**Chi-Squared Concept Questions**

1. The results of flipping a coin 20 times are shown below. Use a chi-squared test to determine if this is evidence that the coin is biased at the 5% level and at the 1% level.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| H | T | H | T | T |
| T | T | T | H | T |
| T | H | T | T | T |
| H | T | T | T | T |

1. The results of flipping another coin 100 times are below. Does this provide evidence at either the 5% level or the 1% level that the coin is biased?

|  |  |
| --- | --- |
| H | T |
| 60 | 40 |

1. A student claims to have recorded the following results of rolling a non-biased tetrahedral die 200 times. Test at the 5% and 1% levels to determine if the students claim is true or if he has simply ‘created’ his results.

|  |  |  |  |
| --- | --- | --- | --- |
| **Blue** | **Green** | **Red** | **Yellow** |
| 80 | 40 | 45 | 35 |

1. When flipping a coin 100 times, between what numbers of heads and tails would a chi-squared test lead to an acceptance of a null hypothesis, that the coin is **not** **biased** at the 5% level and at the 1% level? What about 1000 flips?
2. Is it possible to say between what probabilities would a coin that is biased be deemed to be not biased when chi-testing at the 5% and 1% levels? If so, what are these probability limits?
3. What conditions, or rules, would you advise the student conducting the tetrahedral dice experiment to observe in order to successfully falsify his results?
4. At what percentage level do the totals of the 100 coin flips in question 2, equal the critical value (the value determining the boundary of the critical region) at 5% and 1%?
5. Alice & Bob are discussing rolling a six sided die 60 times. Alice says that the expectation is for the die to land on each side 10 times. Bob disagrees because she thinks this is unlikely. Between what frequencies should Bob expect the die to land on each side? Is either, both or neither, Alice or/nor Bob correct? Should they both just return to their secret messages?
6. Create some similar questions, considering in particular, what issues/questions need to be considered when creating questions.

**Chi-Squared Concept Questions - Answers**

1. The results of flipping a coin 20 times are shown below. Use a chi-squared test to determine if this is evidence that the coin is biased at the 5% level and at the 1% level.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| H | T | H | T | T |
| T | T | T | H | T |
| T | H | T | T | T |
| H | T | T | T | T |

Totals are 15 tails and 5 heads which gives a $χ^{2}$ value of 5. This is greater than the $χ\_{1}^{2}$ 95% value of 3.841 but less than the 99% value of 6.635, and therefore provides evidence to reject null hypothesis (that coin is fair) at the 5% level but accept it at the 1% level.

1. The results of flipping another coin 100 times are below. Does this provide evidence at either the 5% level or the 1% level that the coin is biased?

|  |  |
| --- | --- |
| H | T |
| 60 | 40 |

This gives a $χ^{2}$ value of 4 which is, again, greater than the $χ\_{1}^{2}$ 95% value of 3.841 but less than the 99% value of 6.635, and therefore provides evidence to reject null hypothesis (that coin is fair) at the 5% level but accept it at the 1% level. Note that these proportions of heads:tails differ to previous question yet the $χ^{2}$ values are similar and lead to same accept/reject decisions.

1. A student claims to have recorded the following results of rolling a non-biased tetrahedral die 200 times. Test at the 5% and 1% levels to determine if the students claim is true or if he has simply ‘created’ his results.

|  |  |  |  |
| --- | --- | --- | --- |
| **Blue** | **Green** | **Red** | **Yellow** |
| 70 | 38 | 45 | 47 |

This gives a $χ^{2}$ value of 11.56. This is greater than both the $χ\_{3}^{2}$ 95% value of 7.815 and the 99% value of 11.345, and therefore provides evidence to reject null hypothesis (that the results are from the non-biased die) at both the 5% and 1% levels. The teacher can be very sure that the student is naughty.

1. When flipping a coin 100 times, between what numbers of heads and tails would a chi-squared test lead to an acceptance of a null hypothesis, that the coin is **not** **biased** at the 5% level and at the 1% level? What about 1000 flips?

For 100 flips… The critical $χ\_{1}^{2}$ 95% value of 3.841 must not be exceeded in order to accept the null hypothesis that the coin is not biased. The nearest $χ^{2}$ value below this for a coin is 3.24 which is obtained from throwing 59 of side A and 41 of side B. Similarly, the critical $χ\_{1}^{2}$ 99% value is 6.635 and the max value below this is obtained from throwing in proportions 62:38.

For 1000 flips…. At 5% significance, maximum ratio is 470:530, at 1% significance max ratio is 460:540.

1. Is it possible to say between what probabilities would a coin that is biased be deemed to be not biased when chi-testing at the 5% and 1% levels? If so, what are these probability limits?

No, from answers to questions 1 & 2, we can see that as the number of coin flips changes, the values and proportions of acceptable observed outcomes also changes.

E.g. $n=20$ with results of 15:5 leads to acceptance at 1% level but rejection at 5% level whereas $n=100$ with results of 75:25 (i.e. same ratio/probability as before) leads to rejection at both levels.

1. What conditions, or rules, would you advise the student conducting the tetrahedral dice experiment to observe in order to successfully falsify his results?

(Various answers possible here)

1. At what percentage level do the totals of the 100 coin flips in question 2, equal the critical value (the value determining the boundary of the critical region) at 5% and 1%?

59.8/40.2 gives the $χ^{2}$ value of 3.8416 which matches the 5% value,

62.88/37.12 gives the $χ^{2}$ value of 3.8416 which matches the 1% value.

1. Alice & Bob are discussing rolling a six sided die 60 times. Alice says that the expectation is for the die to land on each side 10 times. Bob disagrees because she thinks this is unlikely. Between what frequencies should Bob expect the die to land on each side? Is either, both or neither, Alice or/nor Bob correct? Should they both just return to their secret messages?

(As per question 6, various answers possible here)

1. Create some similar questions, considering in particular, what issues/questions need to be considered when creating questions.

<http://lifehacker.com/5929611/why-itunes-shuffle-isnt-random-and-how-to-fix-it>

<http://ipod.about.com/od/advanceditunesuse/a/itunes-random.htm>

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<https://prezi.com/u5lsf7h3gku7/stat-project-itunes-shuffling/>

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