Lots of Proof Questions

Algebraic Proof

Prove algebraically that $n^3 + 3n - 1$ is odd for all positive integers n.

[4]

N is an integer that is not divisible by 3. Prove that N^2 is of the form $3p + 1$, where p is an integer.

[5]

Prove that the sum of the squares of any two consecutive integers is of the form $4k + 1$, where k is an integer.

[4]

By considering separately the case when n is odd and the case when n is even, prove that the following statement is true.

n is a positive integer $\Rightarrow n^2 + 1$ is not a multiple of 4.

[4]

Prove that the sum of the squares of any three consecutive positive integers cannot be divided by 3.

[2]

Tom Cruise claims that "n is an **even** positive integer greater than 2 $\Rightarrow 2^n - 1$ is **not** prime".

Prove that Tom's claim is true.

[4]

Counter example and Contradiction

Johnny claims that "If n is any positive integer, then $3^n + 2$ is a prime number."

Prove that Johnny's claim is incorrect.

[3]

Amber says that $x = 3 \Leftrightarrow x^2 = 9$.

Explain why Amber's statement is incorrect and write a corrected version of Amber's statement.

[2]

Prove by counter example that $n^2 + n + 41$ isn't prime for all $n \in \mathbb{Z}$.

[2]

Prove that the following statement is **not** true.

m is an odd number greater than 1 $\Rightarrow m^2 + 4$ is prime.

[1]

It is given that n is an integer. Prove by contradiction that n^2 is even $\Rightarrow n$ is even.

[5]

A student suggests that, for any prime number between 20 and 40, when its digits are squared and then added, the sum is an odd number.

For example, 23 has digits 2 and 3 which gives $2^2 + 3^2 = 13$, which is odd.

Show by counter example that this suggestion is false.

[2]

Prove by contradiction that $\sqrt{7}$ is irrational.

[5]

Exhaustion

Prove by exhaustion that if the sum of the digits of a 2-digit number is 5, then this 2-digit number is not a perfect square.

[3]

Others

Show that, if n is a positive integer, then $(x^n - 1)$ is divisible by $(x - 1)$.

[1]

Hence show that, if k is a positive integer, then $2^{8k} - 1$ is divisible by 17.

[4]

Determine the set of values of n for which $\frac{n^2-1}{2}$ and $\frac{n^2+1}{2}$ are positive integers.

[3]

A 'Pythagorean triple' is a set of three positive integers a, b and c such that $a^2 + b^2 = c^2$.

Prove that, for the set of values of n found in part **(a)**, the numbers n, $\frac{n^2-1}{2}$ and $\frac{n^2+1}{2}$ form a Pythagorean triple.

[2]

Prove that $p^2 - 1$ is prime for all primes, p, greater than 3.

[5]