Hypothesis Testing with the Normal Distribution Exam Questions

The length of a train journey from Norwich to Ipswich follows a normal distribution with a mean of 45 minutes and a standard deviation of 10 minutes.

Commuters have complained in recent weeks that journeys are taking longer. A random sample of 36 journeys is taken, which is found to have a mean time of 48 minutes.

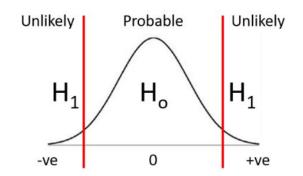
- a) Test at the 5% significance level whether there is evidence of an increase in the train journey time.
- Using the context of the problem, comment on the suitability of the journey times following a normal distribution.

Scientists believe the mass of narwhals $\sim N(900,5^2)$.

Fisherman Bob Mortimer thinks the mean mass of Narwhals isn't 900kg and decides to catch 10 narwhals to test this.

Bob catches 10 Narwhals and conducts a hypothesis test at the 5% significant level. He rejects H_0 , concluding that there is evidence to suggest the mean mass of Narwhals isn't 900kg.

What are the smallest and largest possible mean masses of Bob's catch?



Stringo Balls, a brand of string, are meant to be sold in lengths of 2m and it is claimed that the lengths follow a normal distribution with mean 2m and standard deviation 0.2m. A trading officer inspects a random sample of 22 balls and finds that the sample has a mean length of 1.91m.

Test whether this provides evidence at the 2% significance level, that the mean is less than 2m.

The ages of members of a large bowling club is modelled by a normal distribution with a, previously calculated, average age of 61.4 years and standard deviation of 7.5 years.

The club secretary is concerned about the ageing membership of the club so takes a random sample of 16 members and finds their mean age to be 65.0 years.

- a) Carry out a hypothesis test by finding the critical region at the 5% significance level, to determine whether the mean age of club members has changed.
- b) Comment on the likely number of members aged under 25, giving a numerical reason for your answer.

In previous years, marks obtained by students at Top-Notch College have been modelled by a normal distribution with mean 65 and standard deviation 9.

Teachers are concerned that this year, students are, on average, underachieving so select a random sample of 35 students whose mean score is found to be 61.5.

Investigate the teacher's suspicion at the 5% level.

Writing Frame

		Trains	Narwhals	Stringo Balls	Bowls	Top Notch
1	Define the variable	$\mu =$				
2	Write down the null and alternate hypotheses	H ₀ : μ = H ₁ : μ	H ₀ : μ = H ₁ : μ	H ₀ : μ = H ₁ : μ	H ₀ : μ = H ₁ : μ	H ₀ : μ = H ₁ : μ
3	What is the significance level?					
4	Is it one or two tailed?					
5	Probability of this sample occurring randomly?					
6	Is this probability less than or greater than the significance level?					
7	Accept or reject the null hypothesis?					
8	Conclude and clarify in context					

Writing Frame - answers

		Trains	Narwhals	Stringo Balls	Bowls	Top Notch
1	Define the variable	$\mu = { m average\ train}$ time NOR > IPW	$\mu=$ average mass of narwhals	$\mu = ext{average length}$ of ball of string	$\mu=$ average age of bowling members	$\mu=$ average marks obtained
2	Write down the null and alternate hypotheses	H ₀ : $\mu = 45$ H ₁ : $\mu > 45$	H_0 : $\mu = 900$ H_1 : $\mu \neq 900$	H ₀ : $\mu = 2$ H ₁ : $\mu < 2$	H_0 : $\mu = 61.4$ H_1 : $\mu \neq 61.4$	H ₀ : $\mu = 65$ H ₁ : $\mu < 65$
3	What is the significance level?	5%	5%	2%	5%	5%
4	Is it one or two tailed?	One	Two	One	Two	One
5	Probability of this sample occurring randomly?	0.0359	Smallest and largest mean masses are 896.9 to 903.1	0.0174	Regions at 57.725 and 65.075	0.0107
6	Is this probability less than or greater than the significance level?	It's less than		It's less than	65 is within the accept H_0 region	It's less than
7	Accept or reject the null hypothesis?	Reject H_0		Reject H_0	Accept H_0	Reject H_0
8	Conclude and clarify in context	There is evidence to suggest that average train times are longer than 45 minutes		There is evidence to suggest that average sizes of Stringo balls are smaller than 2m	There is insufficient evidence to suggest that average ages of bowling club members has increased	There is evidence to suggest that students are underachieving