**Parametric Equations**

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| --- | --- |
| The smaller partBe able to find a Cartesian equation / eliminate the third variable (or find an implicit equation). | The bigger partBe able to differentiate, find $\frac{dy}{dx}$ (usually in terms of *t*). |

Simpler examples such as…

|  |  |
| --- | --- |
| $$x=t+1$$ | $$y=t^{2}$$ |

Harder examples such as…

|  |  |
| --- | --- |
| $$x=2sint$$ | $$y=5-4cost$$ |

|  |  |
| --- | --- |
| $$x=3t+\frac{1}{t}$$ | $$y=3t-\frac{1}{t}$$ |

Note that…

|  |  |
| --- | --- |
| $$x^{2}-y^{2}=-8$$ | is an implicit function |
| $$y=\sqrt{x^{2}+8}$$ | is an explicit function |

|  |  |
| --- | --- |
| $$\frac{dy}{dx}=\frac{dy}{dt}×\frac{dt}{dx}$$ | $$\frac{dy}{dx}=\frac{dy}{dt}÷\frac{dx}{dt}$$ |

**Rearrangements of Examples**

The Simpler Example

|  |  |
| --- | --- |
| $$x=t+1$$ | $$y=t^{2}$$ |

$$t=\left(x-1\right)⇒y=\left(x-1\right)^{2}$$

Harder Example 1:

|  |  |
| --- | --- |
| $$x=2sint$$$$⇒\frac{x}{2}=sint$$$$⇒\frac{x}{4}^{2}=sin^{2}t$$ | $$y=5-4cost$$$$⇒y-5=-4cost$$$$⇒\frac{5-y}{4}=cost$$$$⇒\frac{\left(5-y\right)^{2}}{16}=cos^{2}t$$ |

$$\frac{x}{4}^{2}+\frac{\left(5-y\right)^{2}}{16}=1$$

$$4x^{2}+\left(5-y\right)^{2}=16$$

Harder Example 2:

|  |  |
| --- | --- |
| $$x=3t+\frac{1}{t}$$ | $$y=3t-\frac{1}{t}$$ |

$$x+y=6t$$

$$x-y=\frac{2}{t}$$

$$\left(x+y\right)\left(x-y\right)=6t×\frac{2}{t}=12$$

$$\left(x+y\right)\left(x-y\right)=12$$

**Differentiation of Examples**

**E.g. 1**

|  |  |
| --- | --- |
| $$x=2t+1$$$$\frac{dx}{dt}=2$$ | $$y=t^{2}$$$$\frac{dy}{dt}=2t$$ |

And therefore…

$$\frac{dy}{dx}=\frac{dy}{dt}÷\frac{dx}{dt}=2t÷2=t$$

**E.g.2**

|  |  |
| --- | --- |
| $$x=2sint$$$$\frac{dx}{dt}=2cost$$ | $$y=5-4cost$$$$\frac{dy}{dt}=4sint$$ |

And therefore…

$$\frac{dy}{dx}=\frac{dy}{dt}÷\frac{dx}{dt}=\frac{4sint}{2cost}=2tant$$

**E.g.3**

|  |  |
| --- | --- |
| $$x=3t+\frac{1}{t}$$$$\frac{dx}{dt}=3-\frac{1}{t^{2}}$$$$\frac{dt}{dx}=\frac{1}{3-\frac{1}{t^{2}}}$$ | $$y=3t-\frac{1}{t}$$$$\frac{dy}{dt}=3+\frac{1}{t^{2}}$$ |

And therefore…

$$\frac{dy}{dx}=\frac{dy}{dt}×\frac{dt}{dx}=\left(3+\frac{1}{t^{2}}\right)×\left(\frac{1}{3-\frac{1}{t^{2}}}\right)=\frac{3+\frac{1}{t^{2}}}{3-\frac{1}{t^{2}}}=\frac{3t^{2}+1}{3t^{2}-1}$$

Which rearranges to…

$$\frac{3+\frac{1}{t^{2}}}{3-\frac{1}{t^{2}}}×\frac{t^{2}}{t^{2}}=\frac{3t^{2}+1}{3t^{2}-1}$$