**Finding Stationary Points on Graphs of Rational Functions**

**Graphs with** **one stationary point**.



Process to find coordinates of the stationary point:

$$k=\frac{x^{2}+2x-3}{x^{2}+2x+6}$$

$$k\left(x^{2}+2x+6\right)=x^{2}+2x-3$$

$$kx^{2}+2kx+6k-x^{2}-2x+3=0$$

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$$x^{2}\left(k-1\right)+x\left(2k-2\right)+\left(6k+3\right)=0$$

For equal roots, $b^{2}-4ac=0$:

$$\left(2k-2\right)^{2}-4\left(k-1\right)\left(6k+3\right)=0$$

$$-20k^{2}+4k+16=0$$

$$5k^{2}-k-4=0$$

$$\left(5k+4\right)\left(k-1\right)=0$$

$k=\frac{-4}{5}$, $k=1$

No point on graph for which $y=1$, therefore stationary point at $y=-\frac{4}{5}$.

Use \*\*\* to find *x* value:

$$x^{2}\left(\frac{-4}{5}-1\right)+x\left(\frac{-8}{5}-2\right)+\left(\frac{-24}{5}+3\right)=0$$

$$\frac{-9}{5}x^{2}+\frac{-18}{5}x-\frac{9}{5}=0$$

$$x^{2}+2x+1=0$$

$$\left(x+1\right)^{2}=0$$

$$x=-1$$

Coordinates of minimum point are $\left(-1,\frac{-4}{5} \right)$.

**Graphs with** **two stationary points**.



Process to find coordinates of the stationary point:

$$k=\frac{2x^{2}+5x+3}{4x^{2}+5x+3}$$

$$k\left(4x^{2}+5x+3\right)=2x^{2}+5x+3$$

$$4kx^{2}+5kx+3k-2x^{2}-5x-3=0$$

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$$x^{2}\left(4k-2\right)+x\left(5k-5\right)+\left(3k-3\right)=0$$

For equal roots, $b^{2}-4ac=0$:

$$\left(5k-5\right)^{2}-4\left(4k-2\right)\left(3k-3\right)=0$$

$$23k^{2}-22k-1=0$$

$$5k^{2}-k-4=0$$

$$\left(23k+1\right)\left(k-1\right)=0$$

$k=\frac{-1}{23}$, $k=1$

Maximum value is $y=1$, minimum value at $y=-\frac{1}{23}$.

Use \*\*\* to find *x* values:

For $k=1$:

$$x^{2}\left(4×1-2\right)+x\left(5×1-5\right)+\left(3×1-3\right)=0$$

$$2x^{2}+0x+0=0$$

$$x=0$$

For $k=-\frac{1}{23}$:

$$x^{2}\left(-\frac{4}{23}-2\right)+x\left(-\frac{5}{23}-5\right)+\left(-\frac{3}{23}-3\right)=0$$

$$-\frac{50}{23}x^{2}-\frac{120}{23}x-\frac{72}{23}=0$$

$$50x^{2}+120x+72=0$$

$$25x^{2}+60x+36=0$$

$$x=-\frac{6}{5}$$

Coordinates of minimum point are $\left(\frac{-6}{5},\frac{-1}{23} \right)$, coordinates of maximum point are $\left(0,1 \right)$.