**Roots of Quadratics**

$$\left(x-α\right)\left(x-β\right)=x^{2}-\left(α+β\right)+αβ=0$$

$$ax^{2}+bx+c=0⇔x^{2}+\frac{b}{a}x+\frac{c}{a}=0$$

Sum of roots $=α+β=-\frac{b}{a}$

Product of roots $=αβ=\frac{c}{a}$

|  |  |  |
| --- | --- | --- |
| New Roots | New Sum | New Product |
| $$α^{2}, β^{2}$$ | $$α^{2}+β^{2}=\left(α+β\right)^{2}-2αβ$$ | $$α^{2}β^{2}=\left(αβ\right)^{2}$$ |
| $$\frac{1}{α}, \frac{1}{β}$$ | $$\frac{1}{α}+\frac{1}{β}=\frac{α+β}{αβ}$$ | $$\frac{1}{α}×\frac{1}{β}=\frac{1}{αβ}$$ |
| $$α^{3}, β^{3}$$ | $$α^{3}+β^{3}=\left(α+β\right)^{3}-3αβ\left(α+β\right)$$ | $$\left(αβ\right)^{3}$$ |
| $$5α+\frac{1}{α}, 5β+\frac{1}{β}$$ | $$5α+\frac{1}{α}+5β+\frac{1}{β}=5\left(α+β\right)+\frac{α+β}{αβ}$$ | $$\left(5α+\frac{1}{α}\right)\left(5β+\frac{1}{β}\right)$$$$=25αβ+\frac{5\left(α+β\right)^{2}}{αβ}-10+\frac{1}{αβ}$$ |

For a cubic with roots α, β, γ …

$$\left(x-α\right)\left(x-β\right)\left(x-γ\right)=x^{3}-\left(α+β+γ\right)x^{2}+\left(αβ+αγ+βγ\right)x-αβγ=0$$

$$ax^{3}+bx^{2}+cx+d=0⇔x^{3}+\frac{b}{a}x^{2}+\frac{c}{a}x+\frac{d}{a}=0$$

Sum of roots $=α+β+γ=\frac{b}{a}$

Sum of pairs $=αβ+αγ+βγ=-\frac{c}{a}$

Product of roots $=αβγ=\frac{d}{a}$