**Chain, Product, Quotient Rule Recap**

Differentiate these.

**Chain Rule**

|  |  |  |
| --- | --- | --- |
| $$y=\left(3x+4\right)^{3}$$ | $$y=sin\left(2x\right)$$ | $$y=e^{2x}$$ |
| $$y=\left(3x^{3}+1\right)^{3}$$ | $$y=sin\left(x^{2}\right)$$ | $$y=e^{\left(x^{3}\right)}$$ |
| $$y=\frac{4}{4-x^{4}}$$ | $$y=\frac{1}{\left(\frac{1}{cosecx}\right)}$$ | $$y=ln\left(\frac{1}{e^{x}}\right)$$ |
| $$y=\sqrt{\sqrt{x}+1}$$ | $$y=sin\left(cos\left(tanx\right)\right)$$ | $$y=e^{\left(e^{x}\right)}$$ |

**Product Rule**

|  |  |  |
| --- | --- | --- |
| $$y=\left(3x+4\right)\left(2x-3\right)$$ | $$y=x^{2}sinx$$ | $$y=xe^{x}$$ |
| $$y=\left(3x^{2}+4\right)\left(2x^{4}+3\right)$$ | $$y=sinxcosx$$ | $$y=x^{3}e^{x}$$ |
| $$y=\sqrt{3x}\sqrt{2x}$$ | $$y=sinxcosxtanx$$ | $$y=xlnx$$ |
| $$y=\left(\frac{x}{3}\right)^{3}\sqrt[3]{x}$$ | $$y=sin\left(x^{2}\right)cos\left(x^{2}\right)$$ | $$y=e^{x^{2}}ln\left(x^{2}\right)$$ |

**Quotient Rule**

|  |  |  |
| --- | --- | --- |
| $$y=\frac{x+1}{2x+1}$$ | $$y=\frac{3x^{2}}{sinx}$$ | $$y=\frac{2x}{e^{x}}$$ |
| $$y=\frac{x^{2}+1}{x^{2}-1}$$ | $$y=tanx$$ | $$y=\frac{e^{x}}{e^{-x}}$$ |
| $$y=\frac{x^{2}}{\sqrt{x}}$$ | $$y=\frac{x^{2}}{tanx}$$ | $$y=\frac{e^{-4x}}{4e^{4x}}$$ |
| $$y=\sqrt[3]{\frac{x+1}{x-1}}$$ | $$y=\frac{cotx}{2secx}$$ | $$y=\frac{ln\left(x^{2}\right)}{e^{x^{2}}}$$ |

**Chain, Product, Quotient Rule Recap - Answers**

Differentiate these.

**Chain Rule**

|  |  |  |
| --- | --- | --- |
| $$y=\left(3x+4\right)^{3}$$$$\frac{dy}{dx}=9\left(3x+4\right)^{2}$$ | $$y=sin\left(2x\right)$$$$\frac{dy}{dx}=2cos\left(2x\right)$$ | $$y=e^{2x}$$$$\frac{dy}{dx}=2e^{2x}$$ |
| $$y=\left(3x^{3}+1\right)^{3}$$$$\frac{dy}{dx}=3\left(3x^{3}+1\right)^{2}\left(9x^{2}\right)$$$$=27x^{2}\left(3x^{3}+1\right)^{2}$$ | $$y=sin\left(x^{2}\right)$$$$\frac{dy}{dx}=2xcos\left(x^{2}\right)$$ | $$y=e^{\left(x^{3}\right)}$$$$\frac{dy}{dx}=3x^{2}e^{\left(x^{3}\right)}$$ |
| $$y=\frac{4}{4-x^{4}}=4\left(4-x^{4}\right)^{-1}$$$$\frac{dy}{dx}=-4\left(4-x^{4}\right)^{-2}×-4x^{3}$$$$=\frac{16x^{3}}{\left(4-x^{4}\right)^{2}}$$ | $$y=\frac{1}{\left(\frac{1}{cosecx}\right)}=\frac{1}{sinx}$$$$=\left(sinx\right)^{-1}$$$$\frac{dy}{dx}=-\left(sinx\right)^{-2}×cosx$$$$=\frac{-cosx}{\left(sinx\right)^{2}}=cotxcosecx$$ | $$y=ln\left(\frac{1}{e^{x}}\right)$$$$\frac{dy}{dx}=\frac{1}{\frac{1}{e^{x}}}×-e^{-x}$$$$=-e^{x}×e^{-x}$$$$=-e^{0}$$$$=1$$(convince yourself that this is the case) |
| $$y=\sqrt{\sqrt{x}+1}=\left(x^{\frac{1}{2}}+1\right)^{\frac{1}{2}}$$$$\frac{dy}{dx}=\frac{1}{2}\left(x^{\frac{1}{2}}+1\right)^{-\frac{1}{2}}×\frac{1}{2}x^{-\frac{1}{2}}$$$$=\frac{1}{4}×\frac{1}{\sqrt{\sqrt{x}+1}}×\frac{1}{\sqrt{x}}$$$$=\frac{1}{4\sqrt{x}\sqrt{\sqrt{x}+1}}$$ | $$y=sin\left(cos\left(tanx\right)\right)$$$$\frac{dy}{dx}=-cos\left(cos\left(tanx\right)\right)sin\left(tanx\right)sec^{2}x$$ | $$y=e^{e^{x}}$$$$\frac{dy}{dx}=e^{e^{x}}e^{x}=e^{e^{x}+x}$$ |

**Product Rule**

|  |  |  |
| --- | --- | --- |
| $$y=\left(3x+4\right)\left(2x-3\right)$$$$\frac{dy}{dx}=2\left(3x+4\right)+3\left(2x-3\right)$$$$=12x-1$$ | $$y=x^{2}sinx$$$$\frac{dy}{dx}=x^{2}cosx+2xsinx$$ | $$y=xe^{x}$$$$\frac{dy}{dx}= xe^{x}+e^{x}$$$$=\left(x+1\right)e^{x}$$ |
| $$y=\left(3x^{2}+4\right)\left(2x^{4}+3\right)$$$$\frac{dy}{dx}=8x^{3}\left(3x^{2}+4\right)+6x\left(2x^{4}+3\right)$$$$=2x\left[4x^{2}\left(3x^{2}+4\right)+3\left(2x^{4}+3\right)\right]$$$$=2x\left[12x^{4}+16x^{2}+6x^{4}+9\right]$$$$=2x\left[18x^{4}+16x^{2}+9\right]$$ | $$y=sinxcosx$$$$\frac{dy}{dx}=cos^{2}x-sin^{2}x$$ | $$y=x^{3}e^{x}$$$$\frac{dy}{dx}=3x^{2}e^{x}+x^{3}e^{x}$$$$=x^{2}e^{x}\left(x+3\right)$$ |
| $$y=\sqrt{3x}\sqrt{2x}$$$$=\left(6x^{2}\right)^{\frac{1}{2}}=\sqrt{6}x$$$$\frac{dy}{dx}=\sqrt{6}$$(easy method?) | $$y=sinxcosxtanx$$$$=sinxcosx×\frac{sinx}{cosx}$$$$=sin^{2}x$$$$\frac{dy}{dx}=2sinxcosx$$ | $$y=xlnx$$$$\frac{dy}{dx}=lnx+1$$ |
| $$y=\left(\frac{x}{3}\right)^{3}\sqrt[3]{x}$$$$\frac{dy}{dx}=\left(\frac{x}{3}\right)^{3}×\frac{1}{3\sqrt[3]{x}}+\sqrt[3]{x}×\frac{x^{2}}{3}$$$$=\frac{x^{3}}{9\sqrt[3]{x}}+\frac{x^{2}\sqrt[3]{x}}{3}$$$$=\frac{x^{2}}{3}\left(\frac{\sqrt[3]{x^{2}}}{3}+\sqrt[3]{x}\right)$$ | $$y=sin\left(x^{2}\right)cos\left(x^{2}\right)$$$$\frac{dy}{dx}=2x\left[cos^{2}\left(x^{2}\right)+sin^{2}\left(x^{2}\right)\right]$$ | $$y=e^{x^{2}}ln\left(x^{2}\right)$$$$\frac{dy}{dx}=\frac{2e^{x^{2}}}{x}+2xe^{x^{2}}ln\left(x^{2}\right)$$ |

**Quotient Rule**

|  |  |  |
| --- | --- | --- |
| $$y=\frac{x+1}{2x+1}$$$$\frac{dy}{dx}=\frac{2x+1-2\left(x+1\right)}{\left(2x+1\right)^{2}}$$$$=\frac{-1}{\left(2x+1\right)^{2}}$$ | $$y=\frac{3x^{2}}{sinx}$$$$\frac{dy}{dx}=\frac{6xsinx-3x^{2}cosx}{sin^{2}x}$$$$=\frac{3x}{sinx}\left(2-\frac{x}{tanx}\right)$$ | $$y=\frac{2x}{e^{x}}$$$$\frac{dy}{dx}=\frac{2e^{x}-2xe^{x}}{e^{2x}}$$$$=\frac{2-2x}{e^{x}}$$ |
| $$y=\frac{x^{2}+1}{x^{2}-1}$$$$\frac{dy}{dx}=\frac{2x\left(x^{2}-1\right)-2x\left(x^{2}+1\right)}{\left(x^{2}-1\right)^{2}}$$$$=\frac{-4x}{\left(x^{2}-1\right)^{2}}$$ | $$y=tanx=\frac{sinx}{cosx}$$$$\frac{dy}{dx}=\frac{cos^{2}x+sin^{2}x}{cos^{2}x}$$$$=1+sin^{2}x$$$$=sec^{2}x$$ | $$y=\frac{e^{x}}{e^{-x}}=e^{x}×e^{x}=e^{2x}$$$$\frac{dy}{dx}=2e^{2x}$$ |
| $$y=\frac{x^{2}}{\sqrt{x}}$$$$\frac{dy}{dx}=\frac{2x\sqrt{x}-\frac{x^{2}}{2\sqrt{x}}}{x}$$$$=2\sqrt{x}-\frac{\sqrt{x}}{2}$$ | $$y=\frac{x^{2}}{tanx}$$$$\frac{dy}{dx}=\frac{2xtanx-x^{2}secx}{tan^{2}x}$$$$=\frac{2x}{tanx}-\frac{x^{2}}{tan^{2}xcosx}$$$$=\frac{x}{tanx}\left(2-\frac{x}{tanxcosx}\right)$$ | $$y=\frac{e^{-4x}}{4e^{4x}}=\frac{1}{4}\left(e^{-4x}×e^{-4x}\right)$$$$=\frac{e^{-8x}}{4}$$$$\frac{dy}{dx}=\frac{-2}{e^{8x}}$$ |
| $$y=\sqrt[3]{\frac{x+1}{x-1}}=\frac{\left(x+1\right)^{\frac{1}{3}}}{\left(x-1\right)^{\frac{1}{3}}}$$$$\frac{dy}{dx}=\frac{\frac{1}{3}\left(x-1\right)^{\frac{1}{3}}\left(x+1\right)^{\frac{-2}{3}}-\frac{1}{3}\left(x-1\right)^{-\frac{2}{3}}\left(x+1\right)^{\frac{1}{3}}}{\left(x-1\right)^{\frac{2}{3}}}$$$$=\frac{1}{3}\left[\frac{\left(x-1\right)^{-\frac{1}{3}}}{\left(x+1\right)^{\frac{2}{3}}}-\frac{\left(x+1\right)^{\frac{1}{3}}}{\left(x-1\right)^{\frac{4}{3}}}\right]$$$$=\frac{1}{3}\left[\frac{\left(x-1\right)-\left(x+1\right)}{\left(x+1\right)^{\frac{2}{3}}\left(x-1\right)^{\frac{4}{3}}}\right]$$$$=\frac{-2}{3}\left[\frac{1}{\sqrt[3]{\left(x+1\right)^{2}\left(x-1\right)^{4}}}\right]$$ | $$y=\frac{cotx}{2secx}=\frac{cosx}{sinx}÷\frac{2}{cosx}$$$$=\frac{cosx}{sinx}×\frac{cosx}{2}$$$$=\frac{cos^{2}x}{2sinx}$$$$\frac{dy}{dx}=\frac{-4sin^{2}xcosx-2cos^{3}x}{4sin^{2}x}$$$$=-xcosx-\frac{cosx}{2tan^{2}x}$$ | $$y=\frac{ln\left(x^{2}\right)}{e^{x^{2}}}$$$$\frac{dy}{dx}=\frac{\frac{2e^{x^{2}}}{x}-2xe^{x^{2}}lnx^{2}}{e^{2x^{2}}}$$$$=\frac{\frac{2}{x}-2xlnx^{2}}{e^{x^{2}}}$$ |

**Chain, Product, Quotient Rule Recap**

Differentiate these.

**Polynomials**

|  |  |  |
| --- | --- | --- |
| $$y=\left(3x+4\right)^{3}$$ | $$y=\left(3x+4\right)\left(2x-3\right)$$ | $$y=\frac{x+1}{2x+1}$$ |
| $$y=\left(3x^{3}+1\right)^{3}$$ | $$y=\left(3x^{2}+4\right)\left(2x^{4}+3\right)$$ | $$y=\frac{x^{2}+1}{x^{2}-1}$$ |
| $$y=\frac{4}{4-x^{4}}$$ | $$y=\sqrt{3x}\sqrt{2x}$$ | $$y=\frac{x^{2}}{\sqrt{x}}$$ |
| $$y=\sqrt{\sqrt{x}+1}$$ | $$y=\left(\frac{x}{3}\right)^{3}\sqrt[3]{x}$$ | $$y=\sqrt[3]{\frac{x+1}{x-1}}$$ |

**Trigonometry**

|  |  |  |
| --- | --- | --- |
| $$y=sin\left(2x\right)$$ | $$y=x^{2}sinx$$ | $$y=\frac{3x^{2}}{sinx}$$ |
| $$y=sin\left(x^{2}\right)$$ | $$y=sinxcosx$$ | $$y=tanx$$ |
| $$y=\frac{1}{\left(\frac{1}{cosecx}\right)}$$ | $$y=sinxcosxtanx$$ | $$y=\frac{x^{2}}{tanx}$$ |
| $$y=sin\left(cos\left(tanx\right)\right)$$ | $$y=sin\left(x^{2}\right)cos\left(x^{2}\right)$$ | $$y=\frac{cotx}{2secx}$$ |

**Exponentials**

|  |  |  |
| --- | --- | --- |
| $$y=e^{2x}$$ | $$y=xe^{x}$$ | $$y=\frac{2x}{e^{x}}$$ |
| $$y=e^{\left(x^{3}\right)}$$ | $$y=x^{3}e^{x}$$ | $$y=\frac{e^{x}}{e^{-x}}$$ |
| $$y=ln\left(\frac{1}{e^{x}}\right)$$ | $$y=xlnx$$ | $$y=\frac{e^{-4x}}{4e^{4x}}$$ |
| $$y=e^{\left(e^{x}\right)}$$ | $$y=e^{x^{2}}ln\left(x^{2}\right)$$ | $$y=\frac{ln\left(x^{2}\right)}{e^{x^{2}}}$$ |

**Build It Up Practice**

|  |  |  |
| --- | --- | --- |
| $$y=\left(2x+3\right)^{7}$$ | $$y=2x\left(4x+5\right)$$ | $$y=\frac{2x}{4x+5}$$ |
| $$y=\left(2x^{2}+3\right)^{7}$$ | $$y=2x^{2}\left(4x+5\right)$$ | $$y=\frac{2x^{2}}{4x+5}$$ |
| $$y=\left(2x^{2}+3x\right)^{7}$$ | $$y=\left(2x+3\right)\left(4x+5\right)$$ | $$y=\frac{2x+3}{4x+5}$$ |
| $$y=\left(2x^{3}+3x^{2}\right)^{7}$$ | $$y=\left(2x^{2}+3x\right)\left(4x+5\right)$$ | $$y=\frac{2x^{2}+3x}{4x+5}$$ |
| $$y=\frac{1}{2x+3}$$ | $$y=\frac{1}{x}\left(4x+5\right)$$ | $$y=\frac{1}{4x+5}$$ |
| $$y=\frac{1}{2x^{2}+3x}$$ | $$y=\sqrt{x}\left(4x+5\right)$$ | $$y=\frac{\sqrt{x}}{\sqrt{x+1}}$$ |
| $$y=sin\left(2x\right)$$ | $$y=\sin(x)\cos(x)$$ | $$y=\frac{1}{\sqrt{x}\sqrt{x+1}}$$ |
| $$y=sin\left(2x^{2}+3\right)$$ | $$y=\sin(2x)\cos(2x)$$ | $$y=\frac{\sin(x)}{\cos(x)}$$ |
| $$y=\cos(\left(2x^{2}+3\right))$$ | $$y=\sin(\left(2x+3\right))\cos(\left(4x+5\right))$$ | $$y=\frac{\sin(2x)}{\cos(2x)}$$ |
| $$y=sin\left(\frac{1}{2x+3}\right)$$ | $$y=\left(2x+3\right)^{7}\left(4x+5\right)$$ | $$y=\left(\frac{2x+3}{4x+5}\right)^{7}$$ |

**Build It Up Practice - Answers**

|  |  |  |
| --- | --- | --- |
| $y=\left(2x+3\right)^{7}$, $\frac{dy}{dx}=$ | $y=2x\left(4x+5\right)$, $\frac{dy}{dx}=$ | $y=\frac{2x}{4x+5}$, $\frac{dy}{dx}=$ |
| $y=\left(2x^{2}+3\right)^{7}$, $\frac{dy}{dx}=$ | $y=2x^{2}\left(4x+5\right)$, $\frac{dy}{dx}=$ | $y=\frac{2x^{2}}{4x+5}$, $\frac{dy}{dx}=$ |
| $y=\left(2x^{2}+3x\right)^{7}$, $\frac{dy}{dx}=$ | $y=\left(2x+3\right)\left(4x+5\right)$, $\frac{dy}{dx}=$ | $y=\frac{2x+3}{4x+5}$, $\frac{dy}{dx}=$ |
| $y=\left(2x^{3}+3x^{2}\right)^{7}$, $\frac{dy}{dx}=$ | $y=\left(2x^{2}+3x\right)\left(4x+5\right)$, $\frac{dy}{dx}=$ | $y=\frac{2x^{2}+3x}{4x+5}$, $\frac{dy}{dx}=$ |
| $y=\frac{1}{2x+3}$, $\frac{dy}{dx}=$ | $y=\frac{1}{x}\left(4x+5\right)$, $\frac{dy}{dx}=$ | $y=\frac{1}{4x+5}$, $\frac{dy}{dx}=$ |
| $y=\frac{1}{2x^{2}+3x}$, $\frac{dy}{dx}=$ | $y=\sqrt{x}\left(4x+5\right)$, $\frac{dy}{dx}=$ | $y=\frac{\sqrt{x}}{\sqrt{x+1}}$, $\frac{dy}{dx}=$ |
| $y=sin\left(2x\right)$, $\frac{dy}{dx}=$ | $y=\sin(x)\cos(x)$, $\frac{dy}{dx}=$ | $y=\frac{1}{\sqrt{x}\sqrt{x+1}}$, $\frac{dy}{dx}=$ |
| $y=sin\left(2x^{2}+3\right)$, $\frac{dy}{dx}=$ | $y=\sin(2x)\cos(2x)$, $\frac{dy}{dx}=$ | $y=\frac{\sin(x)}{\cos(x)}$, $\frac{dy}{dx}=$ |
| $y=\cos(\left(2x^{2}+3\right))$, $\frac{dy}{dx}=$ | $y=\sin(\left(2x+3\right))\cos(\left(4x+5\right))$, $\frac{dy}{dx}=$ | $y=\frac{\sin(2x)}{\cos(2x)}$, $\frac{dy}{dx}=$ |
| $y=sin\left(\frac{1}{2x+3}\right)$, $\frac{dy}{dx}=$ | $y=\left(2x+3\right)^{7}\left(4x+5\right)$, $\frac{dy}{dx}=$ | $y=\left(\frac{2x+3}{4x+5}\right)^{7}$, $\frac{dy}{dx}=$ |