Interesting Questions

Qu 1… Edexcel unit tests, Parametric Equations -Qu 3. (Link to markscheme)

The curve C has parametric equations $x = 7\sin t - 4$, $y = 7\cos t + 3$, $-\frac{\pi}{2} \leqslant t \leqslant \frac{\pi}{3}$

a Show that the cartesian equation of C can be written as $(x+a)^2 + (y+b)^2 = c$, where a, b and c are integers which should be stated. **(3 marks)**

b Sketch the curve C on the given domain, clearly stating the endpoints of the curve. **(3 marks)**

c Find the length of C. Leave your answer in terms of π. **(2 marks)**

Qu 2… AQA A2 Paper 1, June 2018 -Qu 5. (Link to markscheme)

A curve is defined by the parametric equations

$$x = 4 \times 2^{-t} + 3$$

$$y = 3 \times 2^{t} - 5$$

Show that $\frac{dy}{dx} = -\frac{3}{4} \times 2^{2t}$

[3 marks]

Find the Cartesian equation of the curve in the form $xy + ax + by = c$, where a, b and c are integers.

[3 marks]

Qu 3… AQA A2 Paper 1, June 2018 – Qu 12. (Link to markscheme)

$p(x) = 30x^3 - 7x^2 - 7x + 2$

Prove that $(2x + 1)$ is a factor of $p(x)$

[2 marks]

Factorise $p(x)$ completely.

[3 marks]

Prove that there are no real solutions to the equation

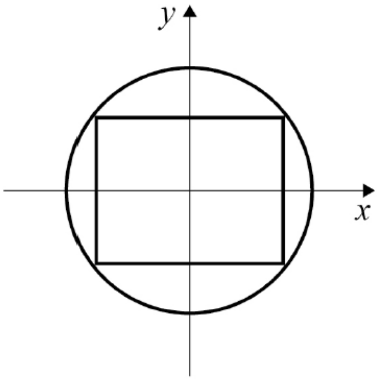
$$\frac{30\sec^2 x + 2\cos x}{7} = \sec x + 1$$

[5 marks]

Qu 4… AQA A2 Paper 1, June 2018 – Qu13. (Link to markscheme)

A company is designing a logo. The logo is a circle of radius 4 inches with an inscribed rectangle. The rectangle must be as large as possible.

The company models the logo on an x-y plane as shown in the diagram.



Use calculus to find the maximum area of the rectangle.

Fully justify your answer.

[10 marks]

Qu 5… AQA A2 Paper 2, June 2018 -Qu 8. (Link to markscheme)

Determine a sequence of transformations which maps the graph of $y = \sin x$ onto the graph of $y = \sqrt{3}\sin x - 3\cos x + 4$

Fully justify your answer.

[7 marks]

Show that the least value of $\dfrac{1}{\sqrt{3}\sin x - 3\cos x + 4}$ is $\dfrac{2-\sqrt{3}}{2}$

[2 marks]

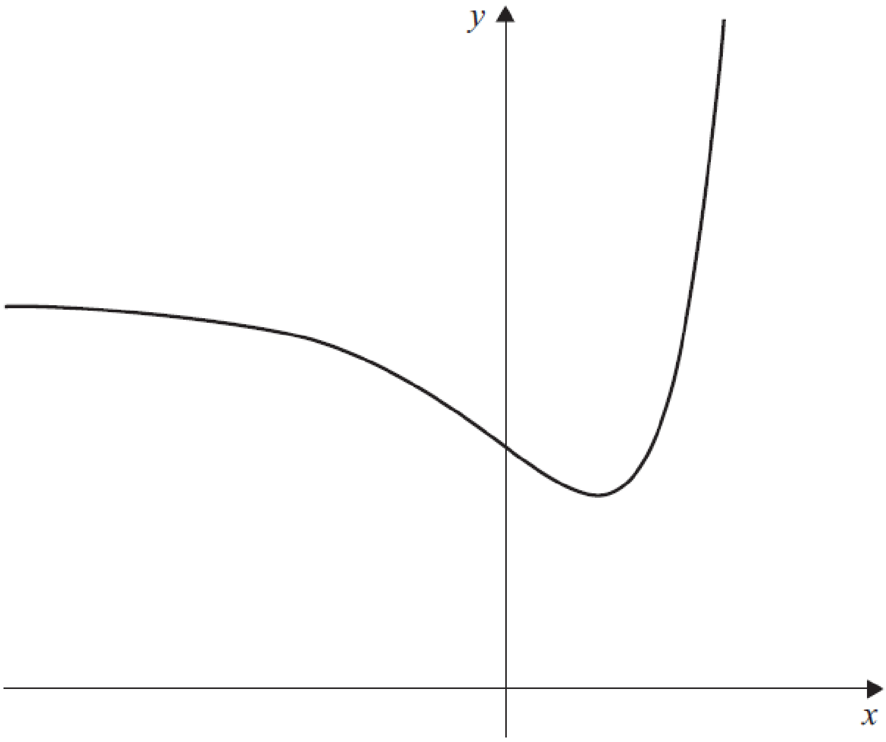
Find the greatest value of $\dfrac{1}{\sqrt{3}\sin x - 3\cos x + 4}$

[1 mark]

Qu 6… AQA A2 Paper 2, June 2018 - Qu 7. (Link to markscheme)

A function f has domain $\mathbb{R}$ and range $\{y \in \mathbb{R} : y \geq \mathrm{e}\}$

The graph of $y = \mathrm{f}(x)$ is shown.



The gradient of the curve at the point (x, y) is given by $\dfrac{\mathrm{d}y}{\mathrm{d}x} = (x - 1)\mathrm{e}^x$

Find an expression for $\mathrm{f}(x)$.

Fully justify your answer.

[8 marks]

Qu 7… Edexcel unit tests, parametric Equations -Qu 6. (Link to markscheme)

A large arch is planned for a football stadium. The parametric equations of the arch are $x = 8(t + 10)$, $y = 100 - t^2$, $-10 \leqslant t \leqslant 10$ where x and y are distances in metres.

a Find the cartesian equation of the arch. **(3 marks)**

b Find the width of the arch. **(2 marks)**

c Find the greatest possible height of the arch. **(2 marks)**

Qu 8… Edexcel Paper 1, June 2018 - Qu7. (Link to markscheme)

Given that $k \in \mathbb{Z}^+$

(a) show that $\int_k^{3k} \frac{2}{(3x-k)}\,\mathrm{d}x$ is independent of k,

(4)

(b) show that $\int_k^{2k} \frac{2}{(2x-k)^2}\,\mathrm{d}x$ is inversely proportional to k.

(3)

Qu 9… AQA Paper 3, June 2018 – Qu 6. (Link to markscheme)

A function f is defined by $\mathrm{f}(x) = \frac{x}{\sqrt{2x-2}}$

State the maximum possible domain of f.

[2 marks]

Qu 10… AQA Paper 3, June 2018 – Qu 8. (Link to markscheme)

Prove the identity $\frac{\sin 2x}{1+\tan^2 x} \equiv 2\sin x \cos^3 x$

[3 marks]

Hence find $\int \frac{4\sin 4\theta}{1+\tan^2 2\theta}\,\mathrm{d}\theta$

[6 marks]

Qu 11… OCR A, Paper 2, June 2018. (Link to markscheme)

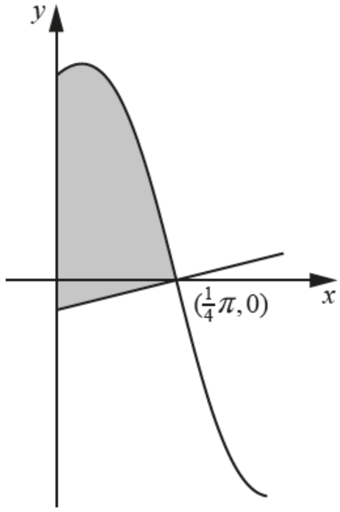
The variable Y has the distribution $\mathrm{N}(\mu, \frac{\mu^2}{9})$. Find $\mathrm{P}(Y > 1.5\mu)$. **[3]**

Qu 12… OCR A, Paper 2, June 2018. (Link to markscheme)

In the expansion of $(0.15 + 0.85)^{50}$, the terms involving 0.15^r and 0.15^{r+1} are denoted by T_r and T_{r+1} respectively.

Show that $\frac{T_r}{T_{r+1}} = \frac{17(r+1)}{3(50-r)}$. **[3]**

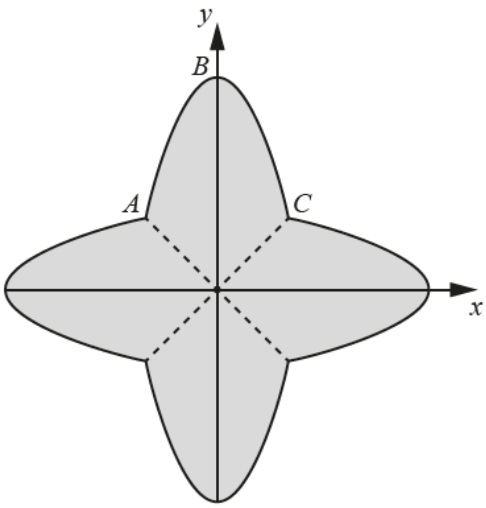
Qu 13… OCR, Paper 1, June 2018. (Link to markscheme)



The diagram shows the curve $y = \frac{4\cos 2x}{3 - \sin 2x}$, for $x \geqslant 0$, and the normal to the curve at the point $(\frac{1}{4}\pi, 0)$. Show that the exact area of the shaded region enclosed by the curve, the normal to the curve and the y-axis is $\ln\frac{9}{4} + \frac{1}{128}\pi^2$. **[10]**

Qu 14… OCR, Paper 2, June 2018. (Link to markscheme)

The diagram shows a part ABC of the curve $y = 3 - 2x^2$, together with its reflections in the lines $y = x$, $y = -x$ and $y = 0$.



Find the area of the shaded region. **[7]**

Qu 15… OCR Practice Papers, Set 2, Paper 3 - Qu3. (Link to markscheme)

A sequence of three transformations maps the curve $y = \ln x$ to the curve $y = e^{3x} - 5$. Give details of these transformations. **[4]**

What about $y = e^{3x-5}$?

Qu 16… OCR Practice Papers, Set 4, Paper 1. (Link to markscheme)

Solve the simultaneous equations

$$e^x - 2e^y = 3$$
$$e^{2x} - 4e^{2y} = 33.$$

Give your answer in an exact form. **[5]**

Qu 17… AQA Core 3, June 2013. (Link to markscheme)

Find $\int (\ln x)^2 \, dx$. *(4 marks)*

Use the substitution $u = \sqrt{x}$ to find the exact value of

$$\int_1^4 \frac{1}{x + \sqrt{x}} \, dx$$ *(7 marks)*

Qu 18… MEI, Paper 1, June 2018 – Qu 10. (Link to markscheme)

Fig. 10 shows the graph of $y = (k - x)\ln x$ where k is a constant ($k > 1$).

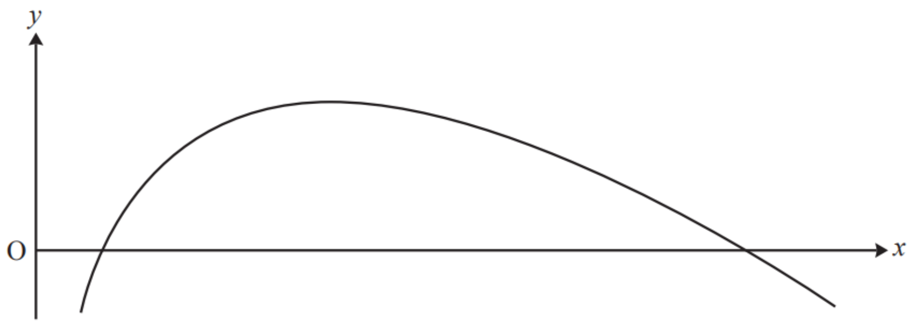


Fig. 10

Find, in terms of k, the area of the finite region between the curve and the x-axis. **[8]**

Qu 19… MEI, Paper 1, June 2018 -Qu 11. (Link to markscheme)

Fig. 11 shows two blocks at rest, connected by a light inextensible string which passes over a smooth pulley. Block A of mass 4.7 kg rests on a smooth plane inclined at 60° to the horizontal. Block B of mass 4 kg rests on a rough plane inclined at 25° to the horizontal. On either side of the pulley, the string is parallel to a line of greatest slope of the plane. Block B is on the point of sliding up the plane.

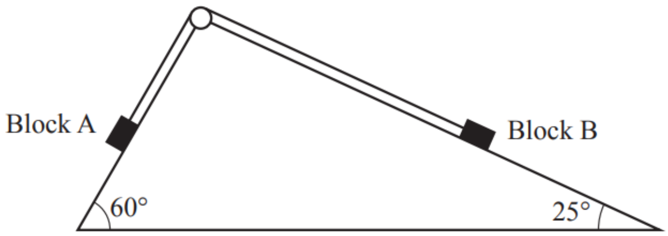


Fig. 11

(i) Show that the tension in the string is 39.9 N correct to 3 significant figures. **[2]**

(ii) Find the coefficient of friction between the rough plane and Block B. **[5]**

Qu 20… MEI, Paper 3, June 2018 – Qu 10. (Link to markscheme)

Point A has position vector $\begin{pmatrix} a \\ b \\ 0 \end{pmatrix}$ where a and b can vary, point B has position vector $\begin{pmatrix} 4 \\ 2 \\ 0 \end{pmatrix}$ and point C has position vector $\begin{pmatrix} 2 \\ 4 \\ 2 \end{pmatrix}$. ABC is an isosceles triangle with AC = AB.

(i) Show that $a - b + 1 = 0$. **[4]**

(ii) Determine the position vector of A such that triangle ABC has minimum area. **[6]**

Qu 21… Edexcel Mock Papers, Paper 1 – Qu 11. (Link to markscheme)

Given that

$$x = 2\tan y \qquad -\frac{\pi}{2} < y < \frac{\pi}{2}$$

show that

$$\frac{dy}{dx} = \frac{k}{4 + x^2}$$

where k is a constant to be found.

(4)

Qu 22… Edexcel Mock Papers, Paper 1 - Qu 10. (Link to markscheme)

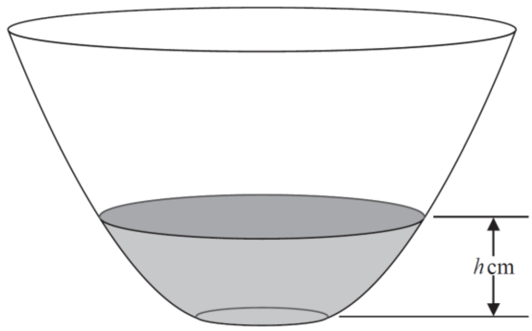


Figure 4

Figure 4 shows a bowl with a circular cross-section.

Initially the bowl is empty. Water begins to flow into the bowl.

At time t seconds after the water begins to flow into the bowl, the height of the water in the bowl is h cm.

The volume of water, V cm^3, in the bowl is modelled as

$$V = 4\pi h(h + 6) \qquad 0 \leqslant h \leqslant 25$$

The water flows into the bowl at a constant rate of 80π cm^3 s^{-1}

(a) Show that, according to the model, it takes 36 seconds to fill the bowl with water from empty to a height of 24 cm.

(1)

(b) Find, according to the model, the rate of change of the height of the water, in cm s^{-1}, when $t = 8$

(8)

Qu 23… Edexcel, A2 Paper 1, June 2019 – Qu 9. (Link to markscheme)

Given that $a > b > 0$ and that a and b satisfy the equation

$$\log a - \log b = \log(a - b)$$

(a) show that

$$a = \frac{b^2}{b-1}$$

(3)

(b) Write down the full restriction on the value of b, explaining the reason for this restriction.

(2)

Qu 24… Edexcel Mock Papers, Paper 1 – Qu 13. (Link to markscheme)

Given that p is a positive constant,

(a) show that

$$\sum_{n=1}^{11} \ln(p^n) = k \ln p$$

where k is a constant to be found,

(2)

(b) show that

$$\sum_{n=1}^{11} \ln(8p^n) = 33\ln(2p^2)$$

(2)

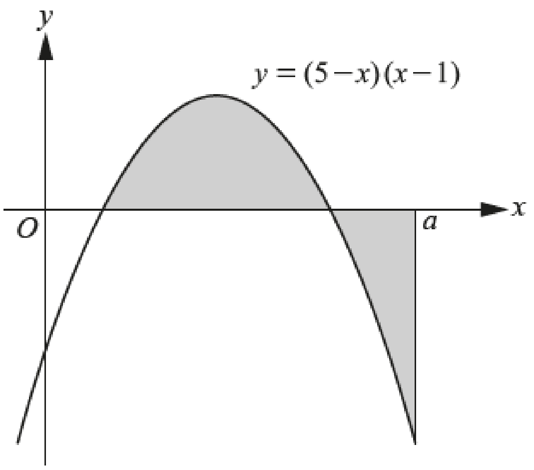
(c) Hence find the set of values of p for which

$$\sum_{n=1}^{11} \ln(8p^n) < 0$$

giving your answer in set notation.

(2)

Qu 25… OCR AS Paper 2, 2019. (Link to markscheme)



The diagram shows part of the curve $y = (5 - x)(x - 1)$ and the line $x = a$.

Given that the total area of the regions shaded in the diagram is 19 units^2, determine the exact value of a.

[8]

Qu 26… Edexcel Mock Papers, Paper 1 - Qu 14. (Link to markscheme)

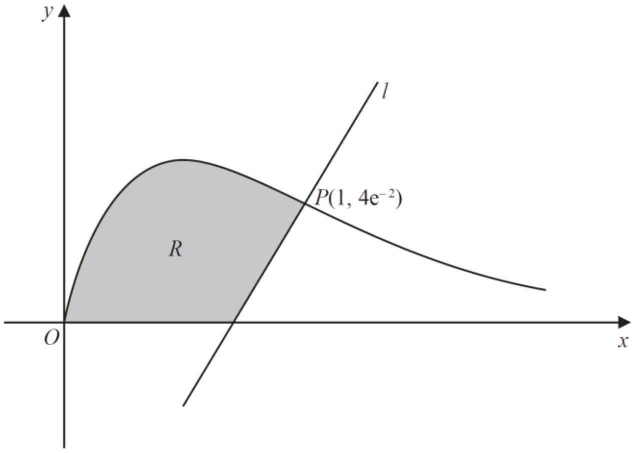


Figure 7

Figure 7 shows a sketch of the curve with equation

$$y = 4xe^{-2x} \qquad x \geqslant 0$$

The line l is the normal to the curve at the point $P(1, 4e^{-2})$

The finite region R, shown shaded in Figure 7, is bounded by the curve, the line l, and the x-axis.

Find the exact value of the area of R.

(Solutions based entirely on graphical or numerical methods are not acceptable.)

(10)

Qu 27… Edexcel A2 Paper 2, 2018 – Qu 4. (Link to markscheme)

(i) Show that $\sum_{r=1}^{16}(3 + 5r + 2^r) = 131\,798$

(4)

(ii) A sequence $u_1, u_2, u_3, \ldots$ is defined by

$$u_{n+1} = \frac{1}{u_n}, \qquad u_1 = \frac{2}{3}$$

Find the exact value of $\sum_{r=1}^{100} u_r$

(3)

Qu 28… AQA, A2 Paper 2, 2019. (Link to markscheme)

Solve the differential equation

$$\frac{\mathrm{d}t}{\mathrm{d}x} = \frac{\ln x}{x^2 t} \qquad \text{for } x > 0$$

given $x = 1$ when $t = 2$

Write your answer in the form $t^2 = \mathrm{f}(x)$

[7 marks]

Qu 29… Edexcel, A2 Paper 1, June 2019 – Qu 5. (Link to markscheme)

$$\mathrm{f}(x) = 2x^2 + 4x + 9 \qquad x \in \mathbb{R}$$

(i) Describe fully the transformation that maps the curve with equation $y = \mathrm{f}(x)$ onto the curve with equation $y = \mathrm{g}(x)$ where

$$\mathrm{g}(x) = 2(x - 2)^2 + 4x - 3 \qquad x \in \mathbb{R}$$

(ii) Find the range of the function

$$\mathrm{h}(x) = \frac{21}{2x^2 + 4x + 9} \qquad x \in \mathbb{R}$$

(4)

Qu 30… Edexcel unit tests, Integration – Qu 8. (Link to markscheme)

Use the substitution $x = 4sin^2\theta$ to find

$$\int_0^3 \sqrt{\left(\frac{x}{4-x}\right)}\,\mathrm{d}x\,,$$

giving your answer in the form $a\pi + b$, where a and b are exact constants.

(9)

Qu 31… Edexcel, A2 Paper 1, June 2019 – Qu 4. (Link to markscheme)

The binomial expansion of

$$\frac{1}{\sqrt{4-x}}$$

Can be used to find an approximation to $\sqrt{2}$.

Possible values of x that could be substituted into this expansion are:

- $x = -14$ because $\dfrac{1}{\sqrt{4-x}} = \dfrac{1}{\sqrt{18}} = \dfrac{\sqrt{2}}{6}$
- $x = 2$ because $\dfrac{1}{\sqrt{4-x}} = \dfrac{1}{\sqrt{2}} = \dfrac{\sqrt{2}}{2}$
- $x = -\dfrac{1}{2}$ because $\dfrac{1}{\sqrt{4-x}} = \dfrac{1}{\sqrt{\dfrac{9}{2}}} = \dfrac{\sqrt{2}}{3}$

Without evaluating your expansion,

(i) state, giving a reason, which of the three values of x should not be used

(1)

(ii) state, giving a reason, which of the three values of x would lead to the most accurate approximation to $\sqrt{2}$

(1)

Qu 32… Edexcel, A2 Paper 1, June 2019 – Qu 14. (Link to markscheme)

The curve C, in the standard Cartesian plane, is defined by the equation

$$x = 4\sin 2y \qquad \frac{-\pi}{4} < y < \frac{\pi}{4}$$

The curve C passes through the origin O

(a) Find the value of $\dfrac{dy}{dx}$ at the origin.

(2)

(b) Show that, for all points (x, y) lying on C,

$$\frac{dy}{dx} = \frac{1}{a\sqrt{b - x^2}}$$

where a and b are constants to be found.

(3)

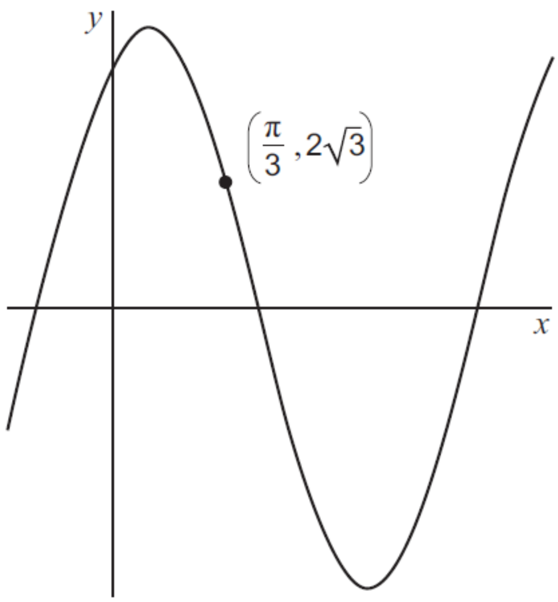
Qu 33… AQA, A2 Paper 2, 2019. ([Link to markscheme](#))

A curve has equation

$$y = a\sin x + b\cos x$$

where a and b are constants.

The maximum value of y is 4 and the curve passes through the point $\left(\frac{\pi}{3}, 2\sqrt{3}\right)$ as shown in the diagram.



Find the exact values of a and b.

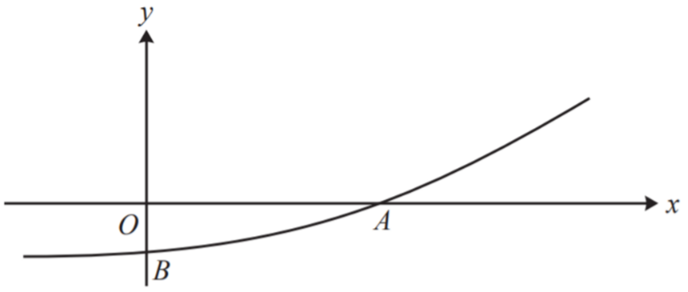
[6 marks]

Qu 34… Edexcel Unit Tests, A2 Stats, Topic 2, Hypothesis Testing. ([Link to markscheme](#))

A random sample of size n is to be taken from a population that is normally distributed with mean 40 and standard deviation 3. Find the minimum sample size such that the probability of the sample mean being greater than 42 is less than 5%.

(Total 5 marks)

Qu 35… OCR A Core 3 June 2013, Differentiation. (Link to markscheme)



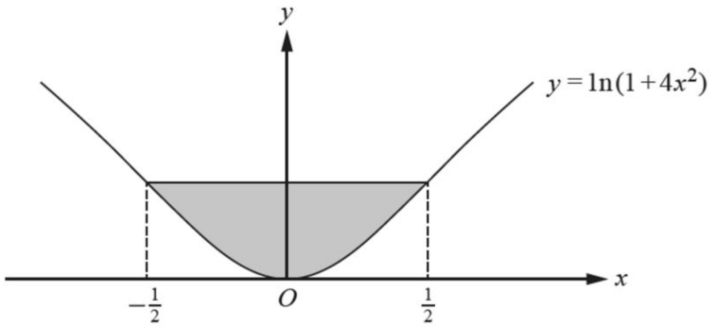
The diagram shows the curve with equation

$$x = (y+4)\ln(2y+3).$$

Find the gradient of the curve at each of the points A and B, giving each answer correct to 2 decimal places.

[8]

Qu 36… OCR A Practice Papers Set 1, Paper 3, Question 6. (Link to markscheme)



The diagram shows the curve $y = \ln(1+4x^2)$. The shaded region is bounded by the curve and a line parallel to the x-axis which meets the curve where $x = \frac{1}{2}$ and $x = -\frac{1}{2}$.

(i) Show that the area of the shaded region is given by

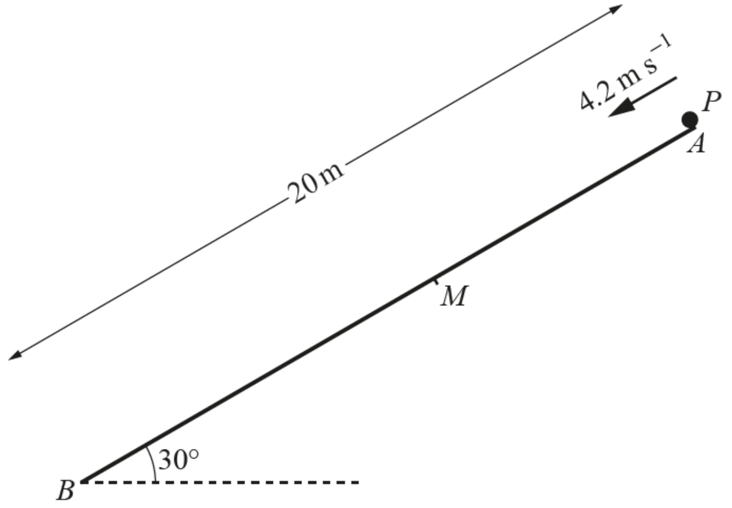
$$\int_0^{\ln 2} \sqrt{e^y - 1}\, dy.$$

[3]

(ii) Show that the substitution $e^y = \sec^2\theta$ transforms the integral in part **(ii)** to $\int_0^{\frac{1}{4}\pi} 2\tan^2\theta \, d\theta$. **[2]**

(iii) Hence find the exact area of the shaded region. **[3]**

Qu 37… OCR A Practice Papers Set 4, Paper 3, Question 10. (Link to markscheme)



A and B are points at the upper and lower ends, respectively, of a line of greatest slope on a plane inclined at 30° to the horizontal. The distance AB is 20 m. M is a point on the plane between A and B. The surface of the plane is smooth between A and M, and rough between M and B.

A particle P is projected with speed $4.2\,\text{m s}^{-1}$ from A down the line of greatest slope (see diagram). P moves down the plane and reaches B with speed $12.6\,\text{m s}^{-1}$. The coefficient of friction between P and the rough part of the plane is $\frac{\sqrt{3}}{6}$.

(a) Find the distance AM. **[8]**

Qu 38… OCR A Practice Papers Set 2, Paper 2, Question 6. (Link to markscheme)

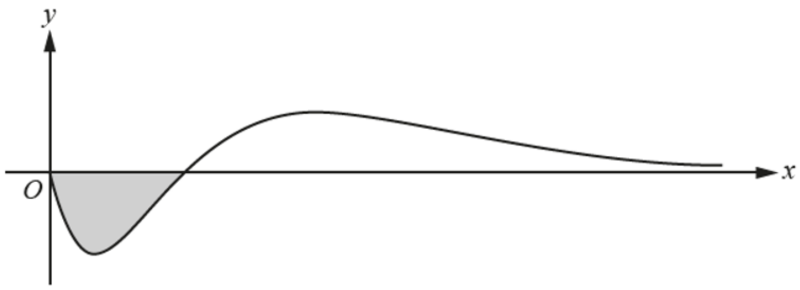
In this question you must show detailed reasoning.

(i) Use the formula for $\tan(A-B)$ to show that $\tan\frac{\pi}{12} = 2-\sqrt{3}$. **[4]**

(ii) Solve the equation $2\sqrt{3}\sin 3A - 2\cos 3A = 1$ for $0° \leqslant A < 180°$. **[7]**

Qu 39… OCR A Practice Papers Set 2, Paper 3, Question 5. (Link to markscheme)

In this question you must show detailed reasoning.



The function f is defined for the domain $x \geqslant 0$ by

$$f(x) = (2x^2 - 3x)e^{-x}.$$

The diagram shows the curve $y = f(x)$.

(i) Find the range of f. **[6]**

(ii) Find the exact area of the shaded region enclosed by the curve and the x-axis. **[7]**

Qu 40… OCR A Sample Assessment Paper, Maths & Statistics, Question 12. (Link to markscheme)

The table shows information for England and Wales, taken from the UK 2011 census.

Total population	Number of children aged 5-17
56 075 912	8 473 617

A random sample of 10 000 people in another country was chosen in 2011, and the number, m, of children aged 5-17 was noted.
It was found that there was evidence at the 2.5% level that the proportion of children aged 5-17 in the same year was higher than in the UK.
Unfortunately, when the results were recorded the value of m was omitted.

Use an appropriate normal distribution to find an estimate of the smallest possible value of m. **[5]**

Qu 41… OCR, A2 Paper 2, 2018, Question 5 (Link to markscheme)

Given that 853 is a prime number, find the square number S such that $S + 853$ is also a square number.

Qu 42… OCR Practice Papers, Set 1, Paper 1, Question 12. (Link to markscheme)

In this question you must show detailed reasoning.

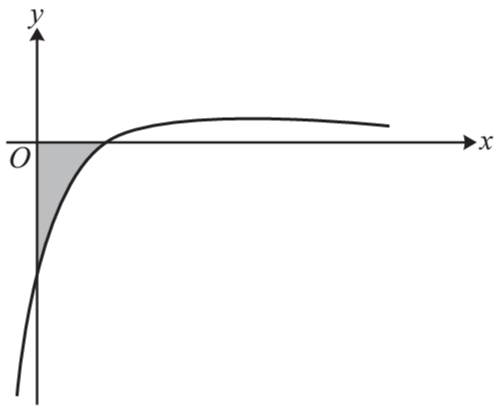
A curve has equation

$$x \sin y + \cos 2y = \frac{5}{2}$$

for $x \geqslant 0$ and $0 \leqslant y < 2\pi$.

Determine the exact coordinates of each point on the curve at which the tangent to the curve is parallel to the y-axis. **[9]**

Qu 43… OCR, A2 Paper 3, 2019, Question 6 (Link to markscheme)



The diagram shows part of the curve $y = \dfrac{2x-1}{(2x+3)(x+1)^2}$.

Find the exact area of the shaded region, giving your answer in the form $p + q\ln r$, where p and q are positive integers and r is a positive rational number. **[10]**

Qu 44… OCR Practice papers Set 2, Paper 2, Question 7 (Link to markscheme)

A tank is shaped as a cuboid. The base has dimensions 10 cm by 10 cm. Initially the tank is empty. Water flows into the tank at 25 cm^3 per second. Water also leaks out of the tank at $4h^2$ cm^3 per second, where h cm is the depth of the water after t seconds. Find the time taken for the water to reach a depth of 2 cm. **[9]**

Qu 45… OCR A2 Paper 1, 2019, Question 2 (link to markscheme)

The point A is such that the magnitude of $\overrightarrow{OA}$ is 8 and the direction of $\overrightarrow{OA}$ is 240°.

(a) **(i)** Show the point A on the axes provided in the Printed Answer Booklet. **[1]**

(ii) Find the position vector of point A.
Give your answer in terms of $\mathbf{i}$ and $\mathbf{j}$. **[3]**

The point B has position vector $6\mathbf{i}$.

(b) Find the exact area of triangle AOB. **[2]**

The point C is such that $OABC$ is a parallelogram.

(c) Find the position vector of C.
Give your answer in terms of $\mathbf{i}$ and $\mathbf{j}$. **[2]**

Qu 46… OCR A2 Paper 2, 2019, Question 9 (link to markscheme)

(a) The masses, in grams, of plums of a certain kind have the distribution N(55, 18).

(i) Find the probability that a plum chosen at random has a mass between 50.0 and 60.0 grams. **[1]**

(ii) The heaviest 5% of plums are classified as extra large.

Find the minimum mass of extra large plums. **[1]**

(iii) The plums are packed in bags, each containing 10 randomly selected plums.

Find the probability that a bag chosen at random has a total mass of less than 530 g. **[4]**

(b) The masses, in grams, of apples of a certain kind have the distribution $N(67, \sigma^2)$. It is given that half of the apples have masses between 62 g and 72 g.

Determine σ. **[5]**

Qu 47… AQA Level 2 Certificate in Further Maths, Paper 2, 2017, Question 24 (Link to markscheme)

Write $12x^2 - 60x + 5$ in the form $a(bx + c)^2 + d$ where a, b, c and d are integers.

[5 marks]

Qu 48… OCR A2 Paper 2, 2020, Question 3 (Link to markscheme)

In this question you should assume that $-1 < x < 1$.

(a) For the binomial expansion of $(1-x)^{-2}$

(i) find and simplify the first four terms, **[2]**

(ii) write down the term in x^n. **[1]**

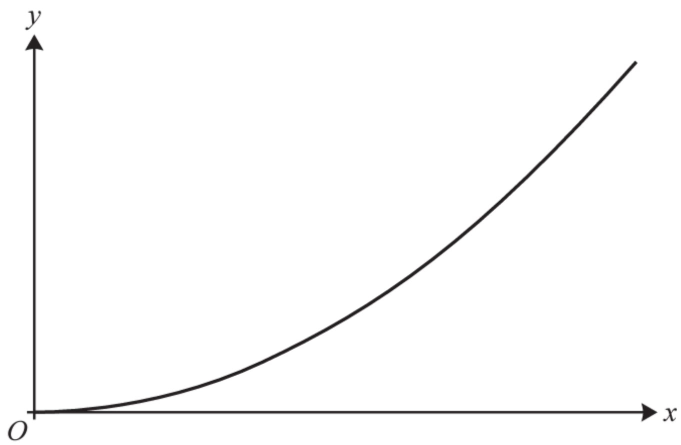
(b) Write down the sum to infinity of the series $1+x+x^2+x^3+\dots$. **[1]**

(c) Hence or otherwise find and simplify an expression for $2+3x+4x^2+5x^3+\dots$ in the form $\frac{a-x}{(b-x)^2}$ where a and b are constants to be determined. **[3]**

Qu 49… OCR A2 Paper 1, 2021, Question 11 (Link to markscheme)

(a) Use the substitution $u^2 = x^2+3$ to show that $\int \frac{4x^3}{\sqrt{x^2+3}}\,dx = \frac{4}{3}(x^2-6)\sqrt{x^2+3}+c$. **[5]**

(b) **In this question you must show detailed reasoning.**



The graph shows part of the curve $y = \frac{4x^3}{\sqrt{x^2+3}}$.

Find the exact area enclosed by the curve $y = \frac{4x^3}{\sqrt{x^2+3}}$, the normal to this curve at the point $(1, 2)$ and the x-axis. **[7]**

Qu 50… Edexcel Specimen Paper 3, Question 3 (Link to markscheme)

For a particular type of bulb, 36% grow into plants with blue flowers and the remainder grow into plants with white flowers. Bulbs are sold in mixed bags of 40.

Russell selects a random sample of 5 bags of bulbs.

(a) Find the probability that fewer than 2 of these bags will contain more bulbs that grow into plants with blue flowers than grow into plants with white flowers **(4)**

Maggie takes a random sample of n bulbs.
Using a normal approximation, the probability that more than 244 of these n bulbs will grow into blue flowers is 0.0521 to 4 decimal places.

(b) Find the value of n. **(6)**

Qu 51… OCR A2 Paper 1, 2021, Question 7 (Link to markscheme)

The curve $y = (x^2 - 2)\ln x$ has one stationary point which is close to $x = 1$.

(a) Show that the x-coordinate of this stationary point satisfies the equation $2x^2 \ln x + x^2 - 2 = 0$. **[2]**

(b) Show that the Newton-Raphson iterative formula for finding the root of the equation in part **(a)** can be written in the form $x_{n+1} = \dfrac{2x_n^2 \ln x_n + 3x_n^2 + 2}{4x_n(\ln x_n + 1)}$. **[4]**

Qu 52… OCR A2 Paper 1, 2019, Question 7 (Link to markscheme)

In this question you must show detailed reasoning.

A sequence $u_1, u_2, u_3 \ldots$ is defined by $u_n = 25 \times 0.6^n$.

Use an algebraic method to find the smallest value of N such that $\sum_{n=1}^{\infty} u_n - \sum_{n=1}^{N} u_n < 10^{-4}$. **[8]**

Qu 53… OCR A2 Paper 2, 2021, Question 5 (Link to markscheme)

In this question you must show detailed reasoning.

Points A, B and C have coordinates (0, 6), (7, 5) and (6, −2) respectively.

(a) Find an equation of the perpendicular bisector of AB. **[3]**

(b) Hence, or otherwise, find an equation of the circle that passes through points A, B and C. **[5]**

Qu 54… OCR A2 Paper 1, 2019, Question 12 (Link to markscheme)

A curve has equation $y = a^{3x^2}$, where a is a constant greater than 1.

(a) Show that $\frac{dy}{dx} = 6xa^{3x^2} \ln a$. **[3]**

(b) The tangent at the point $(1, a^3)$ passes through the point $(\frac{1}{2}, 0)$.

Find the value of a, giving your answer in an exact form. **[4]**

(c) By considering $\frac{d^2y}{dx^2}$ show that the curve is convex for all values of x. **[5]**

Qu 55… OCR A2 Paper 2, 2020, Question 15 (Link to markscheme)

In this question you must show detailed reasoning.

The random variable X has probability distribution defined as follows.

$$P(X = x) = \begin{cases} \frac{15}{64} \times \frac{2^x}{x!} & x = 2, 3, 4, 5, \\ 0 & \text{otherwise.} \end{cases}$$

(a) Show that $P(X = 2) = \frac{15}{32}$. **[1]**

The values of three independent observations of X are denoted by X_1, X_2 and X_3.

(b) Given that $X_1 + X_2 + X_3 = 9$, determine the probability that at least one of these three values is equal to 2. **[6]**

Freda chooses values of X at random until she has obtained $X = 2$ exactly three times. She then stops.

(c) Determine the probability that she chooses exactly 10 values of X. **[3]**

Qu 56… OCR A2 Paper 2, 2020, Question 7 (Link to markscheme)

A and B are fixed points in the x-y plane. The position vectors of A and B are $\mathbf{a}$ and $\mathbf{b}$ respectively.

State, with reference to points A and B, the geometrical significance of

(a) the quantity $|\mathbf{a}-\mathbf{b}|$, **[1]**

(b) the vector $\frac{1}{2}(\mathbf{a}+\mathbf{b})$. **[1]**

The circle P is the set of points with position vector $\mathbf{p}$ in the x-y plane which satisfy

$$\left|\mathbf{p}-\frac{1}{2}(\mathbf{a}+\mathbf{b})\right|=\frac{1}{2}\left|\mathbf{a}-\mathbf{b}\right|.$$

(c) State, in terms of $\mathbf{a}$ and $\mathbf{b}$,

(i) the position vector of the centre of P, **[1]**

(ii) the radius of P. **[1]**

It is now given that $\mathbf{a}=\begin{pmatrix}2\\-1\end{pmatrix}$, $\mathbf{b}=\begin{pmatrix}4\\5\end{pmatrix}$ and $\mathbf{p}=\begin{pmatrix}x\\y\end{pmatrix}$.

(d) Find a cartesian equation of P. **[4]**

Qu 57… Edexcel Mock Set 4, Paper 2, Question 10 (Link to markscheme)

The circle C_1 has Cartesian equation

$$x^2+y^2=10x+k \qquad x\in\mathbb{R} \qquad y\in\mathbb{R}$$

where k is a constant.

The curve C_2 has parametric equations

$$x=t^2 \qquad y=2t \qquad t\in\mathbb{R}$$

The curves C_1 and C_2 intersect at 4 distinct points.

Find the range of possible values for k, giving your answer in set notation.

(6)

Qu 58… MEI Practice Papers Set 4, Paper 2, Question 12 (Link to markscheme)

The day length, Y hours, is defined as the difference between the time the sun rises and the time the sun sets on a particular day. For Manchester, England, the following model is proposed for years which are not leap years.

$$Y = a\sin\left(\frac{2\pi}{365}t+b\right)+c,$$

where t is the time in days since the start of the year and a, b and c are constants.

The maximum value of Y, which is 17.03, occurs on June 21st, when $t = 172$. The minimum value of Y, which is 7.47, occurs on December 21st, when $t = 355$.

(a) Show that $a = 4.78$ and $c = 12.25$. **[2]**

(b) Determine the value of b correct to 3 significant figures. **[2]**

On September 1st, when $t = 244$, the day length is recorded as 13.76 hours.

(c) Show that the model is a good fit for this value. **[2]**

In Reykjavik, Iceland, on June 21st the maximum day length was 21.14 hours and on December 21st the minimum day length was 4.12 hours.

(d) Use this information to refine the model for Manchester to produce a possible model for the day length in Reykjavik. **[1]**

On September 1st the day length in Reykjavik is recorded as 14.56 hours.

(e) Determine whether your possible model for Reykjavik is a good fit for this value. **[1]**

Qu 59… OCR Practice Papers Set 4, Paper 2, Question 6 (Link to markscheme)

The table shows information about three geometric series. The three geometric series have different common ratios.

	First term	Common ratio	Number of terms	Last term
Series 1	1	2	n_1	1024
Series 2	1	r_2	n_2	1024
Series 3	1	r_3	n_3	1024

(a) Find n_1. **[2]**

(b) Given that r_2 is an integer less than 10, find the value of r_2 and the value of n_2. **[2]**

(c) Given that r_3 is **not** an integer, find a possible value for the sum of all the terms in Series 3. **[4]**

Qu 60… Edexcel Mock Papers Set 2 (2020), Paper 2, Question 14 (Link to markscheme)

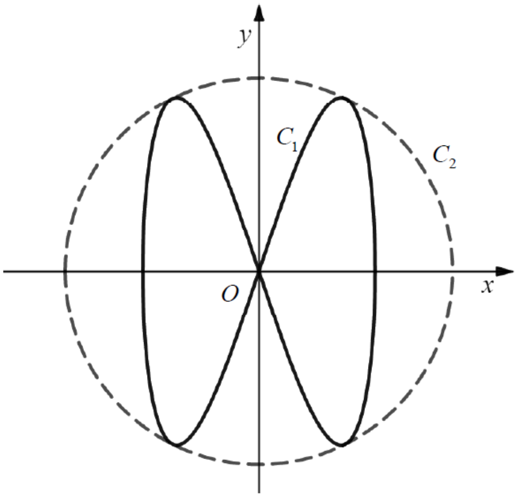


Figure 5

Figure 5 shows a sketch of the curve C_1 with parametric equations

$$x = 2\sin t, \quad y = 3\sin 2t \qquad 0 \leqslant t < 2\pi$$

(a) Show that the Cartesian equation of C_1 can be expressed in the form

$$y^2 = kx^2\left(4 - x^2\right)$$

where k is a constant to be found.

(4)

The circle C_2 with centre O touches C_1 at four points as shown in Figure 5.

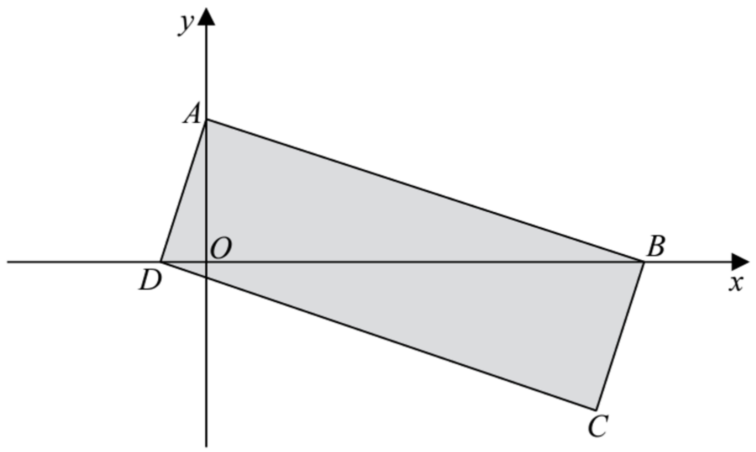
(b) Find the radius of this circle.

(5)

Qu 61… adapted from Edexcel Core 3 June 2012, Question 7b (Link to markscheme)

Given that $x = 3tan2y$ find $\frac{dy}{dx}$ in terms of x without involving any trigonometrical functions.

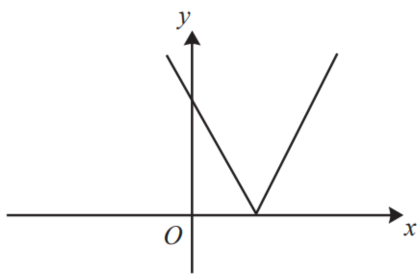
Qu 62… Edexcel Sample Paper 2 June 2012, Question 8 (Link to markscheme)



Given that the straight line through the points A and B has equation $5y + 2x = 10$

find the area of the rectangle $ABCD$. (7)

Qu 63… OCR A2 Paper 1 June 2020 - Question 9 (Link to markscheme)



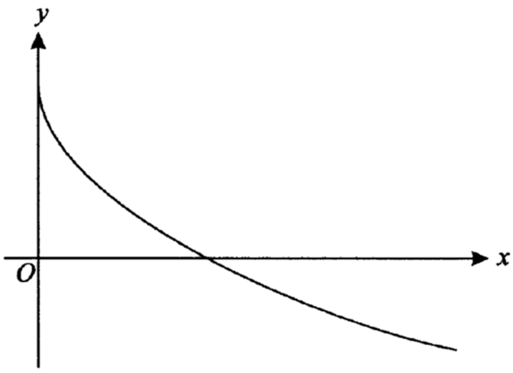
The diagram shows the graph of $y = |2x - 3|$.

Given that the graphs of $y = |2x - 3|$ and $y = ax + 2$ have two distinct points of intersection, determine

(a) the set of possible values of a, **[4]**

(b) the x-coordinates of the points of intersection of these graphs, giving your answers in terms of a. **[3]**

Qu 64… A great question from a long time ago (Link to markscheme)



The function f is defined by $f(x) = 2 - \sqrt{x}$ for $x \geqslant 0$. The graph of $y = f(x)$ is shown above.

(i) State the range of f. [1]

(ii) Find the value of ff(4). [2]

(iii) Given that the equation $|f(x)| = k$ has two distinct roots, determine the possible values of the constant k. [2]

Qu 65… Edexcel A2 Paper 3 Statistics June 2021 - Question 6 (Link to markscheme)

The discrete random variable X has the following probability distribution

x	a	b	c
$P(X = x)$	$\log_{36} a$	$\log_{36} b$	$\log_{36} c$

where

- a, b and c are distinct integers $(a < b < c)$
- all the probabilities are greater than zero

(a) Find

(i) the value of a

(ii) the value of b

(iii) the value of c

Show your working clearly.

(5)

Qu 66… OCR AS Paper 2 June 2023 - Question 8 (Link to markscheme)

In this question you must show detailed reasoning.

Given that $\int_4^a \left(\frac{4}{\sqrt{x}}+3\right)\mathrm{d}x = 7$, find the value of a. **[7]**

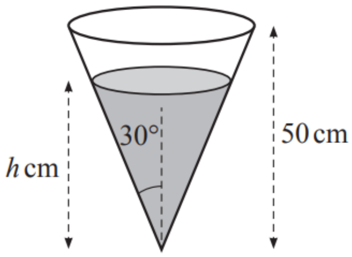
Qu 67… OCR A2 Paper 1 June 2022 - Question 12 (Link to markscheme)

A curve has parametric equations $x = \frac{1}{t}$, $y = 2t$. The point P is $\left(\frac{1}{p}, 2p\right)$.

The tangent to this curve at P crosses the x-axis at the point A and the normal to this curve at P crosses the x-axis at the point B.

Show that the ratio $PA:PB$ is $1:2p^2$. **[12]**

Qu 68… OCR A2 Paper 2 June 2022 - Question 8 (Link to markscheme)



The diagram shows a water tank which is shaped as an inverted cone with semi-vertical angle 30° and height 50 cm. Initially the tank is full, and the depth of the water is 50 cm.

Water flows out of a small hole at the bottom of the tank. The rate at which the water flows out is modelled by $\frac{\mathrm{d}V}{\mathrm{d}t} = -2h$, where $V\,\text{cm}^3$ is the volume of water remaining and h cm is the depth of water in the tank t seconds after the water begins to flow out.

Determine the time taken for the tank to become empty.

[For a cone with base radius r and height h the volume V is given by $\frac{1}{3}\pi r^2 h$.] **[7]**

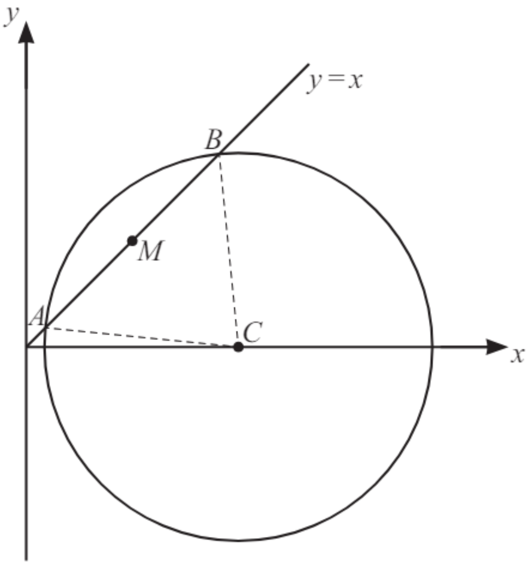
Qu 69… MEI A2 Paper 2 June 2023 - Question 17 (Link to markscheme)

In this question you must show detailed reasoning.

Solve the equation $2\sin x + \sec x = 4\cos x$, where $-\pi < x < \pi$. **[6]**

Qu 70… OCR A2 Paper 2 June 2023 - Question 6 (Link to markscheme)

A circle has centre C which lies on the x-axis, as shown in the diagram. The line $y = x$ meets the circle at A and B. The midpoint of AB is M.



The equation of the circle is $x^2 - 6x + y^2 + a = 0$, where a is a constant.

(a) In this question you must show detailed reasoning.

Show that the area of triangle ABC is $\frac{3}{2}\sqrt{9-2a}$. **[7]**

Qu 71… MEI A2 Paper 3 June 2023 - Question 3 (Link to markscheme)

In this question you must show detailed reasoning.

Find the value of k such that $\dfrac{1}{\sqrt{5}+\sqrt{6}} + \dfrac{1}{\sqrt{6}+\sqrt{7}} = \dfrac{k}{\sqrt{5}+\sqrt{7}}$. **[3]**

Qu 72… OCR A2 Paper 1 June 2023 - Question 12 (Link to markscheme)

Use the substitution $u = e^x - 2$ to show that

$$\int_{\ln 4}^{\ln 6} \frac{7e^x - 8}{(e^x - 2)^2}\,dx = a + \ln b$$

where a and b are rational numbers to be determined. [10]

Qu 73… MEI A2 Paper 2 June 2020 - Question 14 (Link to markscheme)

a) Rewrite $sin(3x)$ in the format $sinAcosB + sinBcosA$.

b)

In this question you must show detailed reasoning.

Fig. 14 shows the graphs of $y = \sin x \cos 2x$ and $y = \frac{1}{2} - \sin 2x \cos x$.

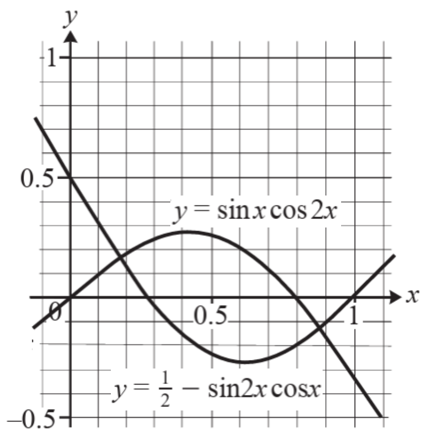


Fig. 14

Use integration to find the area between the two curves, giving your answer in an exact form. **[8]**

Qu 74… OCR A2 Paper 1 June 2020 - Question 7 (Link to markscheme)

A curve has equation $2x^3 + 6xy - 3y^2 = 2$.

Show that there are no points on this curve where the tangent is parallel to $y = x$. **[8]**

Qu 75… OCR A2 Paper 1 June 2020 - Question 12 (Link to markscheme)

A curve has parametric equations $x = \frac{1}{t}$, $y = 2t$. The point P is $\left(\frac{1}{p}, 2p\right)$.

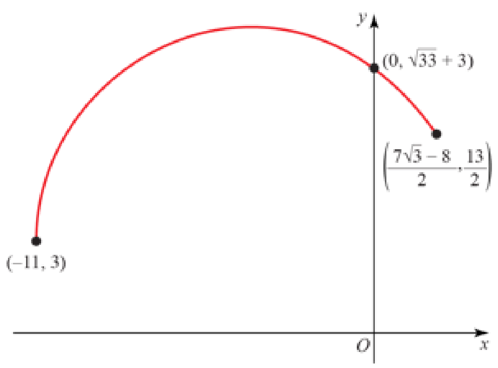
(a) Show that the equation of the tangent at P can be written as $y = -2p^2x + 4p$. **[4]**

The tangent to this curve at P crosses the x-axis at the point A and the normal to this curve at P crosses the x-axis at the point B.

(b) Show that the ratio $PA : PB$ is $1 : 2p^2$. **[8]**

Interesting Questions - Answers

Qu 1… Edexcel unit tests, Parametric Equations - Qu 3. (Link back to question)

a	States $\sin t = \frac{x+4}{7}$ and $\cos t = \frac{y-3}{7}$	**M1**	1.1b	6th Convert between parametric equations and cartesian forms using trigonometry.
	Recognises that the identity $\sin^2 t + \cos^2 t \equiv 1$ can be used to find the cartesian equation.	**M1**	2.2a	
	Makes the substitution to find $(x+4)^2 + (y-3)^2 = 7^2$	**A1**	1.1b	
		(3)		
b	States or implies that the curve is a circle with centre $(-4, 3)$ and radius 7	**M1 ft**	2.2a	6th Sketch graphs of parametric functions.
	Substitutes $t = -\frac{\pi}{2}$ to find $x = -11$ and $y = 3$ $(-11, 3)$ Substitutes $t = \frac{\pi}{3}$ to find $x \approx 2.06$ and $y = 6.5$ $(2.06, 6.5)$ Could also substitute $t = 0$ to find $x = -4$ and $y = 10$ $(-4, 10)$	**M1 ft**	1.1b	
	Figure 1  Draws fully correct curve.	**A1 ft**	1.1b	
		(3)		
c	Makes an attempt to find the length of the curve by recognising that the length is part of the circumference. Must at least attempt to find the circumference to award method mark. $C = 2 \times \pi \times 7 = 14\pi$	**M1 ft**	1.1b	6th Sketch graphs of parametric functions.
	Uses the fact that the arc is $\frac{5}{12}$ of the circumference to write arc length $= \frac{35}{6}\pi$	**A1 ft**	1.1b	
		(2)		

Qu 2… AQA A2 Paper 1, June 2018 -Qu 5. (Link back to question)

Differentiates 2^t or 2^{-t} to obtain $\pm A\ln 2\times 2^{\pm t}$	AO1.1a	M1	
Obtains $\frac{dy}{dt}=(\pm A\ln 2)2^t$ and $\frac{dx}{dt}=(\pm B\ln 2)2^{-t}$	AO1.1b	A1	$\frac{dy}{dt}=(3\ln 2)2^t$ $\frac{dx}{dt}=(-4\ln 2)2^{-t}$
Uses chain rule with correct $\frac{dy}{dt}$ and $\frac{dx}{dt}$ and completes rigorous argument to obtain fully correct printed answer	AO2.1	R1	$\frac{dy}{dx}=\frac{(3\ln 2)2^t}{(-4\ln 2)2^{-t}}$ $=-\frac{3}{4}\times 2^{2t}$
Rearranges to write 2^{-t} in terms of x or 2^t in terms of y Or Writes given expression in terms of t	AO3.1a	M1	$2^t=\frac{y+5}{3}$ $2^{-t}=\frac{x-3}{4}$
Eliminates t Or Compares coefficients PI by a=5 or b=-3	AO1.1a	M1	$1=\left(\frac{y+5}{3}\right)\left(\frac{x-3}{4}\right)$ $12=xy+5x-3y-15$ $xy+5x-3y=27$
Completes rigorous argument to obtain correct values of a, b and c and write the Cartesian equation in the required form ISW	AO2.1	R1	**ALT** $xy+ax+by=(4\times 2^{-t}+3)(3\times 2^t-5)+a(4\times 2^{-t}+3)+b(3\times 2^t-5)$ $=12-15+(4a-20)2^{-t}+(3b+9)2^t+3a-5b$ $a=5, b=-3$ $xy+5x-3y=-3+15+15$ $=27$

Qu 3… AQA A2 Paper 1, June 2018 – Qu 12. (Link back to question)

Begins a proof using a valid method Eg. Factor theorem, algebraic division, multiplication of correct factors	AO1.1a	M1	$p\left(-\frac{1}{2}\right)=30\times\left(-\frac{1}{2}\right)^3-7\left(-\frac{1}{2}\right)^2-7\left(-\frac{1}{2}\right)+2$ $=0$ $\therefore 2x+1$ is a factor of $p(x)$
Constructs rigorous mathematical proof. To achieve this mark: Factor theorem the student must clearly substitute and state that $p(-1/2)=0$ **and** clearly state that this implies that $2x+1$ is a factor Algebraic division OR Multiplication of correct factors The method must be completely correct with a concluding statement	AO2.1	R1	
Obtains quadratic factor PI	AO1.1a	M1	$p(x)=(2x+1)(15x^2-11x+2)$ $=(2x+1)(5x-2)(3x-1)$
Obtains second linear factor	AO1.1b	A1	
Writes $p(x)$ as the product of the correct three linear factors. NMS correct answer 3/3	AO1.1b	A1	
Rearranges to achieve a cubic equation in $\sec x$ (or $\cos x$)	AO3.1a	M1	$\frac{30\sec^2 x+2\cos x}{7}=\sec x+1$ $\Rightarrow 30\sec^2 x+2\cos x=7\sec x+7$ $\Rightarrow 30\sec^3 x+2=7\sec^2 x+7\sec x$ $30\sec^3 x-7\sec^2 x-7\sec x+2=0$ $\Rightarrow(2\sec x+1)(5\sec x-2)(3\sec x-1)=0$ $\Rightarrow \sec x=-\frac{1}{2},\frac{1}{3},\frac{2}{5}$ These values do not fall within the range of $\sec x$ as they are between -1 and 1 $\therefore \frac{30\sec^2 x+2\cos x}{7}=\sec x+1$ has no real solutions.
Equates to zero and uses result from (b) or factorises	AO1.1a	M1	
Deduces that if solutions exist they must be of the form $\sec x$ = -½, $\sec x$ =1/3 or $\sec x$ = 2/5 OE	AO2.2a	A1	
Explains that the range of $\sec x$ is $(-\infty,-1]\cup[1,\infty)$ OE OE for $\cos x$	AO2.4	E1	
Completes argument explaining that there cannot be any real solutions as values are outside of the function's range.	AO2.1	R1	
Total		**10**	

Qu 4… AQA A2 Paper 1, June 2018 – Qu13. (Link back to question)

Q	Marking instructions	AO	Mark	Typical solution
13	Identifies and clearly defines consistent variables for length and width. Can be shown on diagram.	AO3.1b	B1	Width of rectangle = $2x$ Length of rectangle = $2y$
	Models the area of rectangle with an expression of the correct dimensions	AO3.3	M1	$A = 4xy$
	Eliminates either variable to form a model for the area in one variable.	AO1.1a	M1	$x^2 + y^2 = 16$
	Obtains a correct equation to model the area in one variable	AO1.1b	A1	$A = 4x\sqrt{16 - x^2}$
	Differentiates their expression for area. Condone one error	AO3.4	M1	$\frac{dA}{dx} = 4\sqrt{16 - x^2} - \frac{4x^2}{\sqrt{16 - x^2}}$ $\frac{dA}{dx} = \frac{64 - 8x^2}{\sqrt{16 - x^2}}$ For maximum point $\frac{dA}{dx} = 0$
	Explains that their derivative equals zero for a maximum or stationary point.	AO2.4	E1	$\frac{64 - 8x^2}{\sqrt{16 - x^2}} = 0$ $x = 2\sqrt{2}$
	Equates area derivative to zero and obtains correct value for either variable. CAO	AO1.1b	A1	When x = 2.8, $\frac{dA}{dx} = 0.448$ When $x = 2.9$, $\frac{dA}{dx} = -1.191$
	Completes a gradient test or uses second derivative of their area function to determine nature of stationary point	AO1.1a	M1	Therefore maximum
	Deduces that the area is a maximum at $x = 2\sqrt{2}$ or $\theta = \frac{\pi}{4}$ Values need not be exact	AO2.2a	R1	The maximum area is 32 sq in
	Obtains maximum area with correct units AWRT 32	AO3.2a	B1	
	Total		**10**	

Qu 5… AQA A2 Paper 2, June 2018 -Qu 8. (Link back to question)

Compares with $R\cos(x\pm\alpha)$ or $R\sin(x\pm\alpha)$	AO3.1a	M1	$\sqrt{3}\sin x-3\cos x\equiv R\sin(x-\alpha)$ $\equiv R\sin x\cos\alpha-R\cos x\sin\alpha$
Obtains two correct equations for R and α for example $R\cos\alpha=\sqrt{3}$ $R\sin\alpha=3$ Must be explicitly seen	AO3.1a	M1	$R\cos\alpha=\sqrt{3}$ $R\sin\alpha=3$
Obtains correct R Condone AWRT 3.46 PI by description of stretch	AO1.1b	B1	$R=\sqrt{12}=2\sqrt{3}$ $\tan\alpha=\sqrt{3}$
Obtains correct α in radians or degrees PI by description of translation	AO1.1b	B1	$\alpha=\frac{\pi}{3}$ $y=2\sqrt{3}\sin(x-\frac{\pi}{3})+4$
Interprets their values of R and α to form an equation of the form $y=R\sin(x\pm\alpha)+4$ or $y=R\cos(x\pm\alpha)+4$	AO3.2a	B1F	Translation $\begin{pmatrix}\frac{\pi}{3}\\0\end{pmatrix}$ Stretch in the y-direction scale
Interprets ‘their’ equation to identify a transformation	AO3.2a	E1F	factor $2\sqrt{3}$
Identifies all required transformations in a correct order CAO	AO3.2a	A1	Translation $\begin{pmatrix}0\\4\end{pmatrix}$
Deduces the least value occurs when their $\sin(x-\frac{\pi}{3})=1$ Using ‘their’ values of R and α PI by sight of $\frac{1}{2\sqrt{3}+4}$	AO2.2a	M1	$\frac{1}{\sqrt{3}\sin x-3\cos x+4}=\frac{1}{2\sqrt{3}\sin(x-\frac{\pi}{3})+4}$ Least value when $\sin(x-\frac{\pi}{3})=1$ $\therefore$ least value is given by
Completes rigorous argument to obtain $\frac{1}{2\sqrt{3}+4}$ and then the given answer	AO2.1	R1	$\frac{1}{2\sqrt{3}+4}=\frac{2-\sqrt{3}}{2}$
Deduces the greatest value Using ‘their’ values of R and α ACF $\frac{1}{-2\sqrt{3}+4}=\frac{2+\sqrt{3}}{2}$	AO2.2a	B1F	Greatest value = $\frac{2+\sqrt{3}}{2}$
Total		**10**	

Qu 6… AQA A2 Paper 2, June 2018 - Qu 7. (Link back to question)

Integrates using integration by parts	AO3.1a	M1	$y=\int (x-1)e^x dx$ $u=x-1$ $\frac{dv}{dx}=1$ $\frac{dv}{dx}=e^x$ $v=e^x$
Applies integration by parts formula correctly to either of $(x-1)e^x$ or xe^x	AO1.1a	M1	
Obtains fully correct integral, condone missing constant.	AO1.1b	A1	$y=(x-1)e^x-\int e^x dx$ $y=(x-1)e^x-e^x+c$
Explains clearly why the minimum y value is e with reference to the range of the function OE	AO2.4	E1	Range $\geq e \Rightarrow$ at min $y=e$
Uses $\frac{dy}{dx}=0$ to find *x* coordinate of minimum	AO1.1a	M1	Min point when $\frac{dy}{dx}=0 \therefore x=1$ So curve passes through $(1, e)$
Deduces that the curve passes through the point $(1, e)$	AO2.2a	A1	$e=(1-1)e^1-e^1+c$
Uses their minimum point to find their c	AO1.1a	M1	$c=2e$
States the correct equation in any correct form Condone y instead of $f(x)$ CAO	AO1.1b	A1	$\therefore f(x)=(x-2)e^x+2e$

Qu 7… Edexcel unit tests, parametric Equations -Qu 6. (Link back to question)

a	Rearranges $x = 8(t+10)$ to obtain $t = \frac{x-80}{8}$	M1	1.1b	8th Use parametric equations in modelling in a variety of contexts.
	Substitutes $t = \frac{x-80}{8}$ into $y = 100 - t^2$ For example, $y = 100 - \left(\frac{x-80}{8}\right)^2$ is seen.	M1	1.1b	
	Finds $y = -\frac{1}{64}x^2 + \frac{5}{2}x$	A1	1.1b	
		(3)		
b	Deduces that the width of the arch can be found by substituting $t = \pm 10$ into $x = 8(t+10)$	M1	3.4	8th Use parametric equations in modelling in a variety of contexts.
	Finds $x = 0$ and $x = 160$ and deduces the width of the arch is 160 m.	A1	3.2a	
		(2)		
c	Deduces that the greatest height occurs when $\frac{dy}{dt} = 0 \Rightarrow -2t = 0 \Rightarrow t = 0$	M1	3.4	8th Use parametric equations in modelling in a variety of contexts.
	Deduces that the height is 100 m.	A1	3.2a	
		(2)		

Qu 8… Edexcel Paper 1, June 2018 - Qu7. (Link back to question)

Question	Scheme	Marks	AOs
7 (a)	$\int \frac{2}{(3x-k)}\,dx = \frac{2}{3}\ln(3x-k)$	M1 A1	1.1a 1.1b
	$\int_k^{3k} \frac{2}{(3x-k)}\,dx = \frac{2}{3}\ln(9k-k) - \frac{2}{3}\ln(3k-k)$	dM1	1.1b
	$= \frac{2}{3}\ln\left(\frac{8\cancel{k}}{2\cancel{k}}\right) = \frac{2}{3}\ln 4$ oe	A1	2.1
		(4)	
(b)	$\int \frac{2}{(2x-k)^2}\,dx = -\frac{1}{(2x-k)}$	M1	1.1b
	$\int_k^{2k} \frac{2}{(2x-k)^2}\,dx = -\frac{1}{(4k-k)} + \frac{1}{(2k-k)}$	dM1	1.1b
	$= \frac{2}{3k} \quad \left(\propto \frac{1}{k}\right)$	A1	2.1
		(3)	
			(7 marks)

Qu 9… AQA Paper 3, June 2018 – Qu 6. (Link back to question)

Deduces that the lower bound of x is 1	AO2.2a	M1	$\{x \in \mathbb{R} : x > 1\}$
States the domain in a correct form	AO2.5	A1	

Qu 10… AQA Paper 3, June 2018 – Qu 8. (Link back to question)

Recalls a correct trig identity, which could lead to a correct answer	AO1.2	B1	(LHS ≡) $\frac{\sin 2x}{1+\tan^2 x}$ $\equiv \frac{2\sin x\cos x}{1+\tan^2 x}$ $\equiv \frac{2\sin x\cos x}{\sec^2 x}$ $\equiv 2\sin x\cos x\cos^2 x$ $\equiv 2\sin x\cos^3 x$ (≡ RHS)
Demonstrates a strategy for proving the identity, eg by converting all the terms on the LHS to cos and sin.	AO3.1a	M1	
Concludes a rigorous mathematical argument to prove given identity **AG**	AO2.1	R1	
Uses identity to write integrand in the form $a\sin 2\theta\cos^3 2\theta$	AO1.1a	M1	$\int \frac{4\sin 4\theta}{1+\tan^2 2\theta}\,d\theta = \int 8\sin 2\theta\cos^3 2\theta\,d\theta$ Let $u = \cos 2\theta$ then $\frac{du}{d\theta} = -2\sin 2\theta \Rightarrow \sin 2\theta = -\frac{1}{2}\frac{du}{d\theta}$ $I = -4\int u^3 \frac{du}{d\theta}\,d\theta$ $= -4\int u^3\,du$ $= -u^4 + c$ $= -\cos^4 2\theta + c$
Correctly writes integrand as $8\sin 2\theta\cos^3 2\theta$	AO1.1b	A1	
Selects an appropriate method for integrating, e.g. substitution $u = \cos 2\theta$, or by inspection PI by sight of $\cos^4 2\theta$	AO3.1a	M1	
Obtains $k\int u^3\,du$ correctly PI by solution in form $k\cos^4 2\theta$, if by inspection	AO1.1a	M1	
Obtains $-u^4$ or $-\cos^4 2\theta$ OE Only FT value of a	AO1.1b	A1F	
Completes rigorous argument to obtain $-\cos^4 2\theta + c$ OE	AO2.1	R1	
Total		**9**	

Qu 11… OCR A, Paper 2, June 2018. (Link back to question)

$\frac{1.5\mu-\mu}{\mu/3}$	M1	1.1a	$\frac{4.5\sigma-3\sigma}{\sigma}$	SC (eg) Let $\mu = 1$; $N(1,\frac{1}{9})$ M1
$=\frac{3}{2}$	A1	1.1		$X = \frac{3}{2}$ A0
$P(X > 1.5\mu) = 0.0668$ or 0.67 (2 sf)	A1 [3]	1.1		$P(X > \frac{3}{2}) = 0.067$ A1

Qu 12… OCR A, Paper 2, June 2018. (Link back to question)

$\dfrac{\frac{50!}{r!(50-r)!}\times 0.15^{r}\times 0.85^{50-r}}{\frac{50!}{(r+1)!(50-(r+1))!}\times 0.15^{r+1}\times 0.85^{50-(r+1)}}$ oe	M1	1.1a	$\dfrac{{}^{50}C_r\times 0.15^{r}\times 0.85^{50-r}}{{}^{50}C_{r+1}\times 0.15^{r+1}\times 0.85^{50-(r+1)}}$	Fully correct
$\dfrac{\frac{1}{50-r}\times 0.85}{\frac{1}{r+1}\times 0.15}$ or $\dfrac{0.85}{50-r}\times\dfrac{r+1}{0.15}$ oe	A1	2.1	Any correct simplification without factorials **OR** without indices	**or** $\dfrac{17}{20}\times\dfrac{20}{3}\times\dfrac{r+1}{50-r}$
$=\dfrac{17(r+1)}{3(50-r)}$ **AG**	A1	1.1	Any correct simplification without factorials **AND** without indices and correctly obtain result	
	[3]			

Qu 13… OCR, Paper 1, June 2018. (Link back to question)

$\dfrac{dy}{dx}=\dfrac{(-8\sin 2x)(3-\sin 2x)-(4\cos 2x)(-2\cos 2x)}{(3-\sin 2x)^2}$	M1	3.1a	Attempt use of quotient rule	Correct structure, including subtraction in numerator Could be equivalent using the product rule
	A1	1.1	Obtain correct derivative	Award A1 once correct derivative seen even subsequently spoiled by simplification attempt
EITHER when $x=\frac{1}{4}\pi$, gradient $=\dfrac{-16-0}{4}=-4$ **OR** $\dfrac{(-8\sin\frac{\pi}{2})(3-\sin\frac{\pi}{2})-(4\cos\frac{\pi}{2})(-2\cos\frac{\pi}{2})}{(3-\sin\frac{\pi}{2})^2}=-4$	M1	2.4	**DR** Attempt to find gradient at $\frac{1}{4}\pi$	**EITHER** State that $x=\frac{1}{4}\pi$ is being used, and show their fraction with each term (including 0) explicitly evaluated before being simplified ie $x=\frac{1}{4}\pi$, gradient $=-4$ is M0 **OR** Substitute $\frac{1}{4}\pi$ into their derivative and evaluate
gradient of normal is $\frac{1}{4}$	B1ft	2.1	Correct gradient of normal	ft their gradient of tangent
area of triangle is $\frac{1}{2}\times\frac{1}{16}\pi\times\frac{1}{4}\pi\left(=\frac{1}{128}\pi^2\right)$	M1	2.1	Attempt area of triangle ie $\frac{1}{2}\times\frac{1}{4}\pi\times(\text{their } y)$	y coordinate could come from using equation of normal, $y=\frac{1}{4}(x-\frac{1}{4}\pi)$, or from using gradient of normal Could integrate equation of normal
$\displaystyle\int\frac{4\cos 2x}{3-\sin 2x}\,dx=-2\ln\lvert 3-\sin 2x\rvert$	M1*	3.1a	Obtain integral of form $k\ln\lvert 3-\sin 2x\rvert$	Condone brackets not modulus Allow any method, including substitution, as long as integral of correct form
	A1	1.1	Obtain correct integral	Possibly with unsimplified coefficient
$\displaystyle\int_0^{\frac{1}{4}\pi}\frac{4\cos 2x}{3-\sin 2x}\,dx=(-2\ln 2)-(-2\ln 3)$	M1d*	2.1	Attempt use of limits	Using $\frac{1}{4}\pi$ and 0; correct order and subtraction (oe if substitution used) Must see a minimum of $-2\ln 2+2\ln 3$
$2\ln 3-2\ln 2=\ln 9-\ln 4=\ln\frac{9}{4}$ OR $2\ln 3-2\ln 2=2\ln\frac{3}{2}=\ln\frac{9}{4}$	A1	1.1	Correct area under curve	Must be exact At least one log law seen to be used before final answer
hence total area is $\ln\frac{9}{4}+\frac{1}{128}\pi^2$ **A.G.**	A1	2.1	Obtain correct total area	Any equivalent exact form AG so method must be fully correct A0 if the gradient of -4 results from an incorrect derivative having been used A0 if negative area of triangle not dealt with convincingly
	[10]			

Qu 14… OCR, Paper 2, June 2018. (Link back to question)

Summary of marks:			
Attempt find x at intersection of curves	**M1**	**3.1a**	Can be implied
$x = 1$	**A1**	**1.1**	
Correct integral, any limits	**M1**	**3.1a**	
Correct numerical result	**A1**	**1.1**	from correct limits
Attempt area of part or all of 2×2 square	**M1**	**1.1**	
Wholly correct method	**M1**	**2.1**	
$\frac{44}{3}$	**A1**	**1.1**	
	[7]		

Qu 15… OCR Practice Papers, Set 2, Paper 3 - Qu3. (Link back to question)

Reflection, stretch and translation	**B1**	**2.5**	All three correct	Do not accept any other wording
(reflection) in the line $y = x$	**B1**	**1.1**		
(stretch) scale factor $\frac{1}{3}$ parallel to the x-axis	**B1**	**1.1**	Accept 'in the x-direction'; accept 'factor' or 'SF' for 'scale factor'	Do not accept 'in/on/across/up the x-axis' or '$\frac{1}{3}$ units'
(translation) $\begin{pmatrix} 0 \\ -5 \end{pmatrix}$	**B1**	**1.1**	Accept '5 units in the negative y-direction' or '-5 units parallel to the y-axis'	Do not accept 'in/on/across/up the y-axis'
			Order of transformations must be correct for all 4 marks to be awarded	
	[4]			

Qu 16… OCR Practice Papers, Set 4, Paper 1. (Link back to question)

$e^x = 3 + 2e^y$	**M1**	**3.1a**	Attempt to eliminate one variable	
$(3 + 2e^y)^2 - 4e^{2y} = 33$	**A1**	**1.1**	Obtain correct equation in one variable – allow unsimplified	or $e^{2x} - 4(0.5e^x - 1.5)^2 = 33$
$9 + 12e^y + 4e^{2y} - 4e^{2y} = 33$	**M1**	**1.1a**	Simplify and attempt to solve	or $6e^x = 42$
$12e^y = 24$				etc
$e^y = 2$				
$y = \ln 2$	**A1**	**1.1**	Obtain $y = \ln 2$	
$e^x - 4 = 3$				
$e^x = 7$				
$x = \ln 7$	**A1**	**2.1**	Obtain $x = \ln 7$, using either equation.	
	[5]			

Qu 17… AQA Core 3, June 2013. (Link back to question)

$u=(\ln x)^2$ $\frac{dv}{dx}=1$ $\frac{du}{dx}=(2\ln x)\frac{1}{x}$ $v=x$	M1 A1		$\frac{d(\ln x)^2}{dx}$ & $\int dx$ attempted All correct
$\left(\int(\ln x)^2 dx=\right)$ $x(\ln x)^2-\int x\times\frac{2}{x}\ln x\,(dx)$	m1		OE correct substitution of their terms into parts
$=x(\ln x)^2-2(x\ln x-x)+C$ OE	A1	4	All correct (constant needed) including correct use of brackets. **Do not penalise missing constant if already penalised in part (i)** ISW
$\frac{du}{dx}=\frac{1}{2\sqrt{x}}$ or $\frac{1}{2}x^{-\frac{1}{2}}$ $(dx=2u\,du)$	B1		$u=\sqrt{x}$
$\int_{(1)}^{(4)}\frac{1}{x+\sqrt{x}}dx=\int_{(1)}^{(2)}\frac{1}{u^2+u}2u\ (du)$	M1		All in terms of u including attempt at replacing dx (not simply writing du), condone missing limits and du
	A1		Integrand correct unsimplified
$=2\int_{(1)}^{(2)}\frac{1}{u+1}\ (du)$	A1		
$=2\ln(u+1)\Big\|_{(1)}^{(2)}$	A1F		FT *their* $\int\frac{k}{u+1}(du)$
$=2\ln(2+1)-2\ln(1+1)$ or $2\ln(\sqrt{4}+1)-2\ln(\sqrt{1}+1)$	A1F		correct use of correct limits on $k\ln(u+1)$ or $k\ln\left(\sqrt{x}+1\right)$
$=2\ln\frac{3}{2}$ *or* $\ln\frac{9}{4}$ *or* $2\ln3-2\ln2$	A1	7	OE ISW

Qu 18... MEI, Paper 1, June 2018 – Qu 10. (Link back to question)

Curve crosses the x-axis when $y=0$ $y=(k-x)\ln x=0$ Either $k-x=0$ or $\ln x=0$ $x=k$ or 1	M1 A1	3.1a 1.1b	Attempt to solve $y=0$ Both roots required
EITHER Area $=\int_1^k (k-x)\ln x \, dx$ Let $u=\ln x,\ \frac{dv}{dx}=k-x,\ \frac{du}{dx}=\frac{1}{x},\ v=kx-\frac{1}{2}x^2$	M1	2.1	Using integration by parts with $u=\ln x,\ \frac{dv}{dx}=k-x$ clearly argued
Area $=\left[\left(kx-\frac{1}{2}x^2\right)\ln x\right]_1^k-\int_1^k \frac{1}{x}\left(kx-\frac{1}{2}x^2\right) dx$	A1	1.1b	Allow without limits
$\left[\left(kx-\frac{1}{2}x^2\right)\ln x\right]_1^k-\int_1^k\left(k-\frac{1}{2}x\right) dx$	M1	3.1a	Simplifying the integrand
$\left[\left(kx-\frac{1}{2}x^2\right)\ln x-\left(kx-\frac{1}{4}x^2\right)\right]_1^k$	A1	1.1b	Second part correct
$\left(\left(k^2-\frac{1}{2}k^2\right)\ln k-\left(k^2-\frac{1}{4}k^2\right)\right)-\left(\left(k-\frac{1}{2}\right)\ln 1-\left(k-\frac{1}{4}\right)\right)$	M1dep	1.1a	Using limits. Dependent on M mark for integration by parts
$=\frac{1}{2}k^2\ln k-\frac{3}{4}k^2+k-\frac{1}{4}$	A1 [8]	1.1b	Cao

Qu 19... MEI, Paper 1, June 2018 -Qu 11. (Link back to question)

(i)		Component of weight down the plane $4.7g\sin 60°$ Equilibrium equation $T=4.7g\sin 60°$ $=39.889...$ so $T=39.9$ to 3 sf	B1 E1 [2]	2.1 3.3	AG Award if seen Must be clear that 39.9 N is the tension and not just component of weight	
(ii)		Resolve perpendicular to the slope N is the normal reaction between plane and block B $N=4g\cos 25°$	B1	1.1a	Need not be evaluated here $[\approx 35.5]$	
		Resolve up the slope $T-F-4g\sin 25°=0$	M1 A1	3.3 1.1b	Allow only sign errors F need not be evaluated here $[\approx 23.3]$	
		On the point of sliding so $F=\mu N=\mu\times 4g\cos 25°$ $\mu=\frac{4.7g\sin 60°-4g\sin 25°}{4g\cos 25°}=0.656$ to 3sf	M1 A1 [5]	3.1b 1.1b	Do not allow for $F\le\mu N$ unless = used subsequently. FT their values. FT (notice this answer is 0.657 if 39.9 used for T)	If only values are seen used, it must be clear that the values used are friction and normal reaction.

Qu 20… MEI, Paper 3, June 2018 – Qu 10. (Link back to question)

(i)	$\overrightarrow{AC} = \begin{pmatrix} 2-a \\ 4-b \\ 2 \end{pmatrix}$, $\overrightarrow{AB} = \begin{pmatrix} 4-a \\ 2-b \\ 0 \end{pmatrix}$	**M1**	**1.1**	Forming vectors for sides AB and AC
	$(4-a)^2 + (2-b)^2 = (2-a)^2 + (4-b)^2 + 4$ o.e.	**M1**	**1.1a**	Use of AB = AC
	$16 - 8a + a^2 + 4 - 4b + b^2 = 4 - 4a + a^2 + 16 - 8b + b^2 + 4$	**M1**	**1.1**	expanding
	$4a - 4b + 4 = 0 \Rightarrow a - b + 1 = 0$	**A1** **[4]**	**2.1**	**AG** Convincing completion
(ii)	D has position vector $\begin{pmatrix} 3 \\ 3 \\ 1 \end{pmatrix}$ where D is midpoint of BC	**B1**	**3.1a**	Midpoint OR if clearly minimising AC or AB - **M1** for relevant vector using a and b (May be implied by second M1)
	$\overrightarrow{AD} = \begin{pmatrix} 3-a \\ 2-a \\ 1 \end{pmatrix}$	**M1**	**1.1**	Finding relevant vector in terms of a or b only
	Area $= \frac{1}{2} AD.BC = \frac{2\sqrt{3}\sqrt{(3-a)^2 + (2-a)^2 + 1}}{2}$	**M1**	**1.1**	Expression for AD or AD^2 (correct method but may have errors)
	$\sqrt{3}\sqrt{2\left((a-2.5)^2 + 0.75\right)}$	**M1**	**3.1a**	Completion of square
	$a = 2.5$ for min	**A1**	**2.2a**	
	Position vector $\begin{pmatrix} 2.5 \\ 3.5 \\ 0 \end{pmatrix}$	**A1**	**3.2a**	
		[6]		

Qu 21… Edexcel Mock Papers, Paper 1 – Qu 11. (Link back to question)

(ii)	$\dfrac{\mathrm{d}}{\mathrm{d}y}(2\tan y) = 2\sec^2 y$	M1
	$\{x = 2\tan y \Rightarrow\}\ \dfrac{\mathrm{d}x}{\mathrm{d}y} = 2\sec^2 y$ or $1 = (2\sec^2 y)\dfrac{\mathrm{d}y}{\mathrm{d}x}$	A1
	$\dfrac{\mathrm{d}x}{\mathrm{d}y} = 2(1+\tan^2 y)$ or $1 = 2(1+\tan^2 y)\dfrac{\mathrm{d}y}{\mathrm{d}x}$	M1
	E.g. $\dfrac{\mathrm{d}x}{\mathrm{d}y} = 2\left(1+\left(\dfrac{x}{2}\right)^2\right) \Rightarrow \dfrac{\mathrm{d}x}{\mathrm{d}y} = 2\left(1+\dfrac{x^2}{4}\right) \Rightarrow \dfrac{\mathrm{d}x}{\mathrm{d}y} = 2+\dfrac{x^2}{2}$ $\Rightarrow \dfrac{\mathrm{d}x}{\mathrm{d}y} = \dfrac{4+x^2}{2} \Rightarrow \dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{2}{4+x^2}$	A1
		(4)
(ii) **Alt 1**	$\{x = 2\tan y \Rightarrow\}\ y = \arctan\left(\dfrac{x}{2}\right) \Rightarrow \dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{1}{\left(1+\left(\dfrac{x}{2}\right)^2\right)} \times \left(\dfrac{1}{2}\right)$	M1 M1 A1
	$\Rightarrow \dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{1}{2\left(1+\dfrac{x^2}{4}\right)} \Rightarrow \dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{1}{\left(2+\dfrac{x^2}{2}\right)} \Rightarrow \dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{1}{\left(\dfrac{4+x^2}{2}\right)}$ $\Rightarrow \dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{2}{4+x^2}$	A1
		(4)

Qu22... Edexcel Mock Papers, Paper 1 - Qu 10. (Link back to question)

$V = 4\pi h(h+6) = 4\pi h^2 + 24\pi h \quad 0 \leqslant h \leqslant 25\ ;\ \frac{dV}{dt} = 80\pi$	
$\text{Time} = \frac{4\pi(24)(24+6)}{80\pi} = \frac{2880\pi}{80\pi} = 36\ \text{(s)}\ \ *$	B1 *
	(1)
When $t=8,\ V = 80\pi(8) = 640\pi \Rightarrow 640\pi = 4\pi h(h+6)$	M1
$160 = h(h+6) \Rightarrow h^2 + 6h - 160 = 0 \Rightarrow (h+16)(h-10) = 0 \Rightarrow h = ...$	M1
$\{h = -16, \text{reject}\},\ h = 10$	A1
$\frac{dV}{dh} = 8\pi h + 24\pi$	M1
	A1
$\left\{\frac{dV}{dh} \times \frac{dh}{dt} = \frac{dV}{dt} \Rightarrow\right\}\ (8\pi h + 24\pi)\frac{dh}{dt} = 80\pi$	M1
When $h=10,\ \left\{\frac{dh}{dt} = \frac{dV}{dt} \div \frac{dh}{dt} =\right\} \frac{80\pi}{(8\pi(10) + 24\pi)} \left\{= \frac{80\pi}{124\pi}\right\}$	M1
When $h=10,\ \frac{dh}{dt} = \frac{10}{13}\ (\text{cm s}^{-1})$ or awrt $0.769\ (\text{cm s}^{-1})$	A1

Qu 23... Edexcel, A2 Paper 1, June 2019 – Qu 9. (Link back to question)

(a)	States $\log a - \log b = \log\frac{a}{b}$	B1
	Proceeds from $\frac{a}{b} = a - b \rightarrow ...... \rightarrow ab - a = b^2$	M1
	$ab - a = b^2 \rightarrow a(b-1) = b^2 \Rightarrow a = \frac{b^2}{b-1}\ \ *$	A1*
		(3)
(b)	States either $b > 1$ or $b \neq 1$ with reason $\frac{b^2}{b-1}$ is not defined at $b = 1$ oe	B1
	States $b > 1$ and explains that as $a > 0 \Rightarrow \frac{b^2}{b-1} > 0 \Rightarrow b > 1$	B1
		(2)

Qu 24… Edexcel Mock Papers, Paper 1 – Qu 13. (Link back to question)

(a)	$\sum_{n=1}^{11} \ln(p^n) = \ln p + \ln p^2 + \ln p^3 + \ldots + \ln p^{11}$ $= \ln p + 2\ln p + 3\ln p + \ldots + 11\ln p$ $= \frac{11}{2}(2\ln p + (11-1)\ln p)$ or $\frac{1}{2}(11)(12)\ln p$	M1
	$= 66\ln p \qquad \{k = 66\}$	A1
		(2)
b)	$S = \sum_{n=1}^{11} \ln(8p^n) = \ln 8p + \ln 8p^2 + \ln 8p^3 + \ldots + \ln 8p^{11}$ $= 11\ln 8 + 66\ln p$	M1
	e.g. • $11\ln 8 + 66\ln p = 11\ln 2^3 + 66\ln p = 33\ln 2 + 66\ln p$ $= 33(\ln 2 + 2\ln p) = 33(\ln 2 + \ln p^2) = 33\ln(2p^2)$ * • $11\ln 8 + 66\ln p = 11\ln 2^3 + 66\ln p = 33\ln 2 + 66\ln p$ $= \ln(2^{33}p^{66}) = \ln\left((2p^2)^{33}\right) = 33\ln(2p^2)$ *	A1*
		(2)
c)	$S < 0 \Rightarrow 33\ln(2p^2) < 0 \Rightarrow \ln(2p^2) < 0$	
	so either $0 < 2p^2 < 1$ or $2p^2 < 1$	M1
	$\Rightarrow p^2 < \frac{1}{2}$ and $p > 0 \Rightarrow 0 < p < \frac{1}{\sqrt{2}}$	
	In set notation, e.g. $\left\{p : 0 < p < \frac{1}{\sqrt{2}}\right\}$	A1
		(2)

Qu 25... OCR AS Paper 2, 2019 – Qu7. (Link back to question)

$\frac{32}{3}$	B1	1.1	Seen or implied by later working	
	M1*	3.1a	Attempt integration on a 3 term quadratic in x	(increase in power by 1 for at least 1 term but not just multiplying each term by x)
$\int(-x^2+6x-5)\,dx = -\frac{x^3}{3}+3x^2-5x$	A1	1.1	Ignore lack of $+c$	
$-\frac{a^3}{3}+3a^2-5a-\left(-\frac{5^3}{3}+75-25\right)$	Dep*M1	1.1	$\pm(\mathrm{F}(a)-\mathrm{F}(5))$	
$\frac{32}{3}+\frac{a^3}{3}-3a^2+5a+\frac{25}{3}=19$	A1	1.1	oe	
$a^3-9a^2+15a=0 \Rightarrow a^2-9a+15=0 \because a\neq 0$	M1	3.1a	solve their cubic (which comes from attempt at both areas and 19) leading to an exact value for a	**Dependent on both previous M marks**
$a\neq\frac{9-\sqrt{21}}{2} \because a>5$	B1	3.2a	**BC** – must give a reason for rejection of this value of a	Allow rejection of 2.21
$a=\frac{9+\sqrt{21}}{2}$ only	A1	2.2a	**BC**	
	[8]			

Qu 26... Edexcel Mock Papers, Paper 1 -Qu 14. (Link back to question)

$y=4xe^{-2x} \Rightarrow \left\{\begin{matrix} u=4x & v=e^{-2x} \\ \frac{du}{dx}=4 & \frac{dv}{dx}=-2e^{-2x}\end{matrix}\right\},\ \left\{\begin{matrix} u=4x & \frac{du}{dx}=4 \\ \frac{dv}{dx}=e^{-2x} & v=-\frac{1}{2}e^{-2x}\end{matrix}\right\}$	
$\frac{dy}{dx}=4e^{-2x}-8xe^{-2x}$	M1 A1
At $P(1,\,4e^{-2})$, $m_T=4e^{-2}-8e^{-2}=-4e^{-2} \Rightarrow m_N=\frac{-1}{-4e^{-2}}$ or $\frac{1}{4}e^2$	M1
l: $y-4e^{-2}=\frac{e^2}{4}(x-1)$ and $y=0 \Rightarrow -4e^{-2}=\frac{e^2}{4}(x-1) \Rightarrow x=\ldots$	M1
$\{y=0 \Rightarrow x=1-16e^{-4}\}$	
$\int 4xe^{-2x}dx = -2xe^{-2x}-\int -2e^{-2x}\,dx$	M1 A1
$= -2xe^{-2x}-e^{-2x}$	A1
Criteria • $\left[-2xe^{-2x}-e^{-2x}\right]_0^1=\left(-2e^{-2}-e^{-2}\right)-(0-1)\ \{=1-3e^{-2}\}$ • Area triangle = $\frac{1}{2}\left(16e^{-4}\right)\left(4e^{-2}\right)\ \{=32e^{-6}\}$	M1
$\text{Area}(R)=1-3e^{-2}-32e^{-6}$ or $\frac{e^6-3e^4-32}{e^6}$	M1 A1
	(10)

Qu 27… Edexcel A2 Paper 2, 2018 – Qu 4. (Link back to question)

4	(i) $\sum_{r=1}^{16}\left(3+5r+2^r\right)=131\,798$; (ii) $u_1, u_2, u_3, \ldots; u_{n+1}=\frac{1}{u_n}, u_1=\frac{2}{3}$	
(i) Way 1	$\left\{\sum_{r=1}^{16}\left(3+5r+2^r\right)=\right\}\sum_{r=1}^{16}(3+5r)+\sum_{r=1}^{16}\left(2^r\right)$	M1
	$=\frac{16}{2}(2(8)+15(5))\ +\ \frac{2(2^{16}-1)}{2-1}$	M1 M1
	$=728+131\,070\ =\ 131\,798$ *	A1*
		(4)
(i) Way 2	$\left\{\sum_{r=1}^{16}\left(3+5r+2^r\right)=\right\}\sum_{r=1}^{16}3+\sum_{r=1}^{16}(5r)+\sum_{r=1}^{16}\left(2^r\right)$	M1
	$=(3\times16)+\frac{16}{2}(2(5)+15(5))\ +\ \frac{2(2^{16}-1)}{2-1}$	M1 M1
	$=48+680+131\,070\ =\ 131\,798$ *	A1*
		(4)
(i) Way 3	Sum $=\ 10+17+26+39+60+97+166+299+560+1077+2106$ $+4159+8260+16457+32846+65619=131\,798$ *	M1 M1 M1 A1*
		(4)
(ii)	$\left\{u_1=\frac{2}{3}\right\},\ u_2=\frac{3}{2},\ u_3=\frac{2}{3},\ldots$ ***(can be implied by later working)***	M1
	$\left\{\sum_{r=1}^{100}u_r=\right\}\ 50\left(\frac{2}{3}\right)+50\left(\frac{3}{2}\right)$ or $50\left(\frac{2}{3}+\frac{3}{2}\right)$	M1
	$=\frac{325}{3}\ \left(\text{or } 108\frac{1}{3} \text{ or } 108.\dot{3} \text{ or } \frac{1300}{12} \text{ or } \frac{650}{6}\right)$	A1
		(3)

Qu 28… AQA, A2 Paper 2, 2019. (Link back to question)

Separates the variables – one side correct Condone missing integral signs **PI** by correct integration	3.1a	M1	$\int \frac{1}{x^2}\ln x\,dx = \int t\,dt$
Integrates their $\int t\,dt$ correctly	1.1b	A1F	$\int t\,dt = \frac{t^2}{2} + c$
Obtains $u' = \frac{1}{x}$ and $v = -\frac{1}{x}$ **OE**	1.1b	B1	$u = \ln x$ $u' = \frac{1}{x}$
Integrates $\int \frac{1}{x^2}\ln x\,dx$ Substitutes their u, u', v and v' into the correct formula for integration by parts Condone sign errors in formula	1.1a	M1	$v' = x^{-2}$ $v = -x^{-1}$ $-\frac{1}{x}\ln x - \int \frac{1}{x}(-x^{-1})\,dx$ $-\frac{1}{x}\ln x + \int \frac{1}{x^2}\,dx$
Obtains $-\frac{1}{x}\ln x - \frac{1}{x}$	1.1b	A1	$-\frac{1}{x}\ln x - \frac{1}{x}$
Substitutes $t = 2$ and $x = 1$ into their integrated equation to find their $+c$	1.1a	M1	$-\frac{1}{x}\ln x - \frac{1}{x} = \frac{t^2}{2} + c$
Obtains correct solution must have t^2 = …. **ACF**	2.5	A1	$t = 2, x = 1 \Rightarrow -1 = 2 + c$ $c = -3$ $t^2 = 6 - 2\left(\frac{1+\ln x}{x}\right)$

Qu 29… Edexcel, A2 Paper 1, June 2019 – Qu 5. (Link back to question)

(i) Deduces translation with one correct aspect.	M1
Translate $\begin{pmatrix} 2 \\ -4 \end{pmatrix}$	A1
(ii) $\mathrm{h}(x) = \frac{21}{"2(x+1)^2+7"} \Rightarrow$ (maximum) value $\frac{21}{"7"}(=3)$	M1
$0 < \mathrm{h}(x) \leqslant 3$	A1ft
	(4)

Qu 30… Edexcel unit tests, Integration – Qu 8. (Link back to question)

$\{x = 4\sin^2\theta \Rightarrow\}\ \frac{dx}{d\theta} = 8\sin\theta\cos\theta$ or $\frac{dx}{d\theta} = 4\sin 2\theta$ or $dx = 8\sin q\cos q\,dq$	B1
$\int \sqrt{\frac{4\sin^2\theta}{4-4\sin^2\theta}}\,.\,8\sin\theta\cos\theta\,\{d\theta\}$ or $\int \sqrt{\frac{4\sin^2\theta}{4-4\sin^2\theta}}\,.\,4\sin 2\theta\,\{d\theta\}$	M1
$= \int \underline{\underline{\tan\theta}}.\,8\sin\theta\cos\theta\,\{d\theta\}$ or $\int \underline{\underline{\tan\theta}}.\,4\sin 2\theta\,\{d\theta\}$	$\underline{\underline{\text{M1}}}$
$= \int 8\sin^2\theta\,d\theta$	A1
$3 = 4\sin^2\theta$ **or** $\frac{3}{4} = \sin^2\theta$ **or** $\sin\theta = \frac{\sqrt{3}}{2} \Rightarrow \theta = \frac{\pi}{3}$ $\{x = 0 \rightarrow \theta = 0\}$	B1
	(5)
$= \{8\}\int \left(\frac{1-\cos 2\theta}{2}\right) d\theta$ $\left\{= \int (4 - 4\cos 2\theta)\,d\theta\right\}$	M1
$= \{8\}\left(\frac{1}{2}\theta - \frac{1}{4}\sin 2\theta\right)$ $\{= 4\theta - 2\sin 2\theta\}$	M1 A1
$\left\{\int_0^{\frac{\pi}{3}} 8\sin^2\theta\,d\theta = 8\left[\frac{1}{2}\theta - \frac{1}{4}\sin 2\theta\right]_0^{\frac{\pi}{3}}\right\} = 8\left(\left(\frac{\pi}{6} - \frac{1}{4}\left(\frac{\sqrt{3}}{2}\right)\right) - (0+0)\right)$	
$= \frac{4}{3}\pi - \sqrt{3}$ o.e.	A1
	(4)

Qu 31… Edexcel, A2 Paper 1, June 2019 – Qu 4. (Link back to question)

(i)	States $x = -14$ and gives a valid reason. Eg explains that the expansion is not valid for $\lvert x\rvert > 4$	B1
		(1)
(ii)	States $x = -\frac{1}{2}$ and gives a valid reason. Eg. explains that it is closest to zero	B1
		(1)

Qu 32… AQA, A2 Paper 2, 2019. (Link back to question)

(a)	Attempts to differentiate $x = 4\sin 2y$ and inverts $\frac{dx}{dy} = 8\cos 2y \Rightarrow \frac{dy}{dx} = \frac{1}{8\cos 2y}$	M1
	At (0,0) $\frac{dy}{dx} = \frac{1}{8}$	A1
		(2)
(b)	Uses their $\frac{dy}{dx}$ as a function of y and, using both $\sin^2 2y + \cos^2 2y = 1$ and $x = 4\sin 2y$ in an attempt to write $\frac{dy}{dx}$ or $\frac{dx}{dy}$ as a function of x Allow for $\frac{dy}{dx} = k\frac{1}{\cos 2y} = ..\frac{1}{\sqrt{1-(..x)^2}}$	M1
	A correct answer $\frac{dy}{dx} = \frac{1}{8\sqrt{1-\left(\frac{x}{4}\right)^2}}$ or $\frac{dx}{dy} = 8\sqrt{1-\left(\frac{x}{4}\right)^2}$	A1
	and in the correct form $\frac{dy}{dx} = \frac{1}{2\sqrt{16-x^2}}$	A1
		(3)

Qu 33… Edexcel, A2 Paper 1, June 2019 – Qu 14. (Link back to question)

Compares with $R\cos(x \pm \alpha)$ or $R\sin(x \pm \alpha)$ by forming an identity e.g. $R\sin(x+\alpha) \equiv a\sin x + b\cos x$ **OE** or Differentiates correctly and equates to zero **CAO PI** by $a\cos x = b\sin x$ **PI** by $R = 4$ or $a^2 + b^2 = 16$	3.1a	M1	$R\sin(x+\alpha) = a\sin x + b\cos x$ $R = 4$ $4\sin\left(\frac{\pi}{3}+\alpha\right) = 2\sqrt{3}$ $\alpha = \frac{\pi}{3}$ $a = 4\cos\frac{\pi}{3} = 2$ $b = 4\sin\frac{\pi}{3} = 2\sqrt{3}$
Deduces $R = 4$ or $a^2 + b^2 = 16$	2.2a	A1	
Forms a correct equation for α **PI** by correct α or Forms the equation shown below $2\sqrt{3} = \frac{a\sqrt{3}}{2} + \frac{b}{2}$ **OE** **Must** substitute correct exact values for the trig functions	1.1b	B1	
Solves their equation to obtain any correct value of α Correct values are shown below $\alpha = \frac{\pi}{3}$ or 0 for $R\sin(x \pm \alpha)$ $\alpha = \pm\frac{\pi}{6}$ for $R\cos(x \pm \alpha)$ or Eliminates a variable correctly from their two equations – **must** obtain a correct simplified equation	1.1a	M1	
Deduces $a = 2$	2.2a	R1	
Deduces $b = 2\sqrt{3}$	2.2a	R1	

Qu 34… Edexcel Unit Tests, A2 Stats, Topic 2, Hypothesis Testing. (Link to question)

$X \sim \mathrm{N}(40, 3^2)$ $\quad \bar{X} \sim \mathrm{N}\left(40, \frac{9}{n}\right)$ (Condone $Y \sim \mathrm{N}\left(40, \frac{9}{n}\right)$)	B1
$\mathrm{P}(\bar{X} > 42) = \mathrm{P}\left(Z > \frac{42-40}{\sqrt{\frac{9}{n}}}\right)$	M1
$\frac{42-40}{\sqrt{\frac{9}{n}}} \geq 1.6449$	B1 dM1
$n \geq 6.087$	
$n = 7$	A1

Qu 35… OCR A Core 3 June 2013, Differentiation. (Link to question)

Attempt use of product rule	M1	to produce expression of form (something non-zero) $\ln(2y+3)$ + $\frac{\text{linear in } y}{\text{linear in } y}$; ignore what they call their derivative
Obtain $\ln(2y+3)$ …	A1	with brackets included
Obtain … + $\frac{2(y+4)}{2y+3}$	A1	with brackets included as necessary
	[3]	
Substitute $y = 0$ into attempt from part (i) or into their attempt (however poor) at its reciprocal	M1	
Obtain 0.27 for gradient at A	A1	or greater accuracy 0.26558…; beware of 'correct' answer coming from incorrect version $\ln(2y+3)+\frac{8}{3}$ of answer in part (i)
Attempt to find value of y for which $x = 0$	M1	allowing process leading only to $y = -4$
Substitute $y = -1$ into attempt from part (i) or into their attempt (however poor) at its reciprocal	M1	
Obtain 0.17 or $\frac{1}{6}$ for gradient at B	A1	or greater accuracy 0.16666…; value following from correct working
	[5]	

Qu 36… OCR A Practice Papers Set 1, Paper 3, Question 6. (Link to question)

$\text{Area} = 2\int_0^{\lambda} \mathrm{f}(y)\,\mathrm{d}y$	**E1**	**2.1**	Correct integral stated for required area
$y = \ln(1+4x^2) \Rightarrow 4x^2 = \mathrm{e}^y - 1 \Rightarrow \mathrm{f}(y) = \frac{1}{2}\sqrt{\mathrm{e}^y - 1}$	**E1**	**2.1**	Sufficient working for $\mathrm{f}(y) = \frac{1}{2}\sqrt{\mathrm{e}^y - 1}$
$\lambda = \ln\left(1+4\left(\frac{1}{2}\right)^2\right) = \ln 2$	**E1**	**2.1**	Sufficient working for top limit of integral
	[3]		
$\mathrm{e}^y = \sec^2\theta \Rightarrow \mathrm{d}y = 2\tan\theta\,\mathrm{d}\theta$	**M1**	**3.1a**	Allow for any genuine attempt to differentiate the given substitution and express integral entirely in terms of θ
$\text{Area} = 2\int_0^{\frac{1}{4}\pi} \sqrt{\sec^2\theta - 1}\,\tan\theta\,\mathrm{d}\theta = 2\int_0^{\frac{1}{4}\pi} \tan^2\theta\,\mathrm{d}\theta$	**A1**	**2.2a**	**AG**; must show evidence for change of limits
	[2]		

$\text{Area} = 2\int_0^{\frac{1}{4}\pi}(\sec^2\theta - 1)\,d\theta$	M1	3.1a	Reducing to form $\int(a\sec^2\theta + b)\,d\theta$
$= 2[\tan\theta - \theta]_0^{\frac{1}{4}\pi} = 2\left\{\left(\tan\frac{1}{4}\pi - \frac{1}{4}\pi\right) - (\tan 0 - 0)\right\}$	A1ft	1.1	Correctly integrating their $a\sec^2\theta + b$ with correct use of limits
$= 2\left(1 - \frac{1}{4}\pi\right)$	A1 [3]	1.1	

Qu 37… OCR A Practice Papers Set 4, Paper 3, Question 10. (Link to question)

Acceleration component $= g\sin 30°$	B1	1.2	Correct acceleration component seen	
$v_M^2 = 4.2^2 + 2(g\sin 30°)x$	M1	3.3	Use of $v^2 = u^2 + 2as$ for the motion from A to M	x is the distance AM and v_M is the speed of P at M
$R = mg\cos 30°$	B1	3.3	Resolving perpendicular to the plane	R is the normal contact force between P and the plane, m is the mass of P
$F = \frac{\sqrt{3}}{6}mg\cos 30°$	M1	3.4	Use of $F = \mu R$ for the motion of P between M and B	
$mg\sin 30° - F = ma$	M1*	3.3	Use of Newton's 2nd Law for the motion of P between M and B	
$12.6^2 = v_M^2 + 2g\left(\sin 30° - \frac{\sqrt{3}}{6}\cos 30°\right)(20 - x)$	M1dep*	3.4	Correct use of $v^2 = u^2 + 2as$ for the motion from M to B with their a and correct s	
$12.6^2 = 4.2^2 + 2(g\sin 30°)x$ $+ 2g(20 - x)\left(\sin 30° - \frac{\sqrt{3}}{6}\cos 30°\right)$	M1	2.1	Substitute their expression for v_M to obtain an equation in x only	
$x = 8.8$ so the distance AM is 8.8 m	A1 [8]	2.2a	BC	

Qu 38… OCR A Practice Papers Set 2, Paper 2, Question 6. (Link to question)

(i)		**DR**			
		$\tan\frac{\pi}{12} = \tan(\frac{\pi}{3} - \frac{\pi}{4})$	M1	3.1a	Any correct use of double angle formula
		$= \frac{\sqrt{3}-1}{1+\sqrt{3}}$ oe	A1	1.1a	Any correct expression for t (or correct QE)
		$= \frac{\sqrt{3}-1}{1+\sqrt{3}} \times \frac{\sqrt{3}-1}{\sqrt{3}-1}$	M1	1.2	Attempts rationalising (or solve their QE)
		$= \frac{4-2\sqrt{3}}{2}$			This form seen (or both roots)
		$= 2 - \sqrt{3}$ **(AG)**	A1 [4]	2.1	and correct answer alone
(ii)		**DR**			
		$\frac{\sqrt{3}}{2}\sin 3A - \frac{1}{2}\cos 3A = \frac{1}{4}$	M1	1.1a	
		$\sin(3A - 30°) = \frac{1}{4}$	A1	3.1a	
		$3A - 30° = 14.5$	M1	1.1	Use of $\sin^{-1}$ both sides
		$A = 14.8°$	A1	1.1	
		or $3A - 30° = 165.5$ $A = 65.2$ (1 dp)	B1	2.4	
		or $3A - 30° = (14.5 + 360)°$	M1	3.1a	
		$A = 134.8°$	A1f [7]	2.1	ft their 14.8° + 120°

Qu 39… OCR A Practice Papers Set 2, Paper 3, Question 5. (Link to question)

DR Attempt product rule for y	M1	3.1a	Attempt must be of the form $(ax+b)e^{-x} \pm (cx^2+dx)e^{-x}$	
$y' = (4x-3)e^{-x} - (2x^2-3x)e^{-x}$	A1	1.1	Correct derivative, in any form	
$y' = 0 \Rightarrow (4x-3)-(2x^2-3x) = 0$	M1	2.1	Set $y' = 0$ and eliminate exponentials	
Obtain quadratic in x and attempt to solve	M1	1.1	Dependent on both previous M marks	$2x^2-7x+3=0$
$x = \frac{1}{2}, \quad x = 3$	A1	1.1	Correct values from correct equation	
$-e^{-\frac{1}{2}} \le y \le 9e^{-3}$	A1	2.5	Correct range, including correct inequality signs and either y, f or f(x) used for range notation (not x)	Allow 'closed interval' notation $[-e^{-\frac{1}{2}}, 9e^{-3}]$
	[6]			
Use integration by parts with $u = 2x^2-3x$ and $v' = e^{-x}$	M1	1.1	Must obtain result $f(x) \pm \int g(x)\,dx$	
$\int (2x^2-3x)e^{-x}\,dx = -(2x^2-3x)e^{-x} + \int (4x-3)e^{-x}\,dx$	A1	1.1		
Attempt parts again with $u = ax+b$ and $v' = e^{-x}$	M1	1.1	Dependent on previous M mark	
$\int (2x^2-3x)e^{-x}\,dx = -(2x^2+x+1)e^{-x} \quad (+c)$	A1	1.1	oe; accept unsimplified (but all bracketing must be correct)	
$2x^2-3x = 0 \Rightarrow x = \frac{3}{2} \quad$ (and $x=0$)	B1	3.1a		
Correct use of correct limits	M1	1.1	Dependent on both previous M marks	
Integral is $1-7e^{-\frac{3}{2}} < 0$ so area is $7e^{-\frac{3}{2}}-1$	A1	2.2a		
	[7]			

Qu 40… OCR A Sample Assessment Paper, Maths & Statistics, Question 12. (Link to question)

$p = 0.1511$ to 4 s.f.	B1	3.1b		**OR** **B1** $p = 0.1511$ to 4 s.f.
$X \sim \text{Bin}(10000, 0.1511)$	M1	3.3	soi	**B1** $X \sim N(1511, 1283^2)$
$np = 1511 \quad np(1-p) = 1283$			Both; allow 3 s.f.	
$1511 + 1.96 \times \sqrt{1283}$ (or $1511 + 2 \times \sqrt{1283}$)	M1	3.4	their'np'$+ 2 \times \sqrt{\text{their'}np(1-p)\text{'}}$ or their'np'$+1.96 \times \sqrt{\text{their'}np(1-p)\text{'}}$	**M1** $P(X < m) = 0.975$ Then use inverse normal to find…
$=1581$ (or 1583)	A1 FT	1.1	FT *their* 3sf or better values	**A1 FT** 1581.203931… BC
Minimum m is 1581	A1	1.1	Conclusion in context Allow 1580 to 1585	**A1** Minimum m is 1581
	[5]			

Qu 41… OCR, A2 Paper 2, 2018, Question 5. (Link to question)

Let $S = n^2$				$853 = m^2 - n^2$ & $m - n = 1$
$\Rightarrow$ Other square number is $(n+1)^2$	M1	3.1a	or Other square number is $(\sqrt{S}+1)^2$	$\Rightarrow 853 = m + n$
$\Rightarrow 853 = (n+1)^2 - n^2 = 2n+1$	M1	2.2a	$\Rightarrow 853 = (\sqrt{S}+1)^2 - S = 2\sqrt{S}+1$	$\Rightarrow 853 = 2n+1$
$\Rightarrow n = 426$	A1	1.1	$\Rightarrow \sqrt{S} = 426$	$\Rightarrow n = 426$
$\Rightarrow S = 181476$	A1	3.2a	$\Rightarrow S = 181476$ $m-n=1, \quad m+n=853$ M1 $2m = 854$ M1 $m = 427 \quad n = 426$ A1 $n^2 = 181476$ A1	$\Rightarrow S = 181476$ T & I: 426 seen M1M1A1 $S = 181476$ A1

Qu 42… OCR Practice Papers, Set 1, Paper 1, Question 12. (Link to question)

$\left(\frac{7}{2}, \frac{\pi}{2}\right), \left(2\sqrt{3}, \frac{\pi}{3}\right), \left(2\sqrt{3}, \frac{2\pi}{3}\right)$ but be sure to discount $\sin y = -\frac{\sqrt{3}}{2}$ as $x < 0$ isn't allowed

DR $\sin y + x\cos y\frac{dy}{dx} - 2\sin 2y\frac{dy}{dx} = 0$	**B1** **M1** **A1**	**1.1a** **1.1** **1.1**	Correct derivatives of cos*y* and $-2\sin 2y$ Attempt use of product rule for *x*sin*y* Obtain correct derivative	
$\frac{dy}{dx} = \frac{\sin y}{2\sin 2y - x\cos y}$				
$2\sin 2y - x\cos y = 0$ $4\sin y\cos y - x\cos y = 0$ $\cos y(4\sin y - x) = 0$ so $\cos y = 0$ or $x = 4\sin y$	**M1** **M1**	**3.1a** **3.1a**	Rearrange and use denominator = 0 Use $\sin 2y = 2\sin y\cos y$ and attempt solution	
$\cos y = 0$ gives $(\frac{7}{2}, \frac{1}{2}\pi)$	**A1**	**2.1**	Obtain $(\frac{7}{2}, \frac{1}{2}\pi)$	
$x = 4\sin y$ gives $4\sin^2 y + \cos 2y = 2.5$ $4\sin^2 y + 1 - 2\sin^2 y = 2.5$ $\sin y = \pm\frac{1}{2}\sqrt{3}$	**M1**	**3.1a**	Substitute $x = 4\sin y$ into original equation and attempt to solve	Including use of correct identity
$\sin y = \frac{1}{2}\sqrt{3}$ gives $(2\sqrt{3}, \frac{1}{3}\pi)$ and $(2\sqrt{3}, \frac{2}{3}\pi)$	**A1**	**3.2a**	Obtain one correct solution	
$\sin y = -\frac{1}{2}\sqrt{3}$ gives $x < 0$, so no valid solutions	**A1**	**2.4**	Obtain both correct roots	Must discount $\sin y = -\frac{1}{2}\sqrt{3}$
	[9]			

Qu 43… OCR 2019, Paper 3, Question 6. (Link to question)

$\frac{2x-1}{(2x+3)(x+1)^2} \equiv \frac{A}{2x+3} + \frac{B}{x+1} + \frac{C}{(x+1)^2}$	**B1**	**3.1a**	Correct form for partial fractions – may be awarded later or implied by later working	Check carefully for their labelling of their *A*, *B* and *C*
$2x-1 \equiv A(x+1)^2 + B(2x+3)(x+1) + C(2x+3)$	**M1***	**1.1a**	Allow sign errors only – this mark can be implied by at least one correct value www	
$x=-1 \Rightarrow C=-3$	**A1**	**1.1**	www	
$x=-\frac{3}{2} \Rightarrow A=-16$	**A1**	**1.1**	www	
$x=0 \Rightarrow B=8$	**A1**	**1.1**	www $-\frac{16}{2x+3} + \frac{8}{x+1} - \frac{3}{(x+1)^2}$	
$\int\left(\frac{-16}{2x+3} + \frac{8}{x+1} - \frac{3}{(x+1)^2}\right)dx$ $= a\ln(2x+3) + b\ln(x+1) + c(x+1)^{-1}$	**M1dep***	**2.1**	Any non-zero values for *a*, *b* and *c* (from correct form of pf – no additional terms) – allow use of modulus instead of brackets throughout – condone omission of brackets throughout if recovered later	All signs may have been swapped (in advance of calculating area)
$= -8\ln(2x+3) + 8\ln(x+1) + 3(x+1)^{-1}$	**A1**	**1.1**	All correct, may be un-simplified	Limits not required for this or previous mark
$= \left(-8\ln 4 + 8\ln\frac{3}{2} + 2\right) - \left(-8\ln 3 + 3\right)$	**M1dep***	**3.1a**	Correct use of the correct limits of 0 and $\frac{1}{2}$ Allow $\pm\left(F(\frac{1}{2}) - F(0)\right)$	Dependent on all previous M marks
$= 8\ln\frac{3}{2} + 8\ln 3 - 8\ln 4 - 1 = 8\ln\left(\frac{\frac{3}{2}\times 3}{4}\right) - 1$	**M1**	**2.1**	Correctly combining their log terms to a single log term– dependent on correct use of the correct limits and two log terms only (of the form $a\ln(2x+3) + b\ln(x+1)$)	Must be using 0.5 and 0 as limits
Integral is $8\ln\frac{9}{8} - 1 \Rightarrow$ Area $= 1 + 8\ln\frac{8}{9}$	**A1**	**3.2a**	Final answer must be positive (as it is an area) www	$p=1,\ q=8,\ r=\frac{8}{9}$
	[10]			

Qu44… OCR Practice papers Set 2, Paper 2, Question 7 (Link to question)

$V = 100h \Rightarrow \frac{dv}{dh} = 100$	M1	3.4		
$\frac{dv}{dt} = \frac{dv}{dh} \times \frac{dh}{dt} = 100\frac{dh}{dt}$ $[= 25 - 4h^2]$	A1	1.2		
$\Rightarrow 25 - 4h^2 = 100\frac{dh}{dt}$ oe	M1	3.1b	Equate $25 - 4h^2$ to their $\frac{dv}{dh} \times \frac{dh}{dt}$	
$\Rightarrow \int_0^2 \frac{1}{25-4h^2}\,dh = \int_0^t \frac{1}{100}\,dt$	M1	2.5	Attempt integration with correct denominator on LHS	
$\Rightarrow \frac{1}{10}\int_0^2 \frac{1}{5+2h} + \frac{1}{5-2h}\,dh = \int_0^t \frac{1}{100}\,dt$	M1 A1	3.4 2.1	Attempt partial fractions with correct denominators on LHS Correct partial fractions	
$\Rightarrow \frac{1}{10} \times \frac{1}{2}[\ln(5+2h) - \ln(5-2h)]_0^2 = \frac{t}{100}$	M1	1.2	Correct integral; ignore limits	
$\Rightarrow 5\ln 9 = t$ oe Time when depth is 2 cm is 11.0 seconds (3 sf)	A1 A1 [9]	2.2a 3.2a	Any correct numerical expression for t Allow 11 seconds	10.9861…

Qu 45… OCR A2 Paper 1, 2019, Question 12 (Link to question)

(a)	(i)	Show A in third quadrant, with length of 8 and relevant angle marked on given axes	B1 [1]	1.2	Allow any correct angle	Condone A being located by correct **i** and **j** components instead of length and angle – could be stated as a coordinate or values marked on the axes
	(ii)	$x = 8\cos 240° = -4$ $y = 8\sin 240° = -4\sqrt{3}$	M1	1.1a	Attempt both components from magnitude of 8 and an angle	Could use 60° (no need to consider whether positive or negative for this mark) Allow M1 for $8\cos\theta$ and $8\sin\theta$ attempted Condone a value for θ that may not be consistent with their diagram Max of M1 only, if A incorrect on diagram
		A is $-4\mathbf{i} - 4\sqrt{3}\,\mathbf{j}$	A1	1.1	Obtain one correct component	Condone eg $x = -4$ for $-4\mathbf{i}$
			A1 [3]	1.1	Obtain fully correct position vector	Allow 6.93, or better, for $4\sqrt{3}$ A0 if coordinate or column vector
(b)		area $= 0.5 \times 8 \times 6 \times \sin 120°$	M1	3.1a	Attempt area of triangle, using correct formula	M0 if 240° used Allow plausible angle ie 30°, 60°, 120°, 150° Allow other incorrect angles as long as explicit on their diagram Allow multi-step methods as long as fully correct method
		$= 12\sqrt{3}$	A1 [2]	1.1	Obtain $12\sqrt{3}$	Must be exact www eg M1A0 for $12\sqrt{3}$ from A in second quadrant M1A0 for $12\sqrt{3}$ from using 60° without justification that $\sin 120° = \sin 60°$
(c)		$6\mathbf{i} - (-4\mathbf{i} - 4\sqrt{3}\,\mathbf{j})$	M1	3.1a	Attempt $6\mathbf{i}$ – (their OA)	Allow BOD for $6\mathbf{i} - -4\mathbf{i} - 4\sqrt{3}\,\mathbf{j}$, even if final answer is not commensurate with 'invisible brackets'
		C is $10\mathbf{i} + 4\sqrt{3}\,\mathbf{j}$	A1 [2]	1.1	Obtain $10\mathbf{i} + 4\sqrt{3}\,\mathbf{j}$	Allow 6.93, or better, for $4\sqrt{3}$ **SC** B1 for $2\mathbf{i} - 4\sqrt{3}\,\mathbf{j}$ or $-10\mathbf{i} - 4\sqrt{3}\,\mathbf{j}$ ie a valid parallelogram having misinterpreted *OABC*

Qu 46… OCR A2 Paper 2, 2019, Question 9 (Link to question)

(a)	(i)	0.761 or 0.762 (3 sf)	B1 [1]	1.1	**BC** Allow 0.76	
(a)	(ii)	62.0 (3 sf)	B1 [1]	1.1	**BC** Allow 62 or 61.9	Allow $m \geq 62.0$
(a)	(iii)	Use of $\bar{X}$ eg "$\bar{X}$" or "mean" or $\frac{18}{10}$ or $\sqrt{\frac{18}{10}}$	M1	1.1a	$\mu = 550$ seen or implied	
		$\bar{X} \sim \mathrm{N}(55, \frac{18}{10})$	M1	3.3	$\Sigma X \sim \mathrm{N}(550, 180)$ Correct	May be implied
		$\mathrm{P}(\bar{X} < \frac{530}{10})$ **dep** $\sigma^2 = \frac{18}{10}$	M1	3.4	$\mathrm{P}(\Sigma X < 530)$ **dep** $\sigma^2 = 180$	Stated or implied
		$= 0.0680$ (3 sf)	A1 [4]	1.1	$= 0.0680$ (3 sf) **Allow 0.068**	Correct answer from limited (or no) working: M1M1M1A1
(b)		$\mathrm{P}(Y < 72) = 0.75$ / $\mathrm{P}(Y < 62) = 0.25$	M1	3.1b	oe May be implied, eg on diagram	NB $\mathrm{P}(62<Y<72) = 0.5$ no mks yet
		$\Phi^{-1}(0.75)$ or 0.674 / $\Phi^{-1}(0.25)$ or -0.674	M1	2.4	± 0.674 implies M1M1 **Allow 0.67**	
		$\frac{72-67}{\sigma}$ / $\frac{62-67}{\sigma}$	M1	2.1		M1M1M1 may be implied by A1
		$\frac{72-67}{\sigma} = 0.674$ / $\frac{62-67}{\sigma} = -0.674$	A1	1.1	oe, eg $5 = 0.674\,\sigma$ A1 for correct equn, allow 0.67	Must be seen
		$\sigma = 7.41$ or 7.42 (3 sf)	A1	1.1	**SC correct answer with no working or irrelevant working: SC B3** (because "determine" rather than "find")	or SC B2 if correct to 2 sf
		Trial and Improvement $\Phi^{-1}(0.75)$ or 0.674 or $\Phi^{-1}(0.25)$ or -0.674	M2		May be implied	
		eg $\sigma = 8$: $67 - 8\times0.674 = 61.6$	M1		At least one correct trial	
		$\sigma = 7$: $67 - 7\times0.674 = 62.3$	A1		Trials leading to values either side of 62	
		$\sigma = 7.41$: $67 - 7.41\times0.674 = 62.0 \Rightarrow \sigma = 7.41$ or $\sigma = 7.42$: $67 - 7.42\times0.674 = 62.0 \Rightarrow \sigma = 7.42$	A1		Correct trial using $\sigma = 7.41$ or 7.42 and conclusion $\sigma = 7.41$ or 7.42	

Ans 47… AQA Level 2 Certificate in Further Maths, Paper 2, 2017, Question 24 (Link to question)

Alternative method 1

$12(x^2 - 5x)$ … or $12(x - 2.5)^2$ …	M1
$12\{(x - 2.5)^2 - 2.5^2\}$… or $12(x - 2.5)^2 - 75$ …	M1dep
$12(x - 2.5)^2 - 12 \times 2.5^2 + 5$ or $12(x - 2.5)^2 - 70$	M1dep
$12\left(\frac{2x-5}{2}\right)^2 - 12 \times 2.5^2 + 5$	M1dep
$3(2x - 5)^2 - 70$ or $a = 3$ $b = 2$ $c = -5$ $d = -70$ or $3(5 - 2x)^2 - 70$ or $a = 3$ $b = -2$ $c = 5$ $d = -70$	A1

Alternative method 2

$3(4x^2 - 20x)$ … or $3(2x - 5)^2$ …	M1
$3\{(2x - 5)^2 - 5^2\}$ … or $3(2x - 5)^2 - 75$ …	M1dep
$3\{(2x - 5)^2 - 5^2\} + 5$	M1dep
$3(2x - 5)^2 - 3 \times 5^2 + 5$	M1dep
$3(2x - 5)^2 - 70$ or $a = 3$ $b = 2$ $c = -5$ $d = -70$ or $3(5 - 2x)^2 - 70$ or $a = 3$ $b = -2$ $c = 5$ $d = -70$	A1

Ans 48… OCR A2 Paper 2, 2020, Question 3 (Link to question)

(a)	(i)	$1 + (-2)(-x) + \frac{(-2)(-3)}{2!}(-x)^2 + \frac{(-2)(-3)(-4)}{3!}(-x)^3$	M1	1.1	Correct expressions for at least three terms. May be implied
		$\equiv 1 + 2x + 3x^2 + 4x^3$	A1 [2]	1.1	cao
(a)	(ii)	$(n + 1)\, x^n$	B1 [1]	2.2a	Allow $x^n = (n + 1)\, x^n$
(b)		$\frac{1}{1-x}$ oe	B1 [1]	1.1	
(c)		$2 + 3x + 4x^2 + 5x^3 + \ldots$ $= 1 + x + x^2 + x^3 + \ldots$ $+ 1 + 2x + 3x^2 + 4x^3 + \ldots$	M1	3.1a	
		$= \frac{1}{1-x} + \frac{1}{(1-x)^2} = \frac{(1-x)+1}{(1-x)^2}$	M1	3.1a	Their (b)(i) + $\frac{1}{(1-x)^2}$ and attempt single term
		$= \frac{2-x}{(1-x)^2}$	A1 [3]	1.1	cao Unsupported answer, no marks
		$(a - x)(1 - x)^{-2}$	M1		
		$a + 2ax + 3ax^2 + 4ax^3 + \ldots$ $- (x + 2x^2 + 3x^3 + 4x^4 + \ldots..)$ $a = 2$ $\frac{2-x}{(1-x)^2}$	M1		
		Justification for all terms up to infinity	A1		
					NB other correct methods exist

Ans 49… OCR A2 Paper 1, 2021, Question 11 (Link to question)

(a)		$2u\,du = 2x\,dx$	B1	1.1a	Any correct expression linking du and dx	Could be $du = \frac{1}{2}2x\left(x^2+3\right)^{-\frac{1}{2}}dx$ or equiv in terms of u
		$\int \frac{4u\left(u^2-3\right)}{\sqrt{u^2}}du$	M1*	2.1	Attempt to rewrite integrand in terms of u	Not just $dx = du$, unless from a clear attempt at du eg using $u = x+\sqrt{3}$
		$\int\left(4u^2-12\right)du$	A1	1.1	Obtain correct integrand	Allow unsimplified expression
		$\frac{4}{3}u^3-12u(+c)$	M1dep*	1.1	Attempt integration	Simplify to form that can be integrated, then increase all powers by 1
		$\frac{4}{3}u(u^2-9)+c=\frac{4}{3}\left(x^2-6\right)\sqrt{x^2+3}+c$ **A.G.**	A1	2.1	Obtain given answer, with at least one intermediate step seen	Need evidence of common factor (in terms of u or x) being taken out Condone omission of $+c$
			[5]			
(b)		**DR** $\frac{4}{3}\left(\left(-5\times2\right)-\left(-6\times\sqrt{3}\right)\right)$	M1	2.1	Attempt to use limits $x = 0$ and $x = 1$, or $u = \sqrt{3}$ and $u = 2$ in integral in terms of u	Correct order and subtraction Attempt to use both limits in their integral to give two terms DR so just stating decimal area is M0 Either using answer from (a) or their integration attempt with +2 or +3
		$=\frac{4}{3}\left(6\sqrt{3}-10\right)$ or 0.523	A1	1.1	Obtain correct area under curve	Accept exact (inc unsimplified) or decimal Using +2 gives $\frac{4}{3}\left(4\sqrt{2}-3\sqrt{3}\right)$ or 0.614
		$\frac{dy}{dx}=\frac{12x^2\left(x^2+3\right)^{\frac{1}{2}}-4x^3.2x.\frac{1}{2}\left(x^2+3\right)^{-\frac{1}{2}}}{x^2+3}$	M1	3.1a	Attempt derivative using the quotient rule	Or equiv with product rule Need difference of two terms in numerator, at least one term correct, but allow subtraction in incorrect order Using either +2 or +3 equation
			A1	1.1	Obtain correct, unsimplified, derivative	With either +2 or +3
		at $x = 1$, $m = \frac{11}{2}$ hence $m' = -\frac{2}{11}$	M1	2.1	Attempt gradient of normal at $x = 1$	Substitute $x = 1$ and use negative reciprocal Using +2 gives $m' = -\frac{3}{32}\sqrt{3}$ Can be with m found BC
		$y-2=-\frac{2}{11}(x-1)$ when $y = 0$, $x = 12$	M1	1.1	Attempt to find point of intersection of normal with x-axis	Attempt equation of normal with their gradient and either (1, 2) or $\left(1,\frac{4}{3}\sqrt{3}\right)$, and then use $y = 0$ to find x intersection
		area $= 8\sqrt{3}-\frac{40}{3}+11$ $= 8\sqrt{3}-\frac{7}{3}$	A1	3.1a	Obtain correct area Allow any exact (including unsimplified) or decimal equivalent	From combining a correct area under curve and a correct area of triangle (either 11 or $\frac{64}{9}\sqrt{3}$), even if inconsistent Can still get A1 following M0 for area under curve BC and/or m found BC
			[7]			

Ans 50… Edexcel Paper 3, 2022, Question 3 (Link to question)

[A = no. of bulbs that grow into plants with blue flowers,] $A \sim B(40, 0.36)$	M1	3.3
$p = P(A \geq 21) = 0.0240$	A1	1.1b
C = no. of bags with more than 20 bulbs that grow into blue flowers, $C \sim B(5, p)$	M1	3.3
So $P(C \leq 1) = 0.9945\ldots$ awrt 0.995	A1	1.1b
	(4)	
[T ~ number of bulbs that grow into blue flowers] $T \sim B(n, 0.36)$		
T can be approximated by N($0.36n$, $0.2304n$)	B1	3.4
$P\left(Z < \dfrac{244.5 - 0.36n}{\sqrt{0.2304n}}\right) = 0.9479$	M1	1.1b
$\dfrac{244.5 - 0.36n}{\sqrt{0.2304n}} = 1.625$ or $\dfrac{244.5 - 0.36x^2}{0.48x} = 1.625$	M1 A1	3.4 1.1b
$0.36n + 0.78\sqrt{n} - 244.5 = 0$	M1	1.1b
$n = 625$	A1cso	1.1b
	(6)	
(10 marks)		

Ans 51… OCR A2 Paper 1, 2021, Question 7 (Link to question)

(a)		$2x\ln x + \dfrac{x^2 - 2}{x}$	M1	3.1a	Attempt differentiation using product rule	May expand first to give $2x\ln x + \dfrac{x^2}{x} - \dfrac{2}{x}$ (allow middle term as just x)
		$2x\ln x + \dfrac{x^2 - 2}{x} = 0$ $2x^2\ln x + x^2 - 2 = 0$ **A.G.**	A1 **[2]**	1.1	Equate to 0 and obtain given answer	Must be equated to 0 **before** clearing the fractions Must be equation ie … = 0
(b)		$f'(x) = 4x\ln x + 2x^2.\frac{1}{x} + 2x$	B1	1.1	Correct derivative seen	Allow simplified middle term of $2x$
		$x_{n+1} = x_n - \dfrac{2x_n^2\ln x_n + x_n^2 - 2}{4x_n\ln x_n + 2x_n^2.\frac{1}{x_n} + 2x_n}$	M1	1.1	Use correct Newton-Raphson formula, with numerator correct and their derivative in the denominator	Allow fractional term without subscripts **SC** Condone use of N-R on $(x^2 - 2)\ln x$
		$x_{n+1} = \dfrac{x_n(4x_n\ln x_n + 4x_n) - (2x_n^2\ln x_n + x_n^2 - 2)}{4x_n\ln x_n + 4x_n}$ $x_{n+1} = \dfrac{4x_n^2\ln x_n + 4x_n^2 - 2x_n^2\ln x_n - x_n^2 + 2}{4x_n\ln x_n + 4x_n}$	M1	1.1	Attempt rearrangement into single fraction with brackets expanded	Allow without subscripts N-R not necessarily correct, but must be recognisable attempt **SC** Rearrange their N-R on $(x^2 - 2)\ln x$
		$x_{n+1} = \dfrac{2x_n^2\ln x_n + 3x_n^2 + 2}{4x_n(\ln x_n + 1)}$ **A.G.**	A1 **[4]**	2.1	Obtain given answer, with no errors seen	Subscripts needed on RHS at least one step before AG LHS needs x_{n+1} seen

Ans 52… OCR A2 Paper 1, 2019, Question 7 (Link to question)

DR GP, with $a = 15$, $r = 0.6$	**B1**	**3.1a**	Identify GP; correct a and r soi	Stated or implied by use in equation
$S_\infty = \dfrac{15}{1-0.6}$	**B1**	**1.1a**	Correct S_∞, with their a and r	Must be using correct formula Allow $a = 25$, even if not stated explicitly before formula is used Allow $a = 15$, $r = 0.6$ and $\dfrac{a}{1-r} = 37.5$ to imply B1 B0 for 37.5 with no evidence
$S_N = \dfrac{15(1-0.6^N)}{1-0.6}$	**B1**	**1.1a**	Correct S_N, with their a and r	Must be using correct formula Allow $a = 25$, even if not stated explicitly before formula is used
$37.5 - 37.5(1 - 0.6^N) < 10^{-4}$ $37.5 \times 0.6^N < 10^{-4}$	**M1**	**3.1a**	Link $S_\infty - S_N$ to 10^{-4} and attempt to rearrange	As far as $p \times 0.6^N < q$ (q possibly 2 terms) Condone either '=' or any inequality sign M0 for eg $15 \times 0.6^N = 9^N$ or $1 - 0.6^N = 0.4^N$
$0.6^N < 2.67 \times 10^{-6}$	**A1**	**1.1**	Correct equation in useable form	Any linking sign If using logs on 37.5×0.6^N then the product must be dealt with correctly to get both this A1 and the following M1
$N > \log_{0.6}(2.67 \times 10^{-6})$	**M1**	**2.1**	Use logs to solve equation	Either take logs on both sides (consistent base), drop power and rearrange, or take $\log_{0.6}$ on RHS (could be base other than 0.6 if error when manipulating indices) Any linking sign, including an inequality sign that does not change direction
$N > 25.125\ldots$	**A1**	**1.1**	Obtain 25.1 / 25 / 26	Any sign No evidence of use of logs – award B1 instead of M1A1 (and can still get final A1)
hence $N = 26$	**A1**	**2.2a**	Obtain $N = 26$ only (or eg N is 26) www	A0 if inequality eg $N \geq 26$ A0 if it comes from an incorrect inequality eg $N < 25.125\ldots$ unless recovered by testin at least one relevant integer value If solving an equation then must test at least one integer value to justify N
	[8]			If either or both of the second and third B marks are not awarded for lack of **DR** then all other marks are available Answer only is 0/8 T&I could get some credit depending what equations are shown, but question requires both **DR** and an algebraic method so a final answer of 26 will not get credit

Ans 53… OCR A2 Paper 2, 2021, Question 5 (Link to question)

(a)		Midpoint AB is (3.5, 5.5); Gradient $AB = -\frac{1}{7}$ Gradient of perpendicular bisector $-1/(-\frac{1}{7})$ $y - 5.5 = 7(x - 3.5)$ oe ISW	**B1** **M1** **A1**	Both. Allow midpoint $= (\frac{0+7}{2}, \frac{6+5}{2})$ ISW (= 7) cao. Correct answer, no working or inadequate working: SC B2
		Midpt AB is (3.5, 5.5); Gradient $AB = -\frac{1}{7}$ $(y = 7x + c)$ $5.5 = 7\times 3.5 + c$ $y = 7x - 19$	**B1** **M1** **A1**	Both ft their midpt and gradient, NOT $-\frac{1}{7}$ cao. Any correct form
		$x^2 + (y-6)^2 = (x-7)^2 + (y-5)^2$ $-12y + 36 = -14x - 10y + 49 + 25$ ISW	**M1** **M1** **A1**	Attempt expansion cao. Any correct form eg $y = 7x - 19$
			[3]	
(b)		Perpendicular bisector of BC is $x + 7y - 17=0$ OR of CA is $4y = 3x - 1$	**B1**	Any correct form for another perp bisector
		Example method, perp bisectors of AB & BC: $x + 7(7x - 19) - 17 = 0$ $(\Rightarrow x = 3)$	**M1**	Attempt solve simultaneously equations of two perpendicular bisectors. Can be implied
		Alternative method for 1st two marks Grad BC is 7 so BC & AB perpendicular Hence AC is a diameter	**M1** **B1**	
		Centre is (3, 2) eg Radius$^2 = 3^2 + (6-2)^2 = 25$	**B1** **M1**	cao. NB, if centre = (3, 2) without clear working, B0M0B1 Correct method for r^2 or r using their centre & A or B or C
		Equn of circle is $(x-3)^2 + (y-2)^2 = 25$ or $x^2 - 6x + y^2 - 4y = 12$ oe	**A1ft**	ISW. ft their centre & radius, dep both M1 marks
			[5]	

Ans 54… OCR A2 Paper 1, 2019, Question 12 (Link to question)

(a)		$u = 3x^2, y = a^u$ $u' = kx, y' = a^u \ln a$	**M1**	**1.1a**	Attempt use of chain rule, with $y' = a^u \ln a$	Correct use of chain rule, with a^u correctly differentiated No credit for just stating $\frac{d}{dx}\left(a^x\right) = a^x \ln a$ unless clearly used in a correct chain rule
		$\frac{dy}{dx} = 6xa^u \ln a$	**A1**	**2.1**	Use chain rule to obtain correct derivative	Product of their u' and y' May still be in terms of x and u
		$\frac{dy}{dx} = 6xa^{3x^2} \ln a$ **A.G.**	**A1**	**2.4**	Obtain correct derivative	Now fully in terms of x
			[3]			**OR** M1 – attempt differentiation of $\ln y = 3x^2 \ln a$ A1 – obtain $\frac{1}{y}\frac{dy}{dx} = 6x \ln a$ A1 – obtain correct derivative **A.G.** **OR** M1 – attempt differentiation of $y = e^{(3\ln a)x^2}$ A1 – obtain $\frac{dy}{dx} = (6x \ln a)e^{(3\ln a)x^2}$ A1 – obtain correct derivative **A.G.**

(b)		when $x = 1$, $m = 6a^3\ln a$	B1	3.1a	Correct gradient of tangent soi	
		$y - a^3 = 6a^3\ln a(x - 1)$ $0 - a^3 = 6a^3\ln a\,(\frac{1}{2} - 1)$	M1*	1.1a	Use $(\frac{1}{2}, 0)$ in attempt at equation of tangent through $(1, a^3)$, or vice versa	OR $\frac{0-a^3}{\frac{1}{2}-1} = 6a^3\ln a$ M0 if gradient still in terms of x Allow BOD if something other than $x = 1$ was used to find the gradient
		$a^3 = 3a^3\ln a$ $a^3(3\ln a - 1) = 0$ $a = e^{\frac{1}{3}}$	M1d*	1.1a	Attempt to find a	Must go as far as attempting a value for a Condone cancelling by a^3 rather than factorising
			A1	1.1	Obtain correct value for a	Any equivalent exact form eg $a = \sqrt[3]{e}$
			[4]			
(c)		$u = 6x\ln a$, $v = a^{3x^2}$	M1*	3.1a	Attempt use of product rule	On given $\frac{dy}{dx}$, with both parts a function of x If using parts as $u = 6x\,a^{3x^2}$ and $v = \ln a$, then must use product rule properly on u (but condone $\ln a$ differentiating to $\frac{1}{a}$)
		$\frac{d^2y}{dx^2} =$ $(6\ln a)(a^{3x^2}) + (6x\ln a)(6xa^{3x^2}\ln a)$ $= a^{3x^2}(6\ln a)(1 + 6x^2\ln a)$	A1 A1	2.1 2.1	At least one term correct Fully correct second derivative	
		$\ln a > 0$ for $a > 1$ $a^{f(x)} > 0$ for all a and x $6x^2 \geq 0$, so $1 + 6x^2\ln a \geq 1$	M1d*	2.3	Consider sign of each term – must consider each component of each term	Possibly factorised, or possibly considered term by term Domains not needed for the M1 Allow BOD if > not ≥
		$\frac{d^2y}{dx^2} > 0$ for all x hence curve is always convex	A1	2.2a	Correct working only	Second derivative must be correct Domains must be seen Inequality signs must be correct throughout
			[5]			
		OR $\frac{dy}{dx} = 2xe^{x^2}$	M1*		Attempt use of product rule	Substitute their a into $\frac{dy}{dx}$ and attempt to differentiate
		$\frac{d^2y}{dx^2} = 2e^{x^2} + 4x^2e^{x^2}$	A1FT		At least one term correct, FT their a, any a	
			A1FT		Fully correct second derivative, FT their a as long as of form e^k	Expect $6ke^{3kx^2} + 36k^2x^2e^{3kx^2}$
		$e^{x^2} > 0$ for all x, so $2e^{x^2} > 0$ $x^2 \geq 0$ for all x, so $4x^2e^{x^2} \geq 0$ hence $2e^{x^2} + 4x^2e^{x^2} > 0$	M1d*		Consider sign of each term – must consider each component of each term	Possibly factorised, or possibly considered term by term Domains not needed for the M1 Allow BOD if > not ≥
		$\frac{d^2y}{dx^2} > 0$ for all x hence curve is always convex	A1		Correct working only	Second derivative must be correct Domains must be seen Inequality signs must be correct throughout

Ans 55… OCR A2 Paper 2, 2020, Question 15 (Link to question)

(a)		**DR** $\frac{15}{64}\times\frac{2^2}{2!}$ oe eg $\frac{15}{64}\times\frac{4}{2}$ $(=\frac{15}{32}$ **AG**)	**B1** **[1]**	**1.1**	Must see this expression and result
(b)		**DR** 2, 2, 5 2, 3, 4 3, 3, 3	**M1**	**3.1a**	Any two seen, with no more than 2 extra different combinations. eg 0, 4, 5 and 0, 5, 4 count as <u>one</u> extra
		$P(X_1 + X_2 + X_3 = 9) =$ $3\times(\frac{15}{32})^2\times\frac{5}{80}+6\times\frac{15}{32}\times\frac{5}{16}\times\frac{5}{32}+(\frac{5}{16})^3$ $0.0412 + 0.1373 + 0.0305$ $3\times\frac{225}{16384}+6\times\frac{375}{16384}+\frac{125}{4096}$ $\frac{675}{16384}+\frac{1125}{8192}+\frac{125}{4096}$ (= 0.209045)	**M1** **M1**	**2.1** **2.1**	M2: $\geq$ 1 correct product actually seen & all three products correct M1: 1 correct product seen or all correct except omission of, or incorrect, multiple(s) or all three results or total correct, but without working
		$P(X_1 + X_2 + X_3 = 9$ and at least 1 X value = 2) $=3\times(\frac{15}{32})^2\times\frac{5}{80}+6\times\frac{15}{32}\times\frac{5}{16}\times\frac{5}{32}$ (= 0.178528)	**M1**	**1.1**	Allow M1 for 1 correct product or omit, or incorrect, multiple(s) or ft their probabilities from their previous calculation
		$\frac{'0.178528'}{'0.209045'}$	**M1**	**2.1**	÷ their attempted probs of correct events
		= 0.854 (3 sf) or $\frac{117}{137}$	**A1**	**2.2a**	
		$P(X_1 + X_2 + X_3 = 9$ and no X value = 2) $=(\frac{5}{16})^3$ (= 0.030518 or $\frac{125}{4096}$)	**M1**		ft their P(3, 3, 3)
		$1-\frac{'0.030518'}{'0.209045'}$	**M1**		÷ their attempted probabilities of correct events & subtract from 1
		= 0.854 (3 sf) or $\frac{117}{137}$	**A1** **[6]**		NB $1-(\frac{5}{16})^3$ alone scores M1
(c)		P(two 2's in nine vales of X) or 0.094466 or ${}^9C_2\times(1-\frac{15}{32})^7\times(\frac{15}{32})^2$	**M1**	**3.1a**	soi eg by ${}^9C_2$ seen
		P(two 2's in nine vales of X) × P(X = 2) or 0.094466 × $\frac{15}{32}$ or ${}^9C_2\times(1-\frac{15}{32})^7\times(\frac{15}{32})^3$	**M1**	**2.1**	soi NB $\left(\frac{17}{32}\right)^7\times\left(\frac{15}{32}\right)^3$ scores 0, unless multiplied by ${}^9C_2$
		0.0443 (3 sf)	**A1** **[3]**	**1.1**	

Ans 56… OCR A2 Paper 2, 2020, Question 7 (Link to question)

(a)		Length of AB oe	**B1** **[1]**	**1.2**	Magnitude of $\overrightarrow{AB}$ or distance from A to B Allow Magnitude of AB Not magnitude of $\lvert\mathbf{a}-\mathbf{b}\rvert$ or magnitude of $\mathbf{a}-\mathbf{b}$ Not distance from a to b Not distance from position vector A to position vector B
(b)		Midpoint of AB oe	**B1** **[1]**	**1.2**	or Halfway between A and B Allow Midpoint of $\overrightarrow{AB}$ Must refer to A and B, not a and b Not Midpoint of the vectors
(c)	**(i)**	$\frac{1}{2}(\mathbf{a}+\mathbf{b})$	**B1** **[1]**	**2.2a**	
(c)	**(ii)**	$\frac{1}{2}\lvert\mathbf{a}-\mathbf{b}\rvert$ oe	**B1** **[1]**	**2.2a**	
(d)		Centre is (3, 2)	**B1**	**1.1**	Allow this mark for (3, 2) or $\begin{pmatrix}3\\2\end{pmatrix}$ or $\frac{1}{2}\begin{pmatrix}6\\4\end{pmatrix}$ oe seen May be implied by answer
		$r^2 = 10$ or $r=\sqrt{10}$ or 3.16 (3 sf)	**B1**	**1.1**	May be implied by answer. Must imply radius
		$(x-3)^2+(y-2)^2=10$	**M1** **A1** **[4]**	**1.1** **1.1**	M1 for $(x-a)^2+(y-b)^2=r^2$ for any non-zero numerical a, b and r A1 for all correct. **ISW**

Ans 57… Edexcel Mock Set 4, Paper 2, Question 10 (Link to question)

$x=t^2,\ y=2t \Rightarrow t^4+4t^2=10t^2+k \Rightarrow t^4-6t^2-k=0$ or $y=2t \Rightarrow x=\frac{y^2}{4} \Rightarrow \frac{y^4}{16}+y^2=\frac{10y^2}{4}+k \Rightarrow y^4-24y^2-16k=0$ or $x=t^2 \Rightarrow y=2\sqrt{x} \Rightarrow x^2+4x=10x+k \Rightarrow x^2-6x-k=0$	M1 A1	3.1a 1.1b
Roots must be real: $b^2-4ac>0 \Rightarrow 6^2+4k>0 \Rightarrow k>-9$ or e.g. $b^2-4ac>0 \Rightarrow 24^2+64k>0 \Rightarrow k>-9$	dM1 A1	3.1a 1.1b
Both roots must be positive so e.g.: $6-\sqrt{36+4k}>0 \Rightarrow k<0$	B1	2.2a
$\{k:k<0\}\cap\{k:k>-9\}$	A1	2.5
	(6)	

(6 marks)

M1: Makes the key step of using the Cartesian equation with the parametric equations to eliminate 2 of the variables
A1: Correct 3TQ in t^2, y^2 or x
dM1: Recognises the condition that $b^2-4ac>0$ as roots must be real and uses this to find the minimum value for k
A1: For $k>-9$ seen as part of their solution
B1: Deduces that as both roots must be positive, $k<0$
A1: Correct range using the correct notation. Allow equivalents e.g. $\{k:-9<k<0\}$, $k\in(-9,0)$

Ans 58… MEI Practice Papers Set 4, Paper 2, Question 12 (Link to question)

(a)		$a=½(17.03-7.47)=4.78$ $c+4.78=17.03$ so $c=12.25$	**B1** **B1** **[2]**	**3.1b** **3.3**	$\sin\theta=1$ for max $\sin\theta=-1$ for min B1 $17.03=a+c,\ 7.47=-a+c$ B1	BC Sufficient reasoning needed to justify given answers
(b)		$\frac{2\pi}{365}\times172+b=\frac{\pi}{2}$ or $\frac{2\pi}{365}\times355+b=\frac{3\pi}{2}$ $b=-1.39$	**M1** **A1** **[2]**	**3.3** **1.1**		
(c)		$t=244$ used in their formula $Y=13.81$ which is fairly close to 13.75 (out by 3.6 minutes)	**M1** **A1** **[1]**	**3.4** **3.5a**	BC	
(d)		$a=8.51$ and $c=12.63$	**B1** **[1]**	**3.5b**		
(e)		New model gives 15.40 hrs, which is not a good fit	**B1** **[1]**	**3.5a**	NB 15.39828…	

Ans 59… MEI Practice Papers Set 4, Paper 2, Question 12 (Link to question)

(a)		$2^{n_1-1}=1024$ $n_1=11$	**M1** **A1** **[2]**	**1.1** **1.1**		
(b)		$r_2=4$ $4^{n_2-1}=1024$ $n_2=6$	**B1** **B1** **[2]**	**1.1** **2.2a**		
(c)		$r_3=\sqrt{2}$ $(\sqrt{2})^{n_3-1}=1024$ $n_3=21$ $S_{21}=1\times\frac{(\sqrt{2})^{21}-1}{\sqrt{2}-1}$ $=2047+1023\sqrt{2}$ or 3490 (3 sf)	**B1** **M1** **A1** **A1FT** **[4]**	**1.1** **3.1a** **2.2a** **1.1**	Other correct answers score similarly, eg $r_3=\sqrt[4]{2}$ $((\sqrt[4]{2})^{n_3-1}=1024$ $n_3=41$ $S_{21}=1\times\frac{(\sqrt[4]{2})^{41}-1}{\sqrt[4]{2}-1}$ 6430 (3 sf)	ft their r_3 and n_3

Ans 60… Edexcel Mock Papers Set 2 (2020), Paper 2, Question 14 (Link to question)

Uses $y=3\sin 2t=6\sin t\cos t$ and attempts to square	M1
$y^2=9x^2\cos^2 t$	A1
Uses $\cos^2 t=1-\sin^2 t$ with $\sin t=\frac{x}{2}$ $y^2=9x^2\left(1-\frac{x^2}{4}\right)$	M1
$y^2=\frac{9}{4}x^2\left(4-x^2\right)$	A1
	(4)
Deduces that the radius of the circle is given by $r^2=x^2+y^2$	M1
$r^2=x^2+\frac{9}{4}x^2\left(4-x^2\right)$	A1
Circle touches curve when r is a maximum so differentiate $r^2=10x^2-\frac{9}{4}x^4\Rightarrow 2r\frac{dr}{dx}=20x-9x^3$ and set $\frac{dr}{dx}=0\Rightarrow x=\sqrt{\frac{20}{9}}$	M1
Finds $r^2=10x^2-\frac{9}{4}x^4$ with their $x=\sqrt{\frac{20}{9}}$	dM1
$r=\frac{10}{3}$	A1
	(5)

Ans 61… adapted from Edexcel Core 3 June 2012, Question 7b (Link to question)

(b) $x = 3\tan 2y \quad \Rightarrow \dfrac{dx}{dy} = 6\sec^2 2y$	M1A1
$\Rightarrow \dfrac{dy}{dx} = \dfrac{1}{6\sec^2 2y}$	M1
Uses $\sec^2 2y = 1 + \tan^2 2y$ and uses $\tan 2y = \dfrac{x}{3}$	
$\Rightarrow \dfrac{dy}{dx} = \dfrac{1}{6(1+(\frac{x}{3})^2)} = (\dfrac{3}{18+2x^2})$	M1A1
	(5)

Ans 62… adapted from Edexcel Sample Paper 2 June 2012, Question 8 (Link to question)

Gradient $AB = -\dfrac{2}{5}$	B1	2.1
y coordinate of A is 2	B1	2.1
Uses perpendicular gradients $y = +\dfrac{5}{2}x + c$	M1	2.2a
$\Rightarrow 2y - 5x = 4$ *	A1*	1.1b
	(4)	
Uses Pythagoras' theorem to find AB or AD Either $\sqrt{5^2+2^2}$ or $\sqrt{\left(\dfrac{4}{5}\right)^2 + 2^2}$	M1	3.1a
Uses area $ABCD = AD \times AB = \sqrt{29} \times \sqrt{\dfrac{116}{25}}$	M1	1.1b
area $ABCD = 11.6$	A1	1.1b
	(3)	
(7 marks)		

Qu 63… OCR A2 Paper 1 June 2020 - Question 9 (Link to Question)

(a)	$a < 2$ $0 = 1.5a + 2$	B1 M1	3.1a 3.1a	Allow for answer of form $k < a < 2$ Attempt to find value of a at their x intersection	eg Use equation of line to find a Use gradient of line to find a Use a point of intersection of the two lines = their 1.5 Equate two points of intersection and solve for a Square both sides and link discriminant to 0
	$a = -\frac{4}{3}$	A1	1.1	Obtain $-\frac{4}{3}$ (condone any inequality sign, an equals sign or no sign)	Question is 'determine' so method required for this value of a
	$-\frac{4}{3} < a < 2$	A1 [4]	1.1	Correct final inequality	Formal set notation not required
(b)	$2x - 3 = ax + 2$ $x = \dfrac{5}{2-a}$	B1	1.1	Correct point of intersection – allow any exact equiv	**OR** M1 – square both sides and attempt to solve – as far as substituting into quadratic formula A1 A1 for each root
	$3 - 2x = ax + 2$ $(2 + a)x = 1$	M1	1.1a	Attempt to solve linear equation with $2x$ and ax of different signs	Method may be seen in (i), only credit if answers seen in (ii)
	$x = \dfrac{1}{2+a}$	A1 [3]	1.1	Correct point of intersection – allow any exact equiv	Max of 2 out of 3 if additional roots as well.

Qu 64… A great question from a long time ago (Link to Question)

i.	$f(x) \leq 2$
ii.	$ff(4) = 2$
iii.	$0 < k \leq 2$

Qu 65… Edexcel A2 Paper 3 Statistics June 2021 - Question 6 (Link to Question)

[Sum of probs = 1 implies] $\log_{36} a + \log_{36} b + \log_{36} c = 1$	M1
$\Rightarrow \log_{36}(abc) = 1$ so $abc = 36$	A1
All probabilities greater than 0 implies each of a, b and c > 1	B1
$36 = 2^2 \times 3^2$ (or 3 numbers that multiply to give 36 e.g. 2, 2, 9 etc)	dM1
Since a, b and c are distinct must be **2, 3, 6** **($a = 2, b = 3, c = 6$)**	A1
	(5)

Qu 66… OCR AS Paper 2 June 2023 - Question 8 (Link to Question)

DR				
	M1*	**2.1**	**M1** for either term integrated correctly	
$8x^{\frac{1}{2}}+3x$	**A1**	**1.1**		
$\left(8a^{\frac{1}{2}}+3a\right)-(16+12)=7$	**M1dep***	**1.1**	Correct use of correct limits and equating to 7 – allow one substitution error	
$3a+8a^{\frac{1}{2}}-35=0$	**M1**	**1.1**	Forming a 3TQ in $a^{\frac{1}{2}}$	Any three-term form (so terms do not need to be on the same side)
$\left(3a^{\frac{1}{2}}-7\right)\left(a^{\frac{1}{2}}+5\right)=0$	**M1**	**3.1a**	Dependent on all previous **M** marks – correct method for solving for $a^{\frac{1}{2}}$	Or $8a^{\frac{1}{2}}=35-3a$ $9a^2-274a+1225=0$ $(9a-49)(a-25)=0$
$a^{\frac{1}{2}}\neq-5$ as $a^{\frac{1}{2}}$ can't be negative	**A1**	**2.3**	Explicit rejection of –5 No specific justification required	Explicit rejection of $a=25$ No specific justification required
$a^{\frac{1}{2}}=\frac{7}{3}\Rightarrow a=\frac{49}{9}$	**A1**	**2.2a**	Correct value only	
	[7]			

Qu 67… OCR A2 Paper 2 June 2023 - Question 12 (Link to Question)

$\frac{dx}{dt}=\frac{-1}{t^2},\ \frac{dy}{dt}=2$ $\frac{dy}{dx}=\frac{\frac{dy}{dt}}{\frac{dx}{dt}}$	M1	1.1a	Attempt correct process to find gradient in terms of t or p	Correctly combine attempts at two derivatives Need $\frac{dx}{dt}=kt^{-2}$ and $\frac{dy}{dt}=2$ **SC B1** for gradient of $-2x^{-2}$ if it is never seen in terms of t or p
$\frac{dy}{dx}=-2t^2$	A1	2.1	Obtain correct gradient	In terms of t or p
$y-2p=-2p^2\left(x-\frac{1}{p}\right)$	M1	1.1a	Attempt equation of tangent	Condone still working in terms of t Allow mixture of t and p as long as convincingly recovered Using their gradient from a differentiation attempt, but not dependent on first M1 Substitution into $y-y_1=m(x-x_1)$ or equation involving c from $y=mx+c$
$y=-2p^2x+4p$ **A.G.**	A1	2.1	Obtain given answer	Must now be in terms of p Expand brackets and simplify to given answer, or find c and substitute back into equation
$m'=\frac{1}{2p^2}$	B1FT	1.1a	Correct (unsimplified) gradient of normal, following their derivative	Gradient in terms of t or p, but not x Could either FT on their incorrect derivative or deduce the gradient from the equation given in (a)
$y-2p=\frac{1}{2p^2}\left(x-\frac{1}{p}\right)$ $y=\frac{1}{2p^2}x+2p-\frac{1}{2p^3}$	M1	1.1	Attempt equation of normal	Attempt to use their gradient and P Allow mixture of t and p as long as convincingly recovered Substitution into $y-y_1=m(x-x_1)$ or equation involving c from $y=mx+c$
	M1	3.1a	Use $y=0$ to attempt x-coordinate of B	Using their attempt at normal equation As far as finding an expression for x
at B, $y=0$ so $x=2p^2\left(\frac{1}{2p^3}-2p\right)=\frac{1}{p}-4p^3$	A1	2.1	Correct x-coordinate for B	Any equivalent form
at A, $y=0$ so $x=\frac{4p}{2p^2}=\frac{2}{p}$	B1	2.1	Correct x-coordinate for A	Any equivalent form
$PA=\sqrt{\left(\frac{1}{p}\right)^2+(2p)^2}$ $PB=\sqrt{\left(4p^3\right)^2+(2p)^2}$	M1	3.1a	Attempt length of PA or PB	Or **M1** for attempting one of $(PA)^2$ or $(PB)^2$ Must correct distance formula Using the given P, and their coordinates for A and/or B, which must involve a function of p
	A1	2.1	Correct PA and PB	Or correct $(PA)^2$ and $(PB)^2$
$PA:PB=\frac{1}{p}\sqrt{4p^4+1}\ :\ 2p\sqrt{4p^4+1}$ $=\frac{1}{p}:2p$ $=1:2p^2$ **A.G.**	A1	2.1	Simplify ratio to obtain given answer	Must show clear method, such as same expression in each square root before cancelling Could also consider fraction and then cancel to deduce given ratio Could simplify $(PA)^2:(PB)^2$, and then square root to obtain ratio

Qu 68… OCR A2 Paper 2 June 2022 - Question 8 (Link to Question)

Summary method: Express V in terms of h Differentiate V with respect to h Attempt chain rule, Attempt separate variables Correct integrals Substitute correct limits Answer	B1 M1 M1 M1 A1 M1 A1	3.3 3.4	Correct substitution NOT if $h = 50$ or $r = 50\tan 30$ used Resulting equation must involve exactly 2 variables Their equation must involve exactly 2 variables Ignore limits Integrals must be of correct forms (see examples below) Note 1 Candidates who substitute numerical values for h or V or r may be able to score the 2nd and/or 3rd M1 marks, but probably nothing else. See the example of this below. Note 2. There is a special case for candidates who use $r = h\sin 30$ (answer $\frac{625\pi}{4}$ or 491). These can score all 4 M-marks and the final A1 Note 3. The chain rule may be used to find $\frac{dV}{dt}$ or $\frac{dh}{dt}$ or $\frac{dV}{dh}$ or $\frac{dV}{dr}$ or other derivatives. Two of the example methods below illustrate use of $\frac{dV}{dt}$ and $\frac{dV}{dr}$, but use of other derivatives can also lead to correct methods.
Example method 1			
$V = \frac{\pi}{3}(h\tan 30^\circ)^2 h$ or $V = \frac{\pi}{3}\left(\frac{h}{\sqrt{3}}\right)^2 h$ oe	B1	3.3	or $V = \frac{\pi}{9}h^3$ oe
$\frac{dV}{dh} = \frac{\pi}{3}h^2$	M1	3.4	Attempt differentiate their V in terms of h only NOT if $h = 50$ or $r = 50\tan 30$ used.
$\frac{dV}{dt} = \frac{\pi}{3}h^2\frac{dh}{dt}$ oe or $\frac{dh}{dt} = \frac{3}{\pi h^2}\frac{dV}{dt}$ $("\frac{\pi}{3}h^2\frac{dh}{dt}" = -2h$ oe or $\frac{dh}{dt} = \frac{-6}{\pi h})$	M1	2.1	Attempt use chain rule for $\frac{dV}{dt}$ or $\frac{dh}{dt}$ in terms of t & h only (Set their $\frac{dV}{dt} = -2h$)
$\pi\int_{50}^{0} h\,dh = -\int_0^t 6\,dt$ oe	M1	1.1	Attempt separate variables in their equation in terms of h and t only (not V or r). Integral signs not essential
$\left[\frac{\pi h^2}{2}\right]_{50}^{0} = [-6t]_0^t$ oe	A1	2.1	Correct integrals, any limits or none
$-\pi\times\frac{50^2}{2} = -6t$	M1	1.1	Substitute correct limits into integrals of forms ah^2 & bt OR substitute $t = 0$ & $h = 50$ to find c and substitute $h = 0$
Time $= \frac{625\pi}{3}$ secs or 654 secs (3 sf) oe	A1 [7]	3.4	Allow without secs or 10.9 mins or 10 mins 54 secs or: SC. Use of $r = h\sin 30$ (answer $\frac{625\pi}{4}$ or 491) can score all 4 M-marks and final A1

Example method 2			
$V=\frac{\pi}{3}r^2\frac{r}{\tan 30^\circ}$ or $V=\frac{\pi}{\sqrt{3}}r^3$ oe	**B1**		Subst $h=\frac{r}{\tan 30^\circ}$ into correct formula for V
$\frac{dV}{dr}=\sqrt{3}\pi r^2$	**M1**		
$\frac{dV}{dt}=\sqrt{3}\pi r^2\frac{dr}{dt}$ oe	**M1**		Attempt use chain rule to find $\frac{dV}{dt}$ or $\frac{dr}{dt}$ in terms of t and r
("$\sqrt{3}\pi r^2\frac{dr}{dt}$" $=-2r\sqrt{3}$ oe)			(Set their $\frac{dV}{dt}=-2r\sqrt{3}$ oe)
$\pi\int_{\frac{50}{\sqrt{3}}}^{0} r\,dr=-\int_0^t 2\,dt$ oe	**M1**		Attempt separate variables in their equation in terms of r and t only (not V or h). Integral signs not essential
$\left[\frac{\pi r^2}{2}\right]_{\frac{50}{\sqrt{3}}}^{0}=[-2t]_0^t$ oe	**A1**		Correct integrals, any limits or none
$-\frac{\pi\times 50^2}{6}=-2t$	**M1**		Substitute correct limits into integrals of the form ar^2 & bt OR substitute $t=0$ & $r=\frac{50}{\sqrt{3}}$ to find c and substitute $r=0$
Time $=\frac{625\pi}{3}$ secs or 654 secs (3 sf) oe	**A1**	**3.4**	Allow without secs or 10.9 mins or 10 mins 54 secs SC. Use of $r=h\sin 30$ (answer 491) can score M4A1
Example method 3 (NOT using chain rule)			**This method is different from the summary method above**
$V=\frac{\pi}{3}(h\tan 30^\circ)^2 h$ or $V=\frac{\pi}{3}\left(\frac{h}{\sqrt{3}}\right)^2 h$ oe	**B1**	**3.3**	or $V=\frac{\pi}{9}h^3$ oe
$h=\sqrt[3]{\frac{9V}{\pi}}$	**M1**		Allow $h=kV^{1/3}$
$\frac{dV}{dt}=-2\times\sqrt[3]{\frac{9V}{\pi}}$	**M1**		$\frac{dV}{dt}=-2\times$(their h in terms of V)
$\sqrt[3]{\frac{\pi}{9}}\int_{\frac{\pi 50^3}{9}}^{0} V^{-1/3}dV=-2[t]_0^t$	**M1**		Attempt separate variables in their equation in terms of V and t only (not h or r). Integral signs not essential
$\sqrt[3]{\frac{\pi}{9}}\times\frac{3}{2}\left[V^{2/3}\right]_{\frac{\pi 50^3}{9}}^{0}=-2t$	**A1**		Correct integrals, any limits or none
$-\sqrt[3]{\frac{\pi}{9}}\times\frac{3}{2}\times\left(\frac{\pi 50^3}{9}\right)^{2/3}=-2t$	**M1**		Substitute correct limits into integrals of forms $aV^{2/3}$ & bt OR substitute $t=0$ & $V=\frac{\pi 50^3}{9}$ to find c and substitute $V=0$
Time $=\frac{625\pi}{3}$ secs or 654 secs (3 sf) oe	**A1**		Allow without secs or 10.9 mins or 10 mins 54 secs or: SC. Use of $r=h\sin 30$ (answer $\frac{625\pi}{4}$ or 491) can score all 4 M-marks and final A1

Qu 69… MEI A2 Paper 2 June 2023 - Question 17 (Link to Question)

divide through by $\cos x$ to obtain $2\tan x + \sec^2 x = 4$	**B1**	**2.1**	
$2\tan x + \tan^2 x + 1 = 4$	**M1***	**3.1a**	use of Pythagoras to obtain equation in $\tan x$ only; allow 1 sign error
$\tan^2 x + 2\tan x - 3[= 0]$	**A1**	**1.1**	
$\tan x = 1 \text{ or } -3$ $[x =]$ –1.24905 to –1.249 or –1.25 or –1.2	**M1*dep**	**1.1**	2 values obtained for $\tan x$ from their quadratic
$[x =]$ 1.8925 to 1.893 or 1.89 or 1.9 $[x =]$ $\frac{\pi}{4}$ or 0.785 to 0.7854 or 0.79	**A1**	**3.2a**	any two correct
$[x =]$ $-\frac{3\pi}{4}$ or –2.3562 to –2.356 or –2.36 or –2.4	**A1**	**2.2a**	all four correct and **no extra** values in range; ignore **correct** extra values outside range but **A0** if incorrect values outside range

Qu 70… OCR A2 Paper 2 June 2023 - Question 6 (Link to Question)

DR			In this question, if candidates attempt more than one method, review all their working to identify which is their 'final' or 'substantially most complete' answer, then apply one scheme only. Candidates cannot gain marks from more than one scheme.
$2x^2 - 6x + a = 0$	M1	3.1a	Substitute $y = x$ into equation of circle (need not be simplified – may see this quadratic in y)
At A: $x = \frac{6+\sqrt{36-8a}}{4} = \frac{3-\sqrt{9-2a}}{2}$ At B: $x = \frac{3+\sqrt{9-2a}}{2}$	A1	2.2a	A1 for <u>either</u> correct (condone if not specifically identified as A or B may see y= these values)
At M: $x = \frac{3}{2}$	A1	1.1	
$CM^2 = \left(3-\frac{3}{2}\right)^2 + \left(\frac{3}{2}\right)^2 \quad (=\frac{9}{2})$	M1	2.1	Attempt CM using their values (method must be correct) May see $CM = \frac{3\sqrt{2}}{2}$
$BA^2 = \left(\sqrt{9-2a}\right)^2 + \left(\sqrt{9-2a}\right)^2$ $(= 2\left(\sqrt{9-2a}\right)^2)$	M1	1.1	Attempt BA using their values (method must be correct) May see just $\sqrt{2(9-2a)}$ oe, e.g. $\sqrt{18-4a}$ Alternative: $AM = \sqrt{\frac{9}{2}-a}$ (and then $Area = 2 \times \frac{1}{2} \times CM \times AM$)
Area $= \frac{1}{2} \times CM \times BA$	M1	1.1	Attempt at area, in terms of a
$\frac{1}{2} \times \sqrt{2}\sqrt{9-2a} \times \frac{3}{\sqrt{2}} \quad (= 3\sqrt{\frac{9-2a}{4}})$ **AG**	A1	1.1	Must see correct expression before answer
Alternative Method			
DR $(x-3)^2 + y^2 = 9 - a$	M1		Attempt complete the square for x
C is $(3, 0)$; radius $= \sqrt{9-a}$	A1		Both soi
$CM = (3\cos 45 =)\ \frac{3}{\sqrt{2}}$	B1		Possibly coming from $\frac{\lvert 3-0\rvert}{\sqrt{1^2+1^2}}$
$AM^2 = (9-a) - \left(\frac{3}{\sqrt{2}}\right)^2 \quad (=\frac{9}{2}-a)$	M1		Their radius^2 (in terms of a) – their CM^2
Area $= AM \times CM$ or $\frac{1}{2}AB \times CM$	M1		Attempted in terms of a
$= \sqrt{\frac{9}{2}-a} \times \frac{3}{\sqrt{2}}$ oe	A1FT		FT their AM or AB (in terms of a), and their CM
$= \sqrt{\frac{9-2a}{2}} \times \frac{3}{\sqrt{2}} \quad (= 3\sqrt{\frac{9-2a}{4}}$ **AG**$)$	A1		Must see one correct intermediate step
Alternative method for last three marks			
Area $= \frac{1}{2}AB \times r \times \sin BAC$	M1		Attempted in terms of a
$= \sqrt{\frac{9}{2}-a} \times \sqrt{9-a} \times \left(\frac{3}{\sqrt{2}} \div \sqrt{9-a}\right)$	A1FT		FT their r and AM or AB (in terms of a), and their CM
$= \sqrt{\frac{9-2a}{2}} \times \frac{3}{\sqrt{2}} \quad (= 3\sqrt{\frac{9-2a}{4}}$ **AG**$)$	A1		Must see one correct intermediate step
Alternative method for last three marks			
Area $= \frac{1}{2} r \times r \times \sin BCA$	M1		Attempted in terms of a
$= \sqrt{9-a} \times \sqrt{9-a} \times 2\left(\sqrt{\frac{9}{2}-a} \div \sqrt{9-a}\right)\left(\frac{3}{\sqrt{2}} \div \sqrt{9-a}\right)$	A1FT		FT their r and AM or AB (in terms of a), and their CM (using double angle formula for $\sin BCA$)
$= \sqrt{\frac{9-2a}{2}} \times \frac{3}{\sqrt{2}} \quad (= 3\sqrt{\frac{9-2a}{4}})$ **AG**	A1		Must see one correct intermediate step
	[7]		

Qu 71... MEI A2 Paper 3 June 2023 - Question 3 (Link to Question)

DR $\frac{\sqrt{6}-\sqrt{5}}{6-5}+\frac{\sqrt{7}-\sqrt{6}}{7-6}$ oe	**M1**	**3.1a**	Rationalising denominators. This is the minimum working needed for **M1** Accept 1 for "6 – 5" and – 1 for "5 – 6" etc
$\sqrt{7}-\sqrt{5}$ or $\frac{\sqrt{5}-\sqrt{7}}{-1}$	**A1**	**1.1**	
$\frac{k}{\sqrt{5}+\sqrt{7}}=\frac{k\left(\sqrt{7}-\sqrt{5}\right)}{7-5}=\sqrt{7}-\sqrt{5}$ so $k=2$	**A1**	**2.2a**	Finding k convincingly after **M1**
Alternative $\left(\sqrt{6}+\sqrt{7}\right)\left(\sqrt{5}+\sqrt{7}\right)+\left(\sqrt{5}+\sqrt{6}\right)\left(\sqrt{5}+\sqrt{7}\right)$ $=k\left(\sqrt{5}+\sqrt{6}\right)\left(\sqrt{6}+\sqrt{7}\right)$	**M1**		For dealing appropriately with fractions (working with both sides) e.g. clearing the fractions or making k the subject with the RHS as a single fraction. $k=\frac{\left(\sqrt{5}+\sqrt{7}+2\sqrt{6}\right)\left(\sqrt{5}+\sqrt{7}\right)}{\left(\sqrt{5}+\sqrt{6}\right)\left(\sqrt{6}+\sqrt{7}\right)}$
$\left(2\sqrt{5}\sqrt{6}+2\sqrt{5}\sqrt{7}+2\sqrt{6}\sqrt{7}+12\right)$ $=k\left(\sqrt{5}\sqrt{6}+\sqrt{5}\sqrt{7}+\sqrt{6}\sqrt{7}+6\right)$	**A1**		Expanding brackets and collecting like surds $k=\frac{12+2\sqrt{5}\sqrt{7}+2\sqrt{5}\sqrt{6}+2\sqrt{6}\sqrt{7}}{6+\sqrt{5}\sqrt{7}+\sqrt{5}\sqrt{6}+\sqrt{6}\sqrt{7}}$
$2\left(\sqrt{5}\sqrt{6}+\sqrt{5}\sqrt{7}+\sqrt{6}\sqrt{7}+6\right)$ $=k\left(\sqrt{5}\sqrt{6}+\sqrt{5}\sqrt{7}+\sqrt{6}\sqrt{7}+6\right)$ so $k=2$	**A1**		Finding k convincingly after **M1**

Qu 72… OCR A2 Paper 1 June 2023 - Question 12 (Link to Question)

$du = e^x dx$	**B1**	1.1	Correct statement linking du and dx	or $dx=\frac{1}{u+2}du$
$\int\frac{7(u+2)-8}{u^2}\cdot\frac{1}{u+2}du$	**M1**	1.1	Use $e^x = u + 2$ to attempt integrand in terms of u	Must see clear evidence of substitution, including how $e^x dx$ is dealt with **M0** for going straight from $7e^x - 8$ to $7u + 6$ with no justification Must include du
$=\int\frac{7u+14-8}{u^2(u+2)}du=\int\frac{7u+6}{u^2(u+2)}du$	**A1**	2.1	Correct integrand	Including both integral sign and du throughout, as **AG**
$\frac{A}{u}+\frac{B}{u^2}+\frac{C}{u+2}=\frac{7u+6}{u^2(u+2)}$ $Au(u+2)+B(u+2)+Cu^2=7u+6$	**M1**	3.1a	Attempt correct partial fractions May have $\frac{Au+B}{u^2}+\frac{C}{u+2}$ but **M0** for just $\frac{B}{u^2}$ with no $\frac{A}{u}$	Correct method to combine correct fractions, and at least one constant attempted If considering $\frac{7}{u^2}+\frac{-8}{u^2(u+2)}$ then must use partial fractions on the second term to get credit
$\frac{2}{u}+\frac{3}{u^2}-\frac{2}{u+2}$	**A1**	2.1	Correct partial fractions May have $\frac{2u+3}{u^2}-\frac{2}{u+2}$	Possibly implied by their A, B, and C values ie $A = 2$, $B = 3$, $C = -2$
$2\ln\|u\| - 2\ln\|u+2\| - 3u^{-1}$	**M1**	1.1	Attempt integration of $\frac{B}{u^2}$ and at least one of $\frac{A}{u}$ or $\frac{C}{u+2}$, and no others	Allow errors in coefficients only Allow **M1** if only two fractions, as long as of required form If using $\frac{Au+B}{u^2}$ then it must be a correct integration attempt (ie split into two fractions first)

	A1FT	**2.1**	FT on their two or three fractions as long as ku^{-2} and one or two fractions each with a linear denominator	Condone brackets not modulus Condone no brackets as long as implied by later working, eg when limits are used
$(2\ln 4 - 2\ln 6 - \frac{3}{4}) - (2\ln 2 - 2\ln 4 - \frac{3}{2})$	**M1**	**1.1a**	Attempt use of correct limits – correct order and subtraction; u or x but commensurate with their integral	Allow substitution into any function that is clearly attempt at integration
$\left(\frac{3}{2}-\frac{3}{4}\right)+\ln\left(\frac{4\times 4}{6\times 2}\right)^2$	**M1**	**3.1a**	Attempt to rearrange correct numerical integral to required form	Must be correct numerical expression from correct working Terms may have been combined before use of limits, but must still be correct expression to gain **M1** Correct attempt to combine ln terms ie deal with coefficients and correct product / quotient for the sum / differences Allow one slip
$\frac{3}{4}+\ln\frac{16}{9}$	**A1**	**2.1**	Obtain $\frac{3}{4}+\ln\frac{16}{9}$	Condone $\frac{3}{4}+2\ln\frac{4}{3}$ Fractions must be simplified ISW an incorrect attempt to write this answer in a different form, but **A0** if further work done eg multiplying by a constant to clear the fractions

Qu 73… MEI A2 Paper 2 June 2020 - Question 14 (Link to Question)

$\frac{1}{2}-\sin 2x\cos x=\sin x\cos 2x$	**M1**	**3.1a**		
$\sin 3x=\frac{1}{2}$ **oe**	**M1**	**2.1**	from compound angle formula allow sign errors only	*or* $4\sin^3 x - 3\sin x + ½ = 0$ **oe**
$x=\frac{\pi}{18}$ and $x=\frac{5\pi}{18}$	**A1** **A1**	**3.2a** **1.1**	**A1** for each	0.17453… **A1** 0.87266… **A1** to 2 or more sf
$\pm\int\left(\sin x\cos 2x-\left(\frac{1}{2}-\sin 2x\cos x\right)\right)\mathrm{d}x$ **oe**	**M1**	**1.1**	ignore limits	
$\mathrm{F}[x]=-\frac{x}{2}-\frac{\cos 3x}{3}$	**A1**	**1.1**	allow the positive of this	$\pm\left(-\frac{4}{3}\cos^3 x+\cos x-\frac{x}{2}\right)$ **oe** or $\pm(-⅓\cos 2x\cos x + ⅓\sin x\sin 2x - ½x)$ **oe** for **A1**
$\mathrm{F}[\frac{5\pi}{18}] - \mathrm{F}[\frac{\pi}{18}]$	**M1**	**1.1**	F[x] must be one of the correct forms	F[0.87266] – F[0.17453] for **M1**
$\frac{\sqrt{3}}{3}-\frac{\pi}{9}$ or $\frac{3\sqrt{3}-\pi}{9}$ **cao**	**A1** **[8]**	**3.2a**		

Qu 74… OCR A2 Paper 1 June 2022 - Question 7 (Link to Question)

$6x^2 + 6y + 6x\frac{dy}{dx} - 6y\frac{dy}{dx} = 0$	**M1**	**1.1a**	Attempt implicit differentiation	Either of the two $\frac{dy}{dx}$ terms correct, allowing sign errors Condone $6x^2dx + 6ydx + 6xdy - 6ydy$
	B1	**1.1a**	Use product rule correctly on middle term	Both terms correct Must now be $6y + 6x\frac{dy}{dx}$, or implied in a correct expression for $\frac{dy}{dx}$
	A1	**1.1**	Obtain correct derivative on LHS	Condone missing or incorrect RHS Must now have $\frac{dy}{dx}$ and not just dy or dx in terms
$6x^2 + 6y + 6x - 6y = 0$	**M1**	**3.1a**	Use $\frac{dy}{dx} = 1$ in their equation	Must now be equation, but RHS could be incorrect (eg '= 2')
$x^2 + x = 0$ $x = 0,\ x = -1$	**B1**	**1.1a**	Solve correct quadratic in x to obtain two correct roots (possibly **BC**) Quadratic must come from correct implicit differentiation	**B0** if x 'cancelled' in quadratic to give $x = -1$ as only root, but **M1A1** still available
$x = 0$ gives $3y^2 = -2$, but y^2 has to be ≥ 0, so no solutions	**B1**	**2.3**	Explicitly reject $x = 0$, with reasoning $x = 0$ must come from $x^2 + x = 0$	eg negative numbers cannot be square rooted or $y^2 \neq -\frac{2}{3}$ 2 as y is real (just $y^2 \neq -\frac{2}{3}$ is insufficient) Must be sensible reason and not just 'math error' or 'not possible' Could say that there are only imaginary (or not real) roots – condone 'complex' roots
$x = -1$ gives $3y^2 + 6y + 4 = 0$ $b^2 - 4ac = 36 - 48 = -12$	**M1**	**2.1**	Attempt to determine the number of real roots of their 3 term quadratic in y	From substituting their x value into the equation of the curve Consider discriminant, or use quadratic formula, or attempt minimum value of function
$-12 < 0$ hence no (real) roots	**A1** **[8]**	**2.4**	Obtain correct discriminant from correct quadratic and conclude appropriately $x = -1$ must come from $x^2 + x = 0$	If using quadratic formula then it must be fully correct and attention drawn to why there are no real roots

Qu 75… OCR A2 Paper 1 June 2022 - Question 12 (Link to Question)

$\frac{dx}{dt}=\frac{-1}{t^2}$, $\frac{dy}{dt}=2$ $\frac{dy}{dx}=\frac{\frac{dy}{dt}}{\frac{dx}{dt}}$	M1	1.1a	Attempt correct process to find gradient in terms of t or p	Correctly combine attempts at two derivatives Need $\frac{dx}{dt}=kt^{-2}$ and $\frac{dy}{dt}=2$ **SC B1** for gradient of $-2x^2$ if it is never seen in terms of t or p
$\frac{dy}{dx}=-2t^2$	A1	2.1	Obtain correct gradient	In terms of t or p
$y-2p=-2p^2\left(x-\frac{1}{p}\right)$	M1	1.1a	Attempt equation of tangent	Condone still working in terms of t Allow mixture of t and p as long as convincingly recovered Using their gradient from a differentiation attempt, but not dependent on first M1 Substitution into $y-y_1=m(x-x_1)$ or equation involving c from $y=mx+c$
$y=-2p^2x+4p$ **A.G.**	A1	2.1	Obtain given answer	Must now be in terms of p Expand brackets and simplify to given answer, or find c and substitute back into equation
	[4]			
$m'=\frac{1}{2p^2}$	B1FT	1.1a	Correct (unsimplified) gradient of normal, following their derivative	Gradient in terms of t or p, but not x Could either FT on their incorrect derivative or deduce the gradient from the equation given in (a)
$y-2p=\frac{1}{2p^2}\left(x-\frac{1}{p}\right)$ $y=\frac{1}{2p^2}x+2p-\frac{1}{2p^3}$	M1	1.1	Attempt equation of normal	Attempt to use their gradient and P Allow mixture of t and p as long as convincingly recovered Substitution into $y-y_1=m(x-x_1)$ or equation involving c from $y=mx+c$
	M1	3.1a	Use $y=0$ to attempt x-coordinate of B	Using their attempt at normal equation As far as finding an expression for x
at B, $y=0$ so $x=2p^2\left(\frac{1}{2p^3}-2p\right)=\frac{1}{p}-4p^3$	A1	2.1	Correct x-coordinate for B	Any equivalent form
at A, $y=0$ so $x=\frac{4p}{2p^2}=\frac{2}{p}$	B1	2.1	Correct x-coordinate for A	Any equivalent form
$PA=\sqrt{\left(\frac{1}{p}\right)^2+(2p)^2}$ $PB=\sqrt{\left(4p^3\right)^2+(2p)^2}$	M1	3.1a	Attempt length of PA or PB	Or **M1** for attempting one of $(PA)^2$ or $(PB)^2$ Must correct distance formula Using the given P, and their coordinates for A and/or B, which must involve a function of p
	A1	2.1	Correct PA and PB	Or correct $(PA)^2$ and $(PB)^2$
$PA:PB=\frac{1}{p}\sqrt{4p^4+1}:2p\sqrt{4p^4+1}$ $=\frac{1}{p}:2p$ $=1:2p^2$ **A.G.**	A1	2.1	Simplify ratio to obtain given answer	Must show clear method, such as same expression in each square root before cancelling Could also consider fraction and then cancel to deduce given ratio Could simplify $(PA)^2:(PB)^2$, and then square root to obtain ratio
	[8]			