**Recurrence Relations**

(A continuation from finding the rule for the nth term)

|  |  |
| --- | --- |
| Term to Term  ‘Open form equation’ | Position to Term  ‘Closed form equation’ |
| ‘Start at two, then add three each time’ | Multiply the position by three & subtract one |
| , |  |
| 2, 5, 8… | 2, 5, 8… |

**Definitions**

|  |  |  |
| --- | --- | --- |
|  | First Order | Second Order |
| Homogeneous |  |  |
| Non - homogeneous |  |  |

A **linear recurrence relation** is one where the next term is a multiple of the previous term, with no squaring, cubing etc.

**Useful Equations for Solving Non-Homogeneous Equations**

|  |  |
| --- | --- |
| Form of Non-Homogeneous Part | Useful Equation |
| (i.e. a constant) |  |
| (i.e. a linear multiple of |  |
|  |  |
|  |  |
|  |  |

**Example 1** – find the closed form equation of a first order, linear, homogeneous equation

,

The next term is the previous one multiplied by four and the first term is two.

i.e.

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Position |  |  |  |  |  |
| Term |  |  |  |  |  |

**Note…**

In the absence of the first term, we would know the format of the sequence is but wouldn’t know the exact sequence for sure. We could find a general solution but need the starting term to obtain a particular solution.

**Example 2** – find the closed form equation of a first order, linear, homogeneous equation

The next term is the previous one multiplied by five.

i.e…

🡨 this is known as the auxiliary equation

Given …

**Note…**

Equivalently, staring with a first term (where it’s position one but ), this would be…

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**Example 1 again** – a better general method for finding the closed form equation of a first order, linear, homogeneous equation, that we’ll also use later when dealing with non-homogeneous equations

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Er, what power?, try , where also …

🡨 this is the auxiliary equation

🡨 this is the general solution

🡨 this is the particular solution

**Example 3** – find the closed form equation of a first order, linear, non-homogeneous equation

,

We’ll solve the homogeneous part of the equation first, then separately deal with the +4, then put both parts together.

(auxiliary equation)

🡨 non-homogeneous equation so this is known as the complementary function

The non-homogeneous part is a constant so use as the useful equation.

and

Putting both parts together gives…

(general solution)

Initial condition…

(and )

So the particular solution is…

**Example 4** - find the closed form equation of a first order, linear, non-constant non-homogeneous equation

,

We’ll solve the homogeneous part of the equation first, then separately deal with the , then put both parts together.

(auxiliary equation)

(complementary function)

The non-homogeneous part is so use as the useful equation, where .

and

Putting both parts together gives…

(general solution)

Initial condition…

(and )

So the particular solution is…

Your turn…