**Lots of Proof Questions**

**Algebraic Proof**

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| Prove algebraically that $n^{3}+3n-1$ is odd for all positive integers $n$.[4] |
| $N$ is an integer that is not divisible by 3. Prove that $N^{2}$ is of the form $3p+1$, where $p$ is an integer.[5] |
| Prove that the sum of the squares of any two consecutive integers is of the form $4k+1$, where $k$ is an integer.[4] |
| By considering separately the case when $n$ is odd and the case when $n$ is even, prove that the following statement is true.$n$ is a positive integer ⇒ $n^{2}+1 $is not a multiple of 4.[4] |
| Prove that the sum of the squares of any three consecutive positive integers cannot be divided by 3.[2] |
| Tom Cruise claims that “$n$ is an **even** positive integer greater than 2 ⇒ $2^{n}-1$ is **not** prime”.Prove that Tom‘s claim is true.[4] |

**Counter example and Contradiction**

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| Johnny claims that “If $n$ is any positive integer, then $3^{n}+2$ is a prime number.”Prove that Johnny’s claim is incorrect.[3]Amber says that $x=3⟺x^{2}=9$.Explain why Amber’s statement is incorrect and write a corrected version of Amber’s statement.[2] |
| Prove by counter example that $n^{2}+n+41$ isn’t prime for all $n\in Z$.[2] |

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| Prove that the following statement is **not** true.$m$ is an odd number greater than 1 ⇒ $m^{2}+4$ is prime.[1] |
| It is given that n is an integer. Prove by contradiction that $n^{2}$ is even ⇒ $n$ is even.[5] |
| A student suggests that, for any prime number between 20 and 40, when its digits are squared and then added, the sum is an odd number. For example, 23 has digits 2 and 3 which gives 22 + 32 = 13 , which is odd. Show by counter example that this suggestion is false.[2] |
| Prove by contradiction that $\sqrt{7}$ is irrational.[5] |

**Exhaustion**

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| Prove by exhaustion that if the sum of the digits of a 2-digit number is 5, then this 2-digit number is not a perfect square.[3] |

**Others**

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| Show that, if $n$ is a positive integer, then $\left(x^{n}-1\right)$ is divisible by $\left(x-1\right)$.[1]Hence show that, if $k$ is a positive integer, then $2^{8k}-1$ is divisible by 17.[4] |
| Determine the set of values of $n$ for which $\frac{n^{2}-1}{2}$ and $\frac{n^{2}+1}{2} $are positive integers.[3]A ‘Pythagorean triple’ is a set of three positive integers $a$, $b$ and $c$ such that $a^{2}+b^{2}=c^{2}$. Prove that, for the set of values of $n$ found in part **(a)**, the numbers $n$, $\frac{n^{2}-1}{2}$ and $\frac{n^{2}+1}{2} $form a Pythagorean triple.[2] |
| Prove that $p^{2}-1$ is prime for all primes, $p$, greater than 3.[5] |