## Core 3 Function Questions

7 (a) The sketch shows the graph of $y=\sin ^{-1} x$.


Write down the coordinates of the points $P$ and $Q$, the end-points of the graph.

> (2 marks)
(b) Sketch the graph of $y=-\sin ^{-1}(x-1)$.

8 The functions $f$ and $g$ are defined with their respective domains by

$$
\begin{array}{ll}
\mathrm{f}(x)=x^{2} & \text { for all real values of } x \\
\mathrm{~g}(x)=\frac{1}{x+2} & \text { for real values of } x, \quad x \neq-2
\end{array}
$$

(a) State the range of f . (1 mark)
(b) (i) Find $\operatorname{fg}(x)$. (1 mark)
(ii) Solve the equation $\operatorname{fg}(x)=4$. (4 marks)
(c) (i) Explain why the function f does not have an inverse. (1 mark)
(ii) The inverse of g is $\mathrm{g}^{-1}$. Find $\mathrm{g}^{-1}(x)$. (3 marks)

4 (a) Sketch and label on the same set of axes the graphs of:
(i) $y=|x|$; (1 mark)
(ii) $y=|2 x-4|$.
(b) (i) Solve the equation $|x|=|2 x-4|$.
(ii) Hence, or otherwise, solve the inequality $|x|>|2 x-4|$.

8 A function f is defined by $\mathrm{f}(x)=2 \mathrm{e}^{3 x}-1$ for all real values of $x$.
(a) Find the range of f .
(b) Show that $\mathrm{f}^{-1}(x)=\frac{1}{3} \ln \left(\frac{x+1}{2}\right)$.
(c) Find the gradient of the curve $y=\mathrm{f}^{-1}(x)$ when $x=0$.

9 The diagram shows the curve with equation $y=\sin ^{-1} 2 x$, where $-\frac{1}{2} \leqslant x \leqslant \frac{1}{2}$.

(a) Find the $y$-coordinate of the point $A$, where $x=\frac{1}{2}$.

3 The functions $f$ and $g$ are defined with their respective domains by

$$
\begin{aligned}
& \mathrm{f}(x)=3-x^{2}, \quad \text { for all real values of } x \\
& \mathrm{~g}(x)=\frac{2}{x+1}, \quad \text { for real values of } x, x \neq-1
\end{aligned}
$$

(a) Find the range of $f$.
(b) The inverse of g is $\mathrm{g}^{-1}$.
(i) Find $\mathrm{g}^{-1}(x)$.
(ii) State the range of $\mathrm{g}^{-1}$.
(c) The composite function gf is denoted by h .
(i) Find $\mathrm{h}(x)$, simplifying your answer.
(ii) State the greatest possible domain of h .

7 (a) Sketch the graph of $y=|2 x|$.
(b) On a separate diagram, sketch the graph of $y=4-|2 x|$, indicating the coordinates of the points where the graph crosses the coordinate axes.
(c) Solve $4-|2 x|=x$. (3 marks)
(d) Hence, or otherwise, solve the inequality $4-|2 x|>x$.

3 (a) Solve the equation $\operatorname{cosec} x=2$, giving all values of $x$ in the interval $0^{\circ}<x<360^{\circ}$.
(b) The diagram shows the graph of $y=\operatorname{cosec} x$ for $0^{\circ}<x<360^{\circ}$.

(i) The point $A$ on the curve is where $x=90^{\circ}$. State the $y$-coordinate of $A$.
(ii) Sketch the graph of $y=|\operatorname{cosec} x|$ for $0^{\circ}<x<360^{\circ}$. (2 marks)
(c) Solve the equation $|\operatorname{cosec} x|=2$, giving all values of $x$ in the interval $0^{\circ}<x<360^{\circ}$.
(2 marks)

5 The functions $f$ and $g$ are defined with their respective domains by

$$
\begin{aligned}
& \mathrm{f}(x)=\sqrt{x-2} \quad \text { for } x \geqslant 2 \\
& \mathrm{~g}(x)=\frac{1}{x} \quad \text { for real values of } x, \quad x \neq 0
\end{aligned}
$$

(a) State the range of f .
(b) (i) Find $\mathrm{fg}(x)$.
(ii) Solve the equation $\operatorname{fg}(x)=1$.
(c) The inverse of f is $\mathrm{f}^{-1}$. Find $\mathrm{f}^{-1}(x)$.

