**What’s this** $e^{iπ}$ **thing?**

Using Maclaurin Expansions of $e^{x}$, $sinθ$ and $cosθ$ together with $i^{2}=-1$, we derive the following…

$$e^{x}=1+x+\frac{x^{2}}{2!}+\frac{x^{3}}{3!}+\frac{x^{4}}{4!}+\frac{x^{5}}{5!}+…$$

$$e^{iθ}=1+iθ+\frac{\left(iθ\right)^{2}}{2!}+\frac{\left(iθ\right)^{3}}{3!}+\frac{\left(iθ\right)^{4}}{4!}+\frac{\left(iθ\right)^{5}}{5!}+…$$

$$=1+iθ+\frac{-θ^{2}}{2!}+\frac{-iθ^{3}}{3!}+\frac{θ^{4}}{4!}+\frac{iθ^{5}}{5!}+…$$

$$sinθ=θ-\frac{θ^{3}}{3!}+\frac{θ^{5}}{5!}-\frac{θ^{7}}{7!}+…$$

$$isinθ=iθ-\frac{iθ^{3}}{3!}+\frac{iθ^{5}}{5!}-\frac{iθ^{7}}{7!}+…$$

$$cosθ=1-\frac{θ^{2}}{2!}+\frac{θ^{4}}{4!}-\frac{θ^{6}}{6!}+…$$

$$cosθ+isinθ=1-\frac{θ^{2}}{2!}+\frac{θ^{4}}{4!}-\frac{θ^{6}}{6!}+…+ iθ-\frac{iθ^{3}}{3!}+\frac{iθ^{5}}{5!}-\frac{iθ^{7}}{7!}+…$$

$$=1+ iθ-\frac{θ^{2}}{2!}-\frac{iθ^{3}}{3!}+\frac{θ^{4}}{4!}+\frac{iθ^{5}}{5!}-\frac{θ^{6}}{6!}-\frac{iθ^{7}}{7!}+…$$

$$=e^{iθ}$$

$$r\left(cosθ+isinθ\right)=re^{iθ}$$

For the case where $r=1$ and $θ=π$…

$$\left(cosπ+isinπ\right)=\left(-1+0\right)=e^{iπ}=-1$$

And hence…

$$e^{iπ}=-1$$