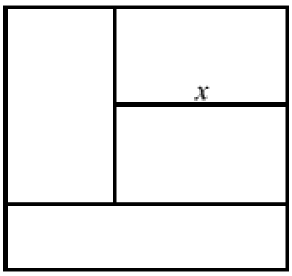
15. For how many integers n is $\frac{n}{100 - n}$ also an integer?

A 1 B 6 C 10 D 18 E 100

2010…

12. The diagram, which is not to scale, shows a square with side length 1, divided into four rectangles whose areas are equal. What is the length labelled x?

A $\frac{2}{3}$ B $\frac{17}{24}$ C $\frac{4}{5}$ D $\frac{49}{60}$ E $\frac{5}{6}$



2011…

1. Which of the numbers below is not a whole number?

A $\frac{2011+0}{1}$ B $\frac{2011+1}{2}$ C $\frac{2011+2}{3}$ D $\frac{2011+3}{4}$ E $\frac{2011+4}{5}$

2. Jack and Jill went up the hill to fetch a pail of water. Having filled the pail to the full, Jack fell down, spilling $\frac{2}{3}$ of the water, before Jill caught the pail. She then tumbled down the hill, spilling $\frac{2}{5}$ of the remainder.

What fraction of the pail does the remaining water fill?

A $\dfrac{11}{15}$ B $\dfrac{1}{3}$ C $\dfrac{4}{15}$ D $\dfrac{1}{5}$ E $\dfrac{1}{15}$

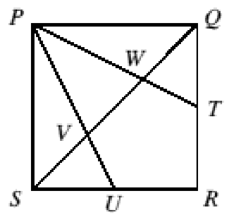
2013…

6. Rebecca went swimming yesterday. After a while she had covered one fifth of her intended distance. After swimming six more lengths of the pool, she had covered one quarter of her intended distance. How many lengths of the pool did she intend to complete?

A 40 B 72 C 80 D 100 E 120

23. *PQRS* is a square. The points *T* and *U* are the midpoints of *QR* and *RS* respectively. The line *QS* cuts *PT* and *PU* at *W* and *V* respectively. What fraction of the area of the square *PQRS* is the area of the pentagon *RTWVU*?

A $\dfrac{1}{3}$ B $\dfrac{2}{5}$ C $\dfrac{3}{7}$ D $\dfrac{5}{12}$ E $\dfrac{4}{15}$



2014…

4. After I had spent $\frac{1}{5}$ of my money and then spent $\frac{1}{4}$ of what was left, I had £15 remaining. How much did I start with?

A £25 B £75 C £100 D £135 E £300

2015…

7. Which of the following has the largest value?

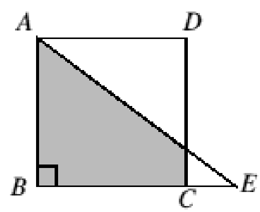
A $\dfrac{\left(\frac{1}{2}\right)}{\left(\frac{3}{4}\right)}$ B $\dfrac{1}{\left(\dfrac{\left(\frac{2}{3}\right)}{4}\right)}$ C $\dfrac{\left(\dfrac{\left(\frac{1}{2}\right)}{3}\right)}{4}$ D $\dfrac{1}{\left(\dfrac{2}{\left(\frac{3}{4}\right)}\right)}$ E $\dfrac{\left(\dfrac{1}{\left(\frac{2}{3}\right)}\right)}{4}$

2016…

6. The diagram shows a square *ABCD* and a right-angled triangle *ABE*. The length of *BC* is 3. The length of *BE* is 4.

What is the area of the shaded region?

A $5\frac{1}{4}$ B $5\frac{3}{8}$ C $5\frac{1}{2}$ D $5\frac{5}{8}$ E $5\frac{3}{4}$



UKMT Fractions Answers

2005…

4. **A** The smaller square has one ninth of the area of the larger square. So the fraction of the larger square which is shaded is half of eight ninths, that is four ninths.

9. **C** The product is $\frac{3}{2} \times \frac{4}{3} \times \frac{5}{4} \times \frac{6}{5} \times \ \ldots \ \times \frac{2005}{2004} \times \frac{2006}{2005} = \frac{2006}{2} = 1003.$

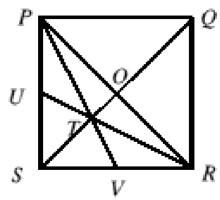
21. **E** Note that $n^2 - 1$ is divisible by $n - 1$. Thus:

$$\frac{n^2 - 9}{n - 1} = \frac{n^2 - 1}{n - 1} - \frac{8}{n - 1} = n + 1 - \frac{8}{n - 1} \qquad (n \neq 1).$$

So, if n is an integer, then $\frac{n^2 - 9}{n - 1}$ is an integer if and only if $n - 1$ divides exactly into 8.
The possible values of $n - 1$ are $-8, -4, -2, -1, 1, 2, 4, 8$, so n is $-7, -3, -1, 0, 2, 3, 5, 9$.
The sum of these values is 8.
(Note that the sum of the 8 values of $n - 1$ is clearly 0, so the sum of the 8 values of n is 8.)

2006…

15. **C** Let O be the centre of square $PQRS$. The medians of triangle PSR intersect at T so $OT = \frac{1}{3}OS$.
Hence the area of triangle PTR is one third of the area of triangle PSR, that is one sixth of the area of square $PQRS$. So the required fraction $= \frac{1}{6} + \frac{1}{2} = \frac{2}{3}$.



20. **D** Adding all three equations: $x + [y] + \{z\} + y + [z] + \{x\} + z + [x] + \{y\} = 4.2 + 3.6 + 2.0 = 9.8$.
Now $[x] + \{x\} = x$, $[y] + \{y\} = y$, $[z] + \{z\} = z$, so $2x + 2y + 2z = 9.8$, that is $x + y + z = 4.9$.
Therefore: $x + y + z - (x + [y] + \{z\}) = 4.9 - 4.2$, that is $\{y\} + [z] = 0.7$. So $[z] = 0$, $\{y\} = 0.7$.
(It is not necessary to find the values of x, y, z to solve this problem, but their values may be shown to be 1.9, 2.7, 0.3 *respectively.)*

22. **A** The terms on the left-hand side of the equation form an arithmetic progression which has $n^3 - 5$ terms. So the sum of these terms is $\frac{n^3 - 5}{2}\left(\frac{3}{n^3} + \frac{n^3 - 3}{n^3}\right) = \frac{n^3 - 5}{2}$. Hence $n^3 - 5 = 120$, so $n = 5$.

2007…

1. **C** $\frac{2007}{9} + \frac{7002}{9} = \frac{9009}{9} = 1001.$

2008…

10. **D** By inspection

$$\frac{3}{4}=\frac{1}{2}+\frac{1}{4}; \quad \frac{3}{5}=\frac{1}{2}+\frac{1}{10}; \quad \frac{3}{6}=\frac{1}{3}+\frac{1}{6}; \quad \frac{3}{8}=\frac{1}{4}+\frac{1}{8}.$$

However $\frac{3}{7} \neq \frac{1}{m}+\frac{1}{n}$. [To see why, suppose that $\frac{3}{7}=\frac{1}{m}+\frac{1}{n}$ and note that $\frac{1}{m} > \frac{1}{n}$ or vice versa. We will suppose the former. Then $\frac{1}{m} \geqslant \frac{3}{14} > \frac{3}{15}$ and so $\frac{1}{m} > \frac{1}{5}$ and $m < 5$. Also $\frac{1}{m} < \frac{3}{7}$ and so $3m > 7$. Hence $m \geqslant 3$. So $m = 4$ or $m = 3$. However $\frac{3}{7}-\frac{1}{4} = \frac{5}{28}$ and $\frac{3}{7}-\frac{1}{3} = \frac{2}{21}$ neither of which has the form $\frac{1}{n}$.]

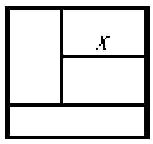
21. **B** Since 2008/1998 lies between 1 and 2, $a = 1$. Subtracting 1 and inverting gives $b+1/(c+1/d) = 1998/10 = 199+4/5$ so that $b = 199$. Then $1/(c + 1/d) = 4/5$ so that $c + 1/d = 5/4$ and this gives $c = 1$ and $d = 4$.
{*Note : This is an example of a continued fraction.*}

2009…

15. **D** Let $\frac{n}{100 - n} = x$ where x is an integer. Hence $n = 100x - nx$.
Hence $n(1 + x) = 100x$ giving $n = \frac{100x}{1 + x}$.
Now x and $1 + x$ can have no common factors. Therefore $1 + x$ must be a factor of 100 and can be any of them.
Hence $1 + x \in \{\pm1, \pm2, \pm4, \pm5, \pm10, \pm20, \pm25, \pm50, \pm100\}$ thus the number of possible integers n is 18.

2010…

12. **A** As the square has side length 1 its area is $1 \times 1 = 1$.
Thus the area of each of the four rectangles is $\frac{1}{4}$.
The length of the bottom rectangle is 1 hence its width is $\frac{1}{4}$.
Thus the width of each of the two congruent rectangles is $\frac{1}{2}(1 - \frac{1}{4}) = \frac{3}{8}$.
Hence the area of one of these congruent rectangles is $\frac{3}{8}x$.
But we know this area is $\frac{1}{4}$, therefore $\frac{3}{8}x = \frac{1}{4}$ and hence $x = \frac{2}{3}$.



2011…

1. **D** Every integer is divisible by 1; 2012 is divisible by 2 since it is even; 2013 is divisible by 3 since its digits total to a multiple of 3; and 2015 is divisible by 5 since its last digit is 5. However, 2014 is not divisible by 4 because 14 is not.

2. **D** After the first spill, $\frac{1}{3}$ of the water remains.
After the second spill, $\frac{3}{5} \times \frac{1}{3}$ of the water remains, hence $\frac{1}{5}$ of the pail had water left in it.

2013…

6. **E** Let d be the number of lengths that Rebecca intended to swim. Then $6 = \frac{d}{4} - \frac{d}{5} = \frac{d}{20}$ and therefore $d = 6 \times 20 = 120$.

23. **A** The pentagon $RTWVU$ is the remainder when triangles SUV and WTQ are removed from the bottom right half of the square. Draw in the diagonal PR and consider the triangle PRS. The medians of triangle PRS join each vertex P, R and S to the midpoint of its opposite side, i.e. P to U and S to the middle of the square. The medians intersect at V and therefore the height of V above SR is $\frac{1}{3}$ of PS.
The area of triangle SUV is therefore $\frac{1}{2} \times \frac{1}{2}SR \times \frac{1}{3}PS = \frac{1}{12}$ of the area of the square. By symmetry, this is also the area of triangle WTQ. The area of the pentagon $RTWVU$ is then $\frac{1}{2} - \left(\frac{1}{12} + \frac{1}{12}\right) = \frac{1}{3}$ of the area of the square $PQRS$.

2014…

4. **A** Let the original amount of money be x (in pounds). If I spend $\frac{x}{5}$ then $\frac{4x}{5}$ remains. When I spend $\frac{1}{4}$ of that, $\frac{3}{4}$ of it remains. So $\frac{4x}{5} \times \frac{3}{4}$ is what is left and that is £15. As $\frac{4x}{5} \times \frac{3}{4} = 15$, we have $x = \frac{5}{3} \times 15 = 25$. So the original amount of money is £25.

2015…

7. **B** Evaluating each option gives

A $\frac{\left(\frac{1}{2}\right)}{\left(\frac{3}{4}\right)} = \frac{1}{2} \times \frac{4}{3} = \frac{2}{3}$ B $\frac{1}{\left(\frac{\left(\frac{2}{3}\right)}{4}\right)} = \frac{1}{\left(\frac{2}{12}\right)} = \frac{12}{2} = 6$ C $\frac{\frac{\left(\frac{1}{2}\right)}{3}}{4} = \frac{\left(\frac{1}{6}\right)}{4} = \frac{1}{24}$

D $\frac{1}{\frac{2}{\left(\frac{3}{4}\right)}} = \frac{1}{\left(\frac{8}{3}\right)} = \frac{3}{8}$ E $\frac{\left(\frac{1}{\left(\frac{2}{3}\right)}\right)}{4} = \frac{\left(\frac{3}{2}\right)}{4} = \frac{3}{8}$.

So B has the largest answer.

2016…

6. **D** Let F be the point of intersection of the lines AE and CD. Let the length of CF be h. Then, using similar triangles, $\frac{CF}{CE} = \frac{BA}{BE}$, so $\frac{h}{1} = \frac{3}{4}$ giving $h = \frac{3}{4}$. The shaded region $ABCF$ is a trapezium, so has area $\frac{1}{2}\left(3 + \frac{3}{4}\right) \times 3 = \frac{45}{8}$ which is $5\frac{5}{8}$.