**Lots of Proof Questions**

**Algebraic Proof**

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| Prove algebraically that is odd for all positive integers .  [4] |
| is an integer that is not divisible by 3. Prove that is of the form , where is an integer.  [5] |
| Prove that the sum of the squares of any two consecutive integers is of the form , where is an integer.  [4] |
| By considering separately the case when is odd and the case when is even, prove that the following statement is true.  is a positive integer ⇒ is not a multiple of 4.  [4] |
| Prove that the sum of the squares of any three consecutive positive integers cannot be divided by 3.  [2] |
| Tom Cruise claims that “ is an **even** positive integer greater than 2 ⇒ is **not** prime”.  Prove that Tom‘s claim is true.  [4] |

**Counter example and Contradiction**

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| Johnny claims that “If is any positive integer, then is a prime number.”  Prove that Johnny’s claim is incorrect.  [3]  Amber says that .  Explain why Amber’s statement is incorrect and write a corrected version of Amber’s statement.  [2] |
| Prove by counter example that isn’t prime for all .  [2] |

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| Prove that the following statement is **not** true.  is an odd number greater than 1 ⇒ is prime.  [1] |
| It is given that n is an integer. Prove by contradiction that is even ⇒ is even.  [5] |
| A student suggests that, for any prime number between 20 and 40, when its digits are squared and then added, the sum is an odd number.  For example, 23 has digits 2 and 3 which gives 22 + 32 = 13 , which is odd.  Show by counter example that this suggestion is false.  [2] |
| Prove by contradiction that is irrational.  [5] |

**Exhaustion**

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| Prove by exhaustion that if the sum of the digits of a 2-digit number is 5, then this 2-digit number is not a perfect square.  [3] |

**Others**

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| Show that, if is a positive integer, then is divisible by .  [1]  Hence show that, if is a positive integer, then is divisible by 17.  [4] |
| Determine the set of values of for which and are positive integers.  [3]  A ‘Pythagorean triple’ is a set of three positive integers , and such that .  Prove that, for the set of values of found in part **(a)**, the numbers , and form a Pythagorean triple.  [2] |
| Prove that is prime for all primes, , greater than 3.  [5] |