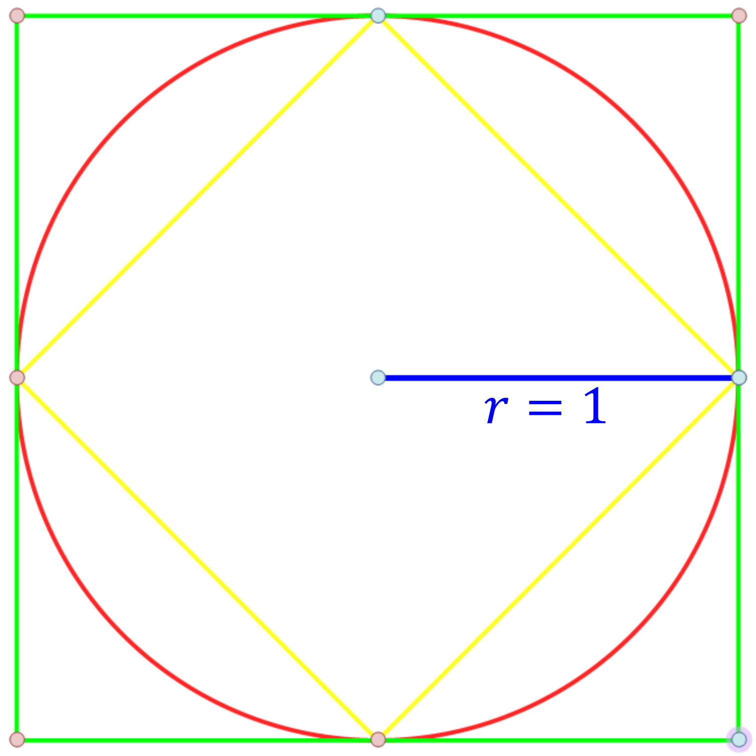
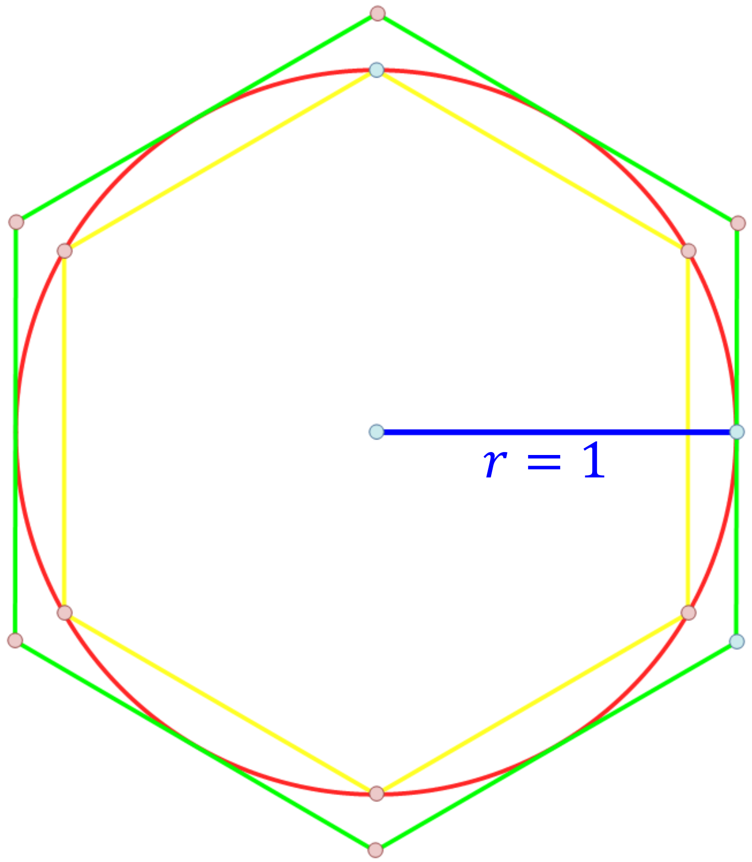
Use the yellow and green shapes below to find upper and lower bounds for π, refining your accuracy each time.

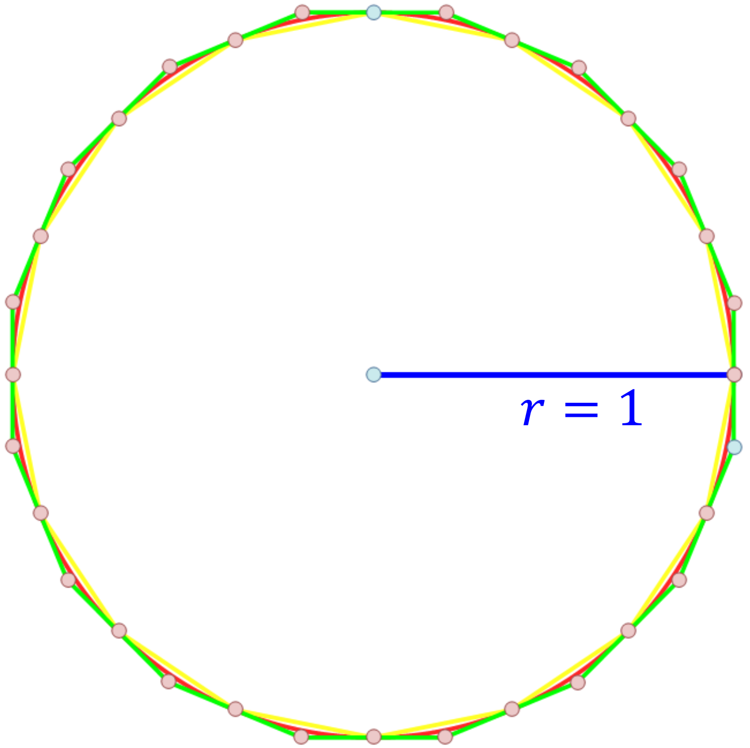
Squares



Hexagons



Hexadecagons



How about a general formula for n sides?

Application of Binomial Expansion

1.c

a) Given $f(x) = \sqrt{1-x^2}$, write down the first four terms of the binomial expansion of $f(x)$.

b) Integrate your binomial expansion between the limits 0 and 1, and then multiply this by four.

c) Sketch the graph of $x^2 + y^2 = 1$

2.

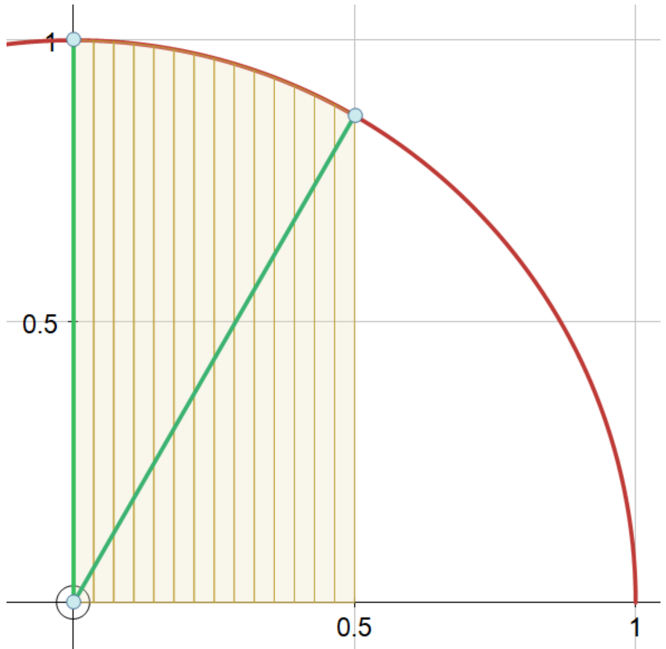
a) Integrate your binomial expansion between the limits 0 and $\frac{1}{2}$.

b) Find the area of the triangle with vertices at the origin, $\left(\frac{1}{2}, 0\right)$ and $\left(\frac{1}{2}, f\left(\frac{1}{2}\right)\right)$.

c) Find the angle between the positive y axis and the line joining the origin to $f\left(\frac{1}{2}\right)$.

d) Hence find a new approximation for the value that you found in question 1.

e) Compare and contrast.



3. Explore more at https://www.youtube.com/watch?v=gMlf1ELvRzc.

We're gonna need a quicker method

1.

a) Rearrange $x^2 + y^2 = 1$ to make y the subject.

b) Write down the first four terms of the binomial expansion for your equation for y above.

c) Integrate your binomial expansion between the limits 0 and 1, and then multiply this by four.

d) Compare the accuracy of your result with the actual value.

2.

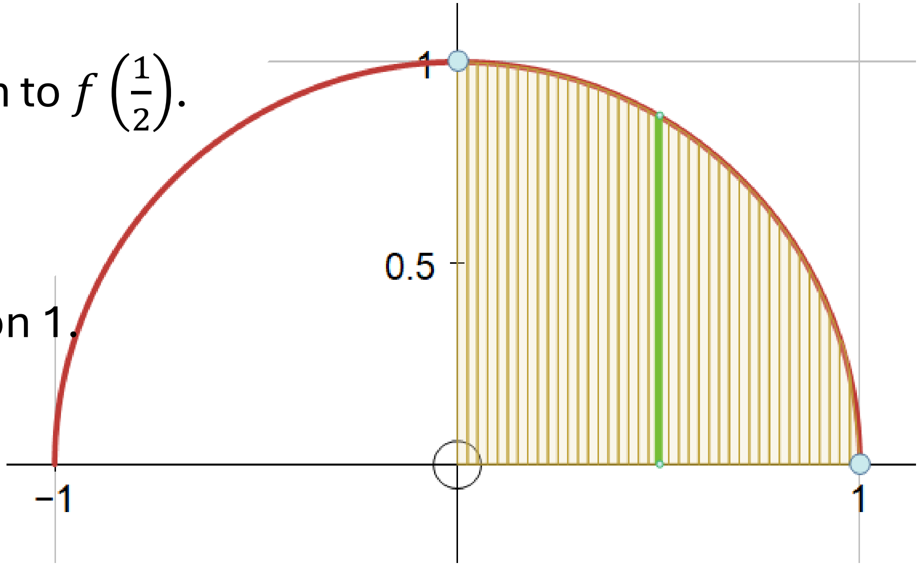
a) Integrate your binomial expansion between the limits 0 and $\frac{1}{2}$.

b) Find the angle between the positive y axis and the line joining the origin to $f\left(\frac{1}{2}\right)$.

c) Find the area of the triangle with vertices at the origin, $x = \frac{1}{2}$ and $f\left(\frac{1}{2}\right)$.

d) Hence find a new approximation for the value that you found in question 1.

e) Compare and contrast.



3. Explore more at https://www.youtube.com/watch?v=gMlf1ELvRzc.

We’re gonna need a quicker method

Use the substitution $x = sin\theta$, to find

$$\int_0^1 \sqrt{1-x^2}\,dx$$

Why can’t we just use this result to find the exact value of π?