Write down the proof that $\sqrt{2}$ is irrational

Write down the proof by first principles that
$y = \sin x$ differentiates to $\cos x$

Write down the proof that $\sqrt{2}$ is irrational

Assume $\sqrt{2}$ is rational, i.e. $\sqrt{2} = \frac{a}{b}$ where $\frac{a}{b}$ is a fraction in lowest terms.

$$\sqrt{2} = \frac{a}{b}$$
$$\Longrightarrow 2 = \frac{a^2}{b^2}$$
$$\Longrightarrow 2b^2 = a^2$$

$\Longrightarrow a^2$ must be even

$\Longrightarrow a$ is even

$$\Longrightarrow a = 2p$$

$$\Longrightarrow a^2 = (2p)^2 = 4p^2$$

$$\Longrightarrow 2b^2 = 4p^2$$

$$\Longrightarrow b^2 = 2p^2$$

$\Longrightarrow a$ and b are both even

$\Longrightarrow$ contradiction.

$\Longrightarrow$ our original assumption must be wrong…

$\sqrt{2}$ is irrational.

Write down the proof by first principles that $y = \sin x$ differentiates to $\cos x$

$$y = \sin(x)$$

$$\frac{dy}{dx} = \lim_{h \to 0} \frac{\sin(x+h) - \sin(x)}{h}$$

$$\frac{dy}{dx} = \lim_{h \to 0} \frac{\sin(x)\cos(h) + \cos(x)\sin(h) - \sin(x)}{h}$$

$$\frac{dy}{dx} = \lim_{h \to 0} \frac{\sin(x)\left(1 - \frac{h^2}{2}\right) + \mathrm{h}\cos(x) - \sin(x)}{h}$$

$$\frac{dy}{dx} = \lim_{h \to 0} \frac{-\frac{h^2}{2}\sin(x) + h\cos(x)}{h}$$

$$\frac{dy}{dx} = \lim_{h \to 0} (-\frac{h}{2}\sin(x) + \cos(x))$$

$$\frac{dy}{dx} = \cos(x)$$