FP3 Introduction to Differential Equations Questions

3 (a) Show that $y = x^3 - x$ is a particular integral of the differential equation

$$\frac{dy}{dx} + \frac{2xy}{x^2 - 1} = 5x^2 - 1$$ *(3 marks)*

(b) By differentiating $(x^2 - 1)y = c$ implicitly, where y is a function of x and c is a constant, show that $y = \frac{c}{x^2 - 1}$ is a solution of the differential equation

$$\frac{dy}{dx} + \frac{2xy}{x^2 - 1} = 0$$ *(3 marks)*

(c) Hence find the general solution of

$$\frac{dy}{dx} + \frac{2xy}{x^2 - 1} = 5x^2 - 1$$ *(2 marks)*

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3 (a) Show that $\sin x$ is an integrating factor for the differential equation

$$\frac{dy}{dx} + (\cot x)y = 2\cos x$$ *(3 marks)*

(b) Solve this differential equation, given that $y = 2$ when $x = \frac{\pi}{2}$. *(6 marks)*

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3 (a) Show that x^2 is an integrating factor for the first-order differential equation

$$\frac{dy}{dx} + \frac{2}{x}y = 3(x^3 + 1)^{\frac{1}{2}}$$ *(3 marks)*

(b) Solve this differential equation, given that $y = 1$ when $x = 2$. *(6 marks)*

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3 By using an integrating factor, find the solution of the differential equation

$$\frac{dy}{dx} + (\tan x)y = \sec x$$

given that $y = 3$ when $x = 0$. *(8 marks)*
