Trigonometry

WORKSHEETS

Trigonometry

The worksheets available in this unit DO NOT constitute a course since no instructions or worked examples are offered, and there are far too many of them. They are offered here in the belief that the teacher is the one best placed to formulate and provide the 'teaching element', but that practice will be needed in support.

So, here are many sheets which will provide that practice. A selection will have to be made to suit the enviroment in which the teaching is taking place. The brief commentary given with the index is meant only to provide guidance on what the sheets contain, it does not imply any particular ordering or structure of a course. Whether for normal practice, supplementary exercises, revision purposes or homework, it should be possible to find something suitable here.

As a rough guide, the contents are grouped under two main headings

- **Basic Trigonometry** deals only with right-angled triangles and the use of Pythagoras, Tangents and Sines.
- **Further Trigonometry** deals with the general triangle; Sine and Cosine Rules; Heron's and the $\frac{1}{2}\,ab\sin C$ formulas for area; Circumscribed, Inscribed and Escribed Circles; angles greater than 90º.

General Notes

It is assumed that all working is done on a calculator which implies (for trigonometry) that it must be of the 'scientific' variety. Some particular thoughts to bear in mind about calculators are

First, is the vexed business of how much working needs to be shown. In many elementary problems it can all be done on the calculator so that only the answer appears on the paper. There are obvious drawbacks to this - particularly in an examination - but it is difficult to define the rules as to what is a necessary minimum.

Second, is that of “degree of accuracy”. A final value to 3 significant figures nearly always fits the bill, but many pupils have difficulty with this idea. It is better left to the teacher to decide. Where possible it should be 3 s.f., otherwise 1 or 2 decimal places are usually adequate, and perhaps angles could always be given to 1 decimal place. A related difficulty is whether crude truncation or correct rounding is sought. Again, it is up to the teacher. Probably the single most important idea concerning calculators that all pupils should leave school with, is that there is nothing intrinsically “more accurate” in writing down every figure in a calculator display. Together with the continual thought that, just because a number appears on a calculator display it does not mean that it is necessarily correct!

Third is when, in solving more complex problems, it is necessary to work in stages, using the information gained in one stage to work out something in the next stage, and so on. This notion is not difficult to grasp. What is a little more difficult to understand and handle, is the idea of “cumulative error”. Even if no mistake is made, we do introduce an error when we truncate the reading on the calculator, especially if correct rounding is not being used. This error is then carried forward and used in the next stage. The only (relatively) safe and reasonable way of dealing with this is to say that, whatever degree of accuracy we might require in our final answer, then we should carry forward at least 2 more figures in our intermediate stages. Since, in practice, the calculator is doing all the arithmetic, it is usually quite simple to carry all possible figures forward just by leaving them on the display and working with that. However, this does demand a degree of sophistication and organisation in calculator usage that many pupoils find difficult, especially if it requires the use of the calculator memory.

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Notes on Trigonometry

Some brief reminders.

Pupils do need to be familiar with two forms of notation for triangles.

- The descriptive AB, BC etc for identifying edges, and $\angle$ABC for angles
- The formulaic a, b, c for edges with A, B, C for angles.

The first is necessary to be able to identify any triangle and its constituent parts no matter how complex the diagram, while the second enables the relevant formulas to be stated in the most economical way.

When dealing only with right-angled triangles there is no need to use cosines at all. Pythagoras, tans and sines can cope with every situtation. So, in the interests of simplicity, cosines have not been specifically mentioned here in the early work. Of course they can always be used if wished.

Before applying trigonometry to 'real' problems, some thought must be given to the background knowledge needed. The most commonly needed knowledge concerns angles of elevation and depression, together with compass bearings and directions. Then there is the usual assumption that the ground is level and horizontal, and walls and poles etc are vertical.

In problems, dimensions must always be borne in mind of course, and units must be consistent. But the usual problems set for pupils do not have inconsistencies in them, and there are none here. (Try putting in a problem about finding the angle of descent of an aeroplane which loses 1000 metres of height along a glide-path of 5 kilometres!) In the graphical work given here, units are not stated at all.

Keep in mind there is an order of difficulty for pupils working with trig-ratios. Finding the size of an angle is the easiest. Finding an edge provides two cases. With tans, for example, finding the 'opposite' (the multiplication case) is easier than finding the 'adjacent' (the division case). Far too many just go for multiplication, whether it appropriate or not.

Abstracting the necessary right-triangles from 3-dimensional situations provides difficulties for most pupils. Encourage the use of primitive constructions, like holding some pens and pencils together at the top to represent the edges of a pyramid. Have some 'wire' models available to view. Make up a big model for demonstration purposes. Eight one-metre sticks and some lumps of plasticine enable a pyramid to be assembled very quickly, and the various right-triangles can pointed out using more sticks or some string. (Don't attempt cuboids, they are not rigid enough, and provide more amusement than instruction!)

A Diversion

Do you use the 'meaningless' string of letters

SOHCAHTOA

with its attendant mnemonic

Should **O**ld **H**arry **C**atch **A**ny **H**errings **T**rawling **O**ff **A**merica

or some other sentence which you know?

Try getting pupils to invent their own mnemonic.

Having explained what is required, possibly by using an example, get them to write their own before the next lesson. Then have them read their efforts out in turn. (There can be quite a lot of merriment in this.)

Also, you can get them to ask their parents what mnemonics, if any, they learnt at school, possibly in other subjects - the order of the planets seems to be a favourite for this.

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List of Contents

The **TESTS** which are listed, together with answers to all the problems, are not contained in this unit but are provided separately. See the note on **Tests and Answers** about this.

Graphical means that the relevant triangle is drawn with all information written against the appropriate edges or angles, and a **?** indicates what is to be found.

Problems means that the exercises are stated in words and related to a 'real' problem. This usually means the relevant triangle has to be picked out from the given physical situation. A diagram may, or may not, be included.

Basic Trigonometry

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T/22	Tans & Sines	***Graphical***	Finding size of one angle.
T/23	Tans & Sines	***Graphical***	Finding length of one edge.
T/24	Tans & Sines	***Graphical***	Mixture of previous 2 sheets
T/25	Tans & Sines	**TEST**	J & K based on T/24
T/26	Tans & Sines	***Graphical***	Compound shapes
T/27		***Problems***	
T/28		***Problems***	
T/29			*unallocated*
T/30	Tans & Sines	**TEST**	L & M based on ALL previous work

Further Trigonometry

T/31	The General Angle		
T/32	Area of Triangle **ABC** = $\frac{1}{2}\boldsymbol{ab}\sin\mathbf{C}$		
T/33	Sine rule		
T/34	Cosine rule		
T/35	Heron's formula and circles		
T/36		***Problems***	
T/37		***Problems***	
T/38		**TEST**	N & P parallel to T/37

Tests and Answers

Mathematics is a particularly difficult subject to test in the classroom and ensure that the minimum of copying takes place. At this level, with so much working 'lost' in the calculator, it only needs a quick look at somebody else's figures to render a whole question useless - for assessment purposes.

To overcome this, as far as is practicable, all tests are provided in sets of two parallel copies. If these are dealt out properly, then no two adjacent pupils will be working with the same set of figures. Of course, the principles being tested are the same, but it is much easier for an observer to see when copying of that degree is going on.

It is clearly not desirable that tests and answers should be openly available from the Internet. For that reason, the file containing those is locked and needs a password in order to be accessed. The file can be downloaded by anyone, but it needs a password in order to open it.

The route to that file starts at the bottom of the **trol** index page under

Index to Restricted Files

and details about obtaining the necessary password can also be found there.

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Trigonometry

Use Pythagoras' theorem to decide whether each of these triangles is right-angled or not.

Drawings are NOT to scale..

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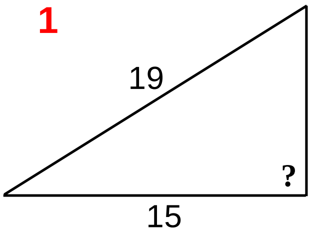

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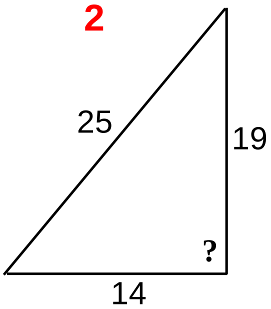

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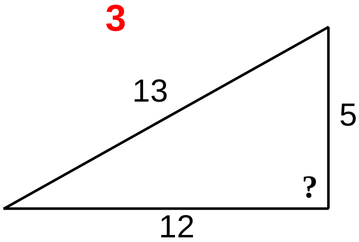

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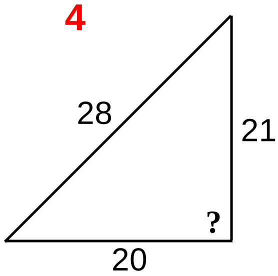

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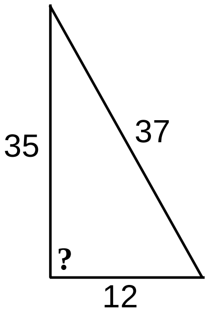

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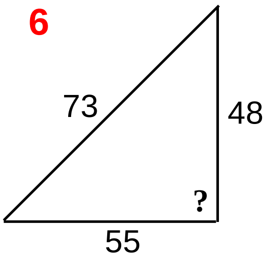

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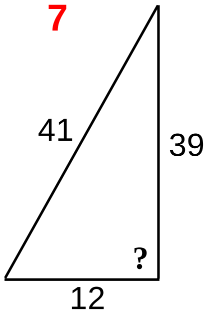

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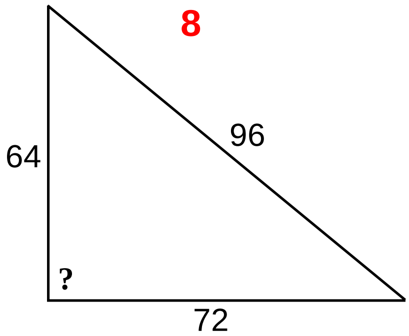

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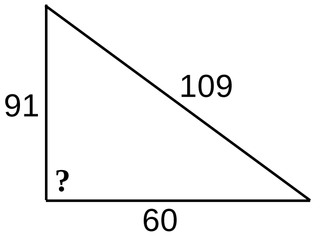

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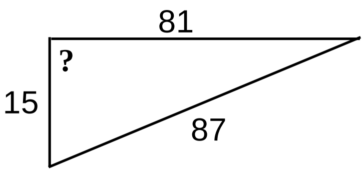

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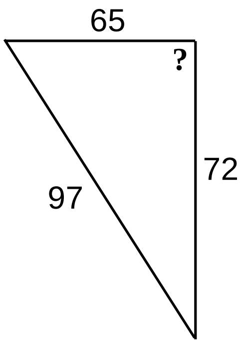

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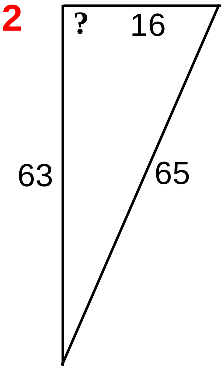

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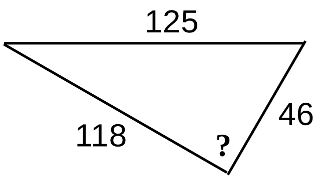

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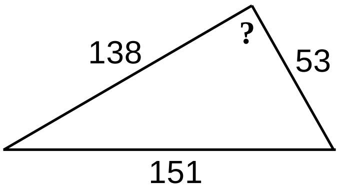

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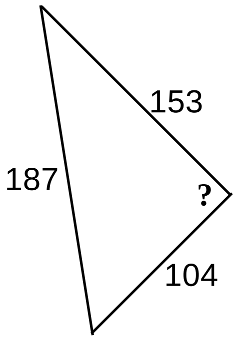

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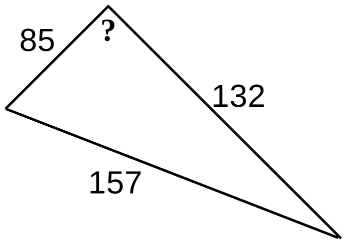

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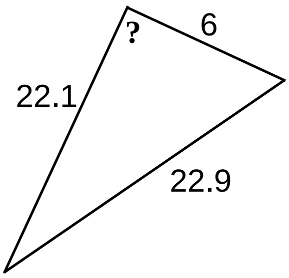

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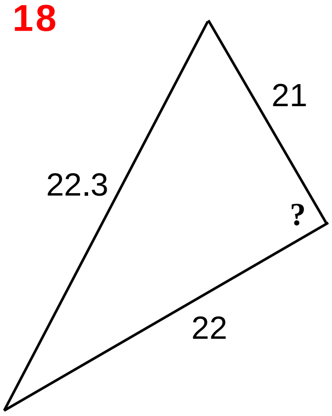

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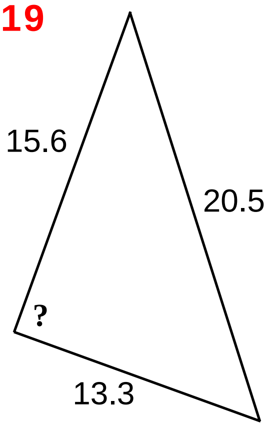

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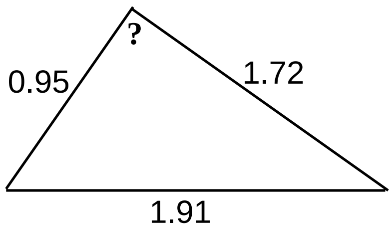

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Trigonometry

Use Pythagoras' theorem to find the length of the **hypotenuse** marked **?** in each of these **right-angled** triangles.

Drawings are NOT to scale.

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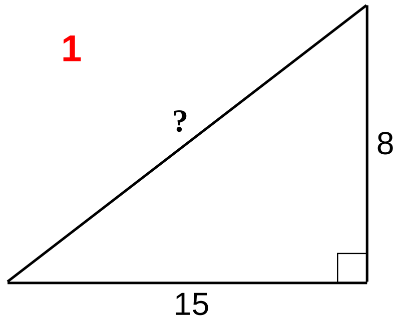

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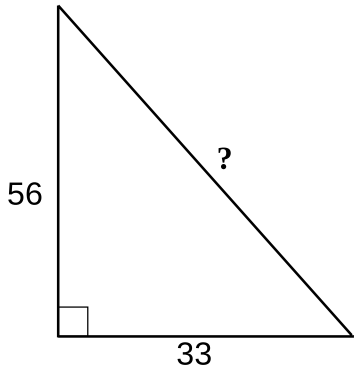

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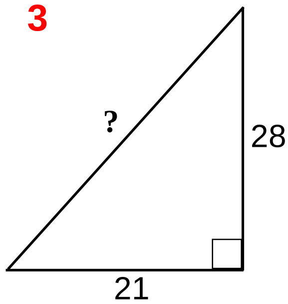

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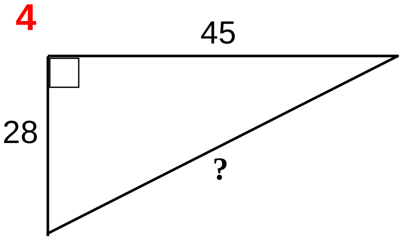

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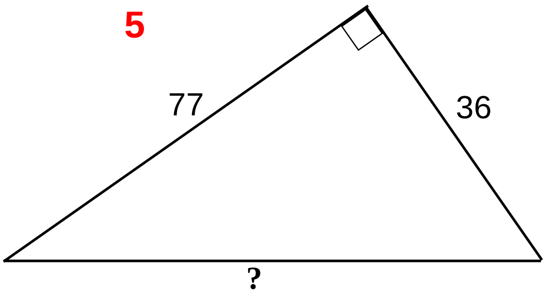

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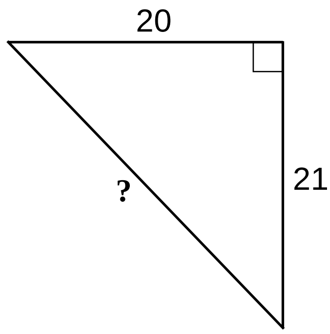

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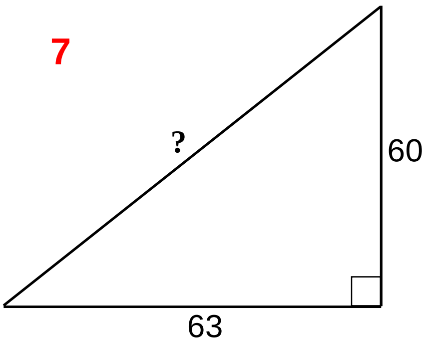

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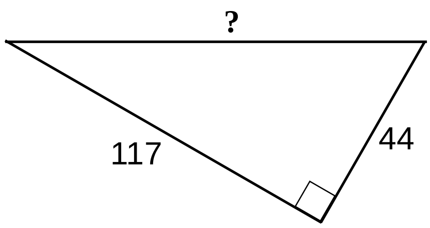

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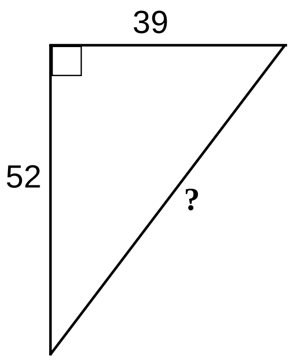

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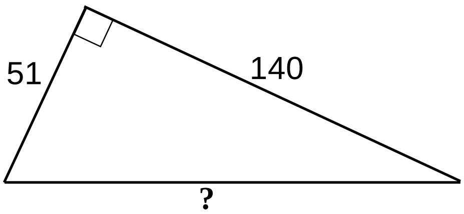

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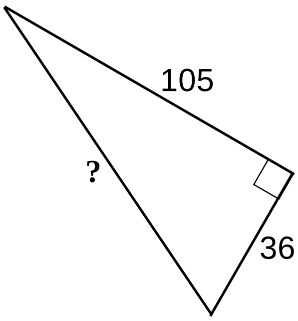

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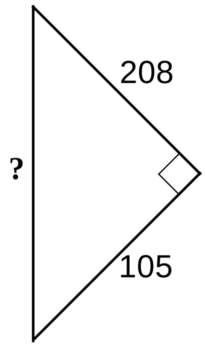

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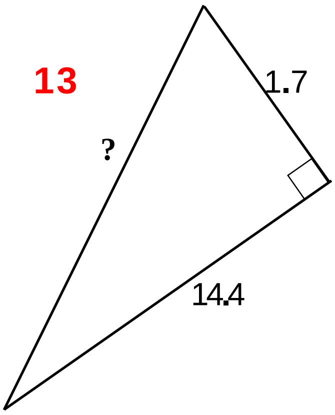

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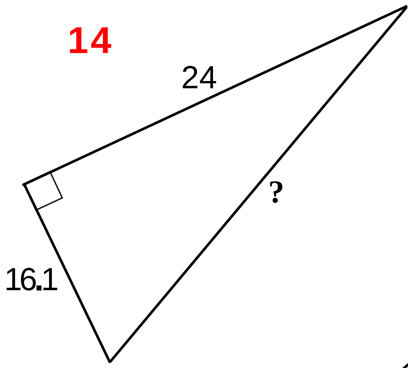

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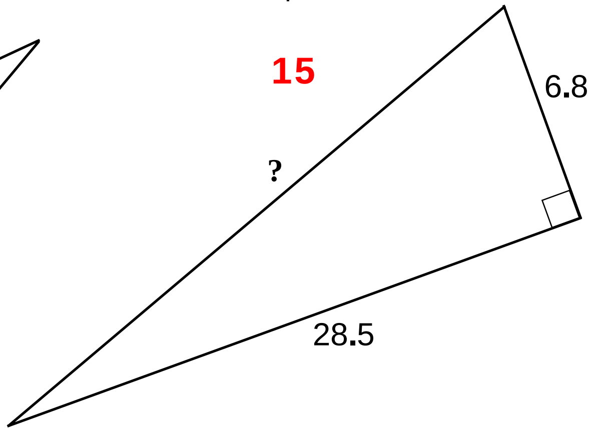

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Trigonometry

Use Pythagoras' theorem to find the length of the edge marked **?** in each of these **right-angled** triangles.

Drawings are NOT to scale.

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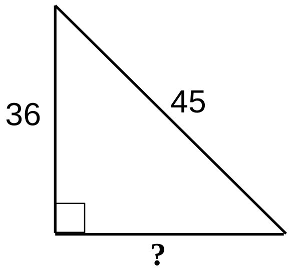

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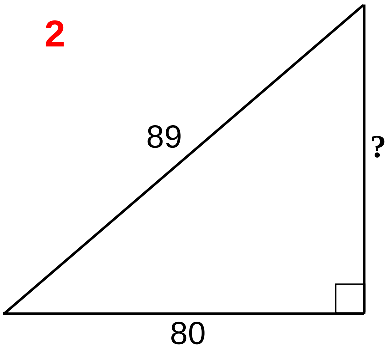

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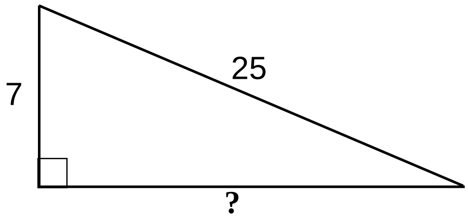

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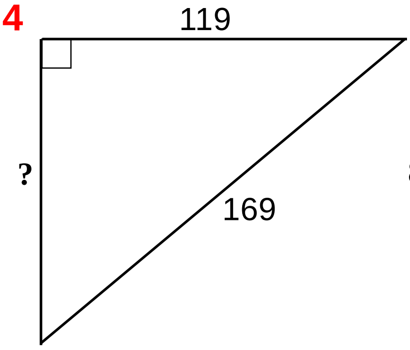

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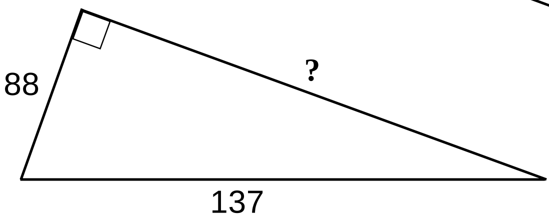

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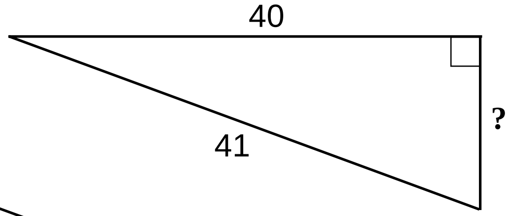

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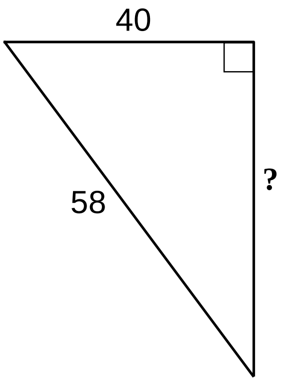

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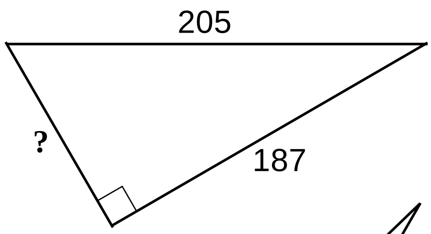

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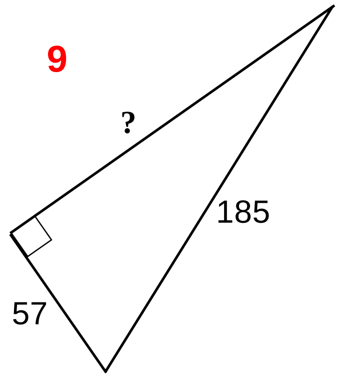

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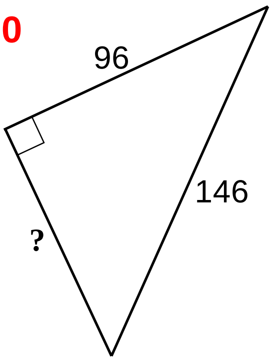

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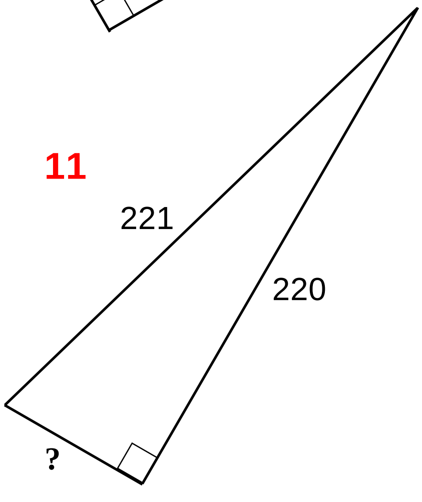

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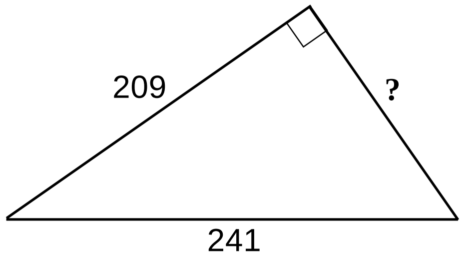

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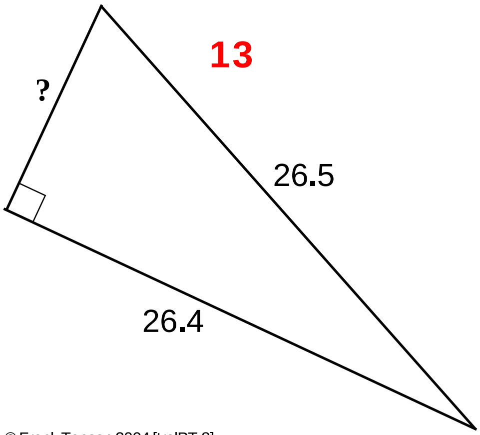

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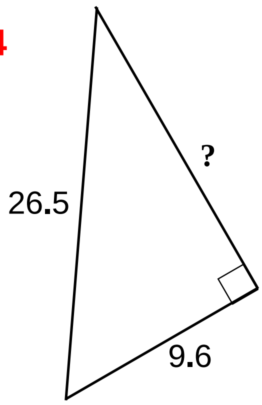

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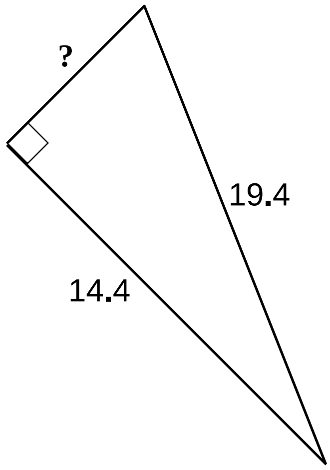

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Trigonometry

Use Pythagoras' theorem to find the length of the edge marked **?, OR** decide whether the triangle is right-angled or not

Drawings are NOT to scale.

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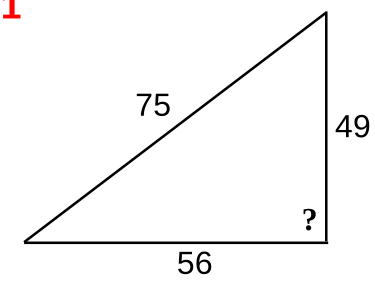

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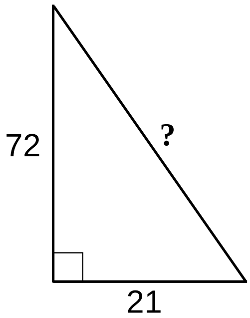

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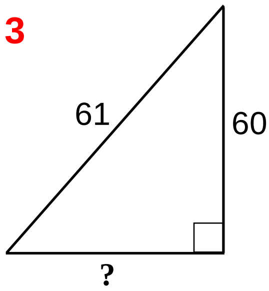

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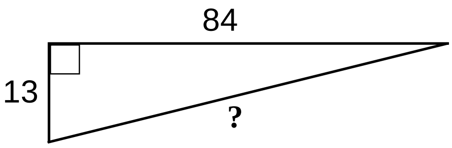

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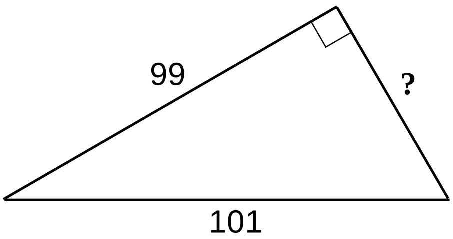

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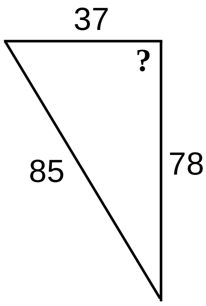

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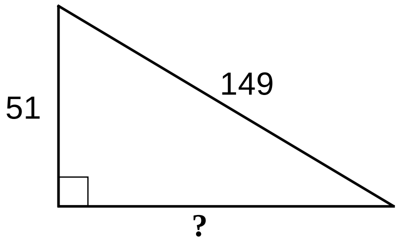

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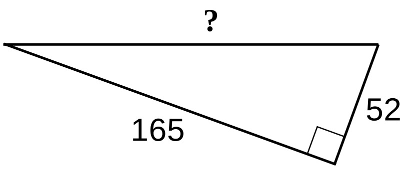

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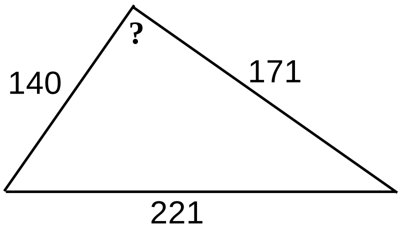

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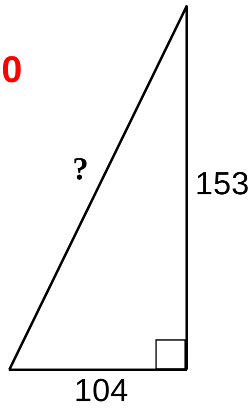

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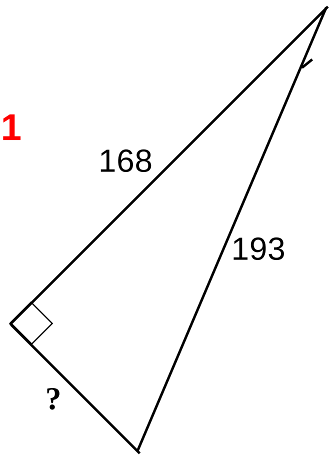

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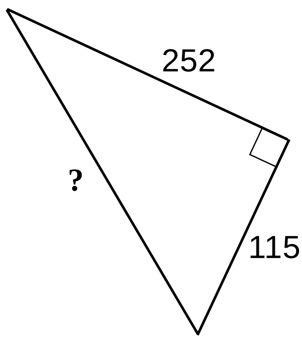

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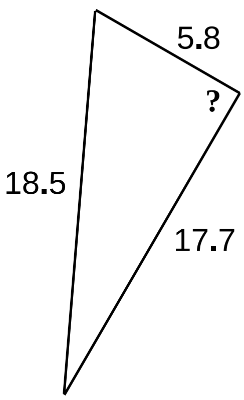

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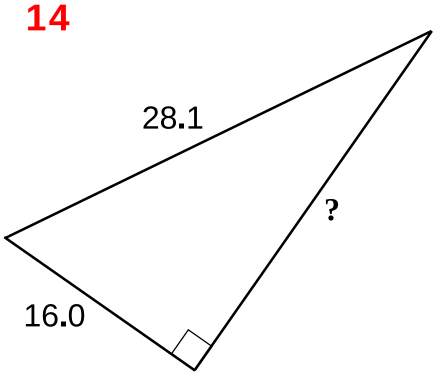

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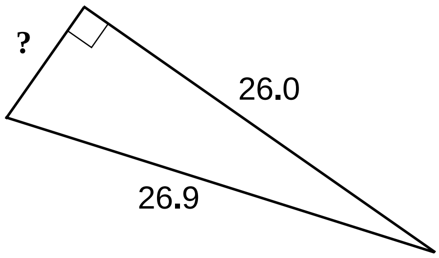

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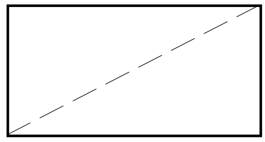
Trigonometry

Pythagorean problems in 2-dimensions.

Drawings are NOT to scale.

1. Find the length of the diagonal of an oblong measuring 5.7 cm by 17.6 cm.



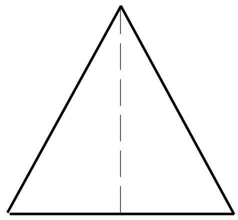
2. A rectangle measures 27 cm by 34 cm. What is the length of its diagonal?

3. The diagonal of a rectangle is 6.7 cm long, and one edge measures 2.3 cm.
   What is the other edge-length of the rectangle?

4. What is the length of the diagonal of a square whose edge-length is 9.6 cm?

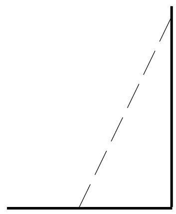
5. Find the length of the edge of a square whose diagonal measures 96 mm

6. An isosceles triangle has base edge of length 70 mm.
   Its perpendicular height is 110 mm.
   Calculate the length of one of its other edges.



7. An isosceles triangle has a base length of 13.8 cm.
   Its other edge length is 21.4 cm.
   What is its perpendicular height?

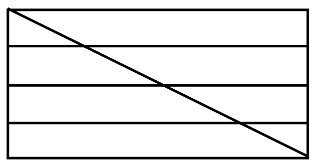
8. A ladder is 7 metres long. It leans against a wall with the foot
   of the ladder 2 metres out from the bottom of the wall.
   How far up the wall does the ladder reach?



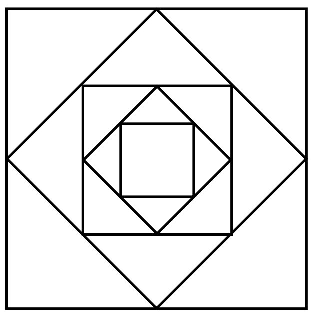
9. In the previous question, if the foot of the ladder is moved another
   2 metres away from the wall, by how much will the top slide down the wall?

10. A straight road 8 miles long runs from A to B.
    Another straight road goes from B to C and is 3.5 miles long.
    The angle between these two roads is a right angle.
    It is proposed to build a new straight road from A to C.
    What distance would be saved on the journey from A to C?

11. A traditional 5-bar gate is 3.5 metres wide.
    The vertical distance between the horizontal bars is 0.5 metres.
    What is the length of the diagonal strut?



12. A pattern is made by drawing a large square, then marking the
    middle-point of each edge and joining them to make another square.
    This is repeated on each square in turn until they become so small
    it is impossible to draw them.
    In one case the 1st square has an edge-length of 14 cm.
    What is the edge-length of the 2nd square?
    And the 3rd?
    How many squares must be drawn until one has an edge less than 2 cm?
    By looking at the sequence of edge-lengths,
    can you see a simple relationship between them?
    If so, then use that to determine the size of the 10th square to be drawn.



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Trigonometry

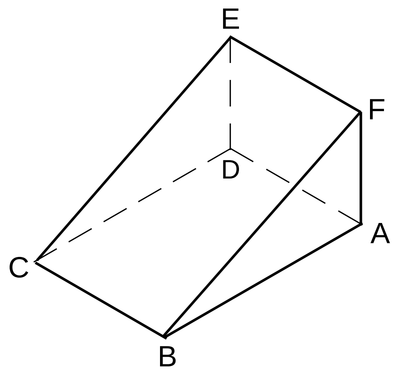
Pythagorean problems in 3-dimensions.

Drawings are NOT to scale.

1. Find the length of the space-diagonal of cuboid measuring 4 metres by 5 metres by 7 metres.

2. What is the greatest length of a thin rod, which must measure an exact number of centimetres, and can fitted into a cuboidal box measuring 14 cm by 8 cm by 5 cm?

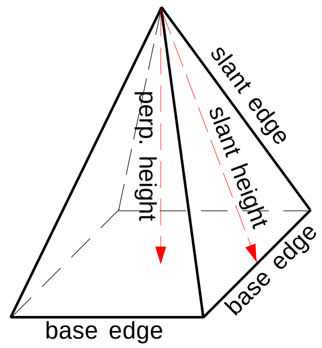
3. The drawing on the right represents a wooden wedge. Face ABCD is in the horizontal plane, face ADEF is in the vertical plane. BCEF is the sloping face.
BC is 10 cm; AB is 14 cm; AF is 4 cm.
If an insect set out to walk up the sloping face, starting from C, it would find that the steepest slope was along the line CE; while the least slope was along the line CF. What is the difference in lengths between the lines of the least and the steepest slopes?



4. A right pyramid has a square base with an edge-length of 15 cm, and a perpendicular height of 20 cm. Calculate its slant height.

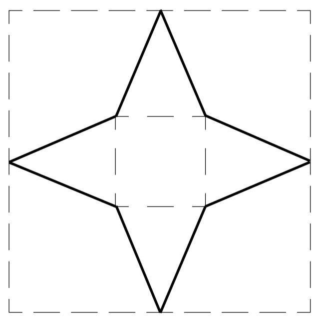
5. What is the perpendicular height of a right square-based pyramid having a base-edge of length 9.8 cm and a slant height of 13.6 cm?

6. A square-based pyramid has a base-edge of length 53 mm and a slant height of 117 mm. Find the length of one its slant edges.



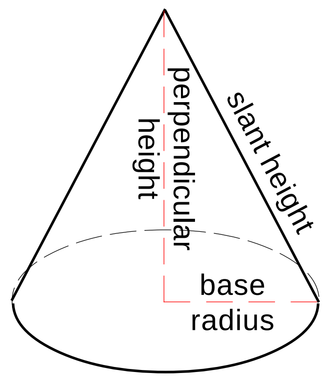
7. The great pyramid of Gizeh (built about 2500 BC) has a square base which measures 226 metres along each edge, and a perpendicular height of 144 metres. What is the length of one of its slant edges?

8. The net of a pyramid can be cut from a square piece of card as shown in the drawing on the right. What size of square would be needed to cut out the net of a square-based pyramid having a base-edge of 10 cm and a perpendicular height of 15 cm?
The necessary square can be made smaller if the net is rotated through 45° relative to the square. What size of square would be needed for the above pyramid in that case?



9. This table gives some details of 3 different right-cones. Calculate the missing values shown as (a) (b) (c)

	base radius	perpendicular height	slant height
Cone 1	5.4 cm	7.3 cm	(a)
Cone 2	2.1 cm	(b)	10.3 cm
Cone 3	(c)	13.6 cm	19.7 cm



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Trigonometry

Find the size of the angle marked **?** in each of these **right-angled** triangles.

Drawings are NOT to scale.

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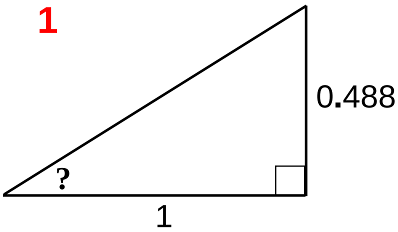

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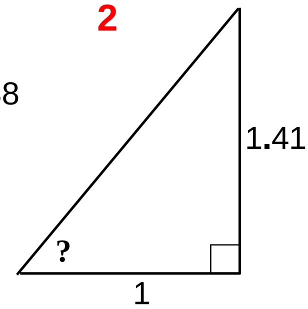

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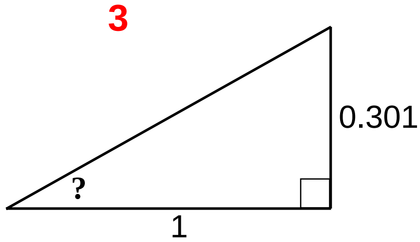

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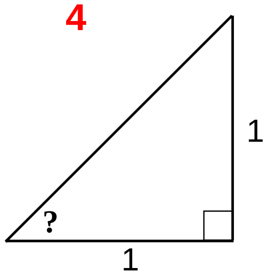

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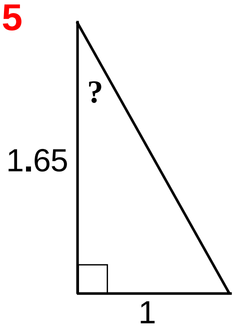

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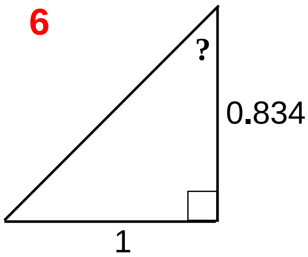

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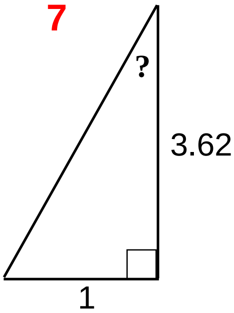

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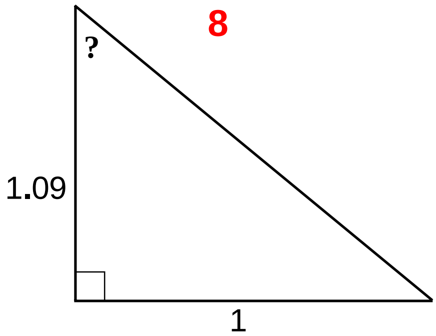

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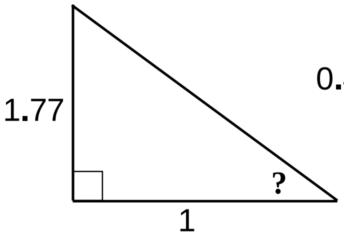

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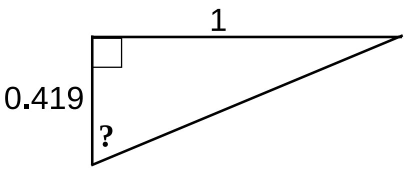

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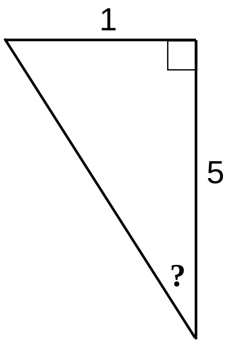

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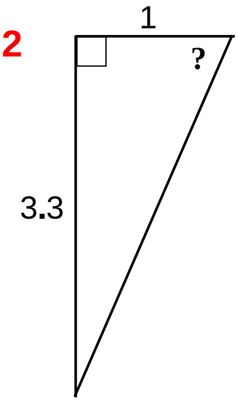

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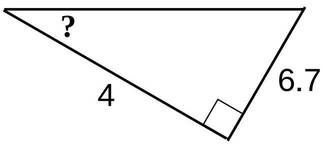

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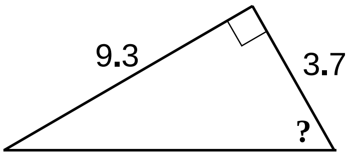

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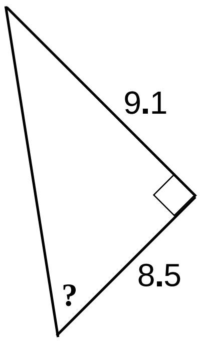

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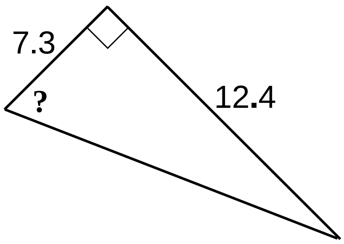

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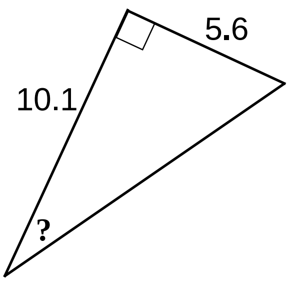

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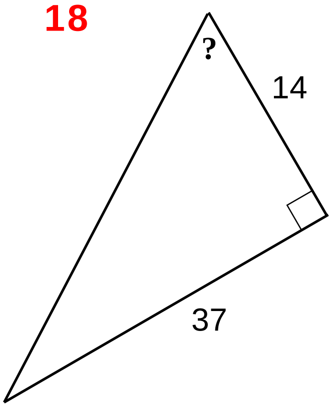

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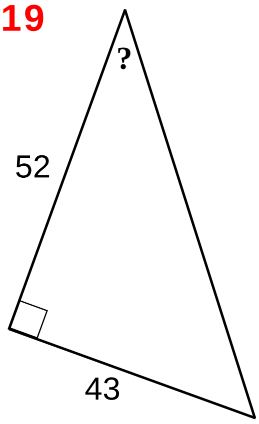

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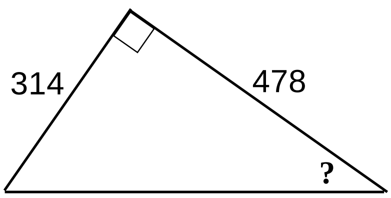

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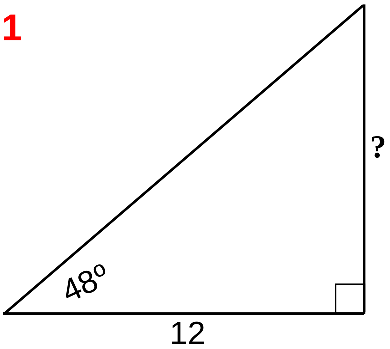
Trigonometry

Find the length of the edge marked **?** in each of these **right-angled** triangles.

Drawings are NOT to scale.

1



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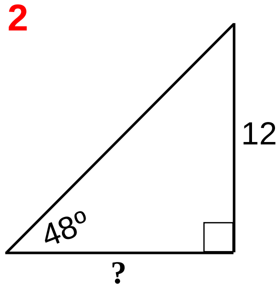

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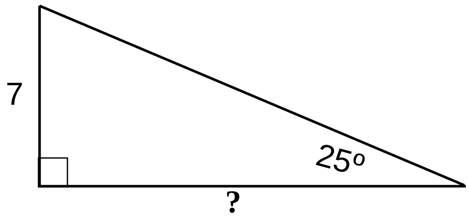

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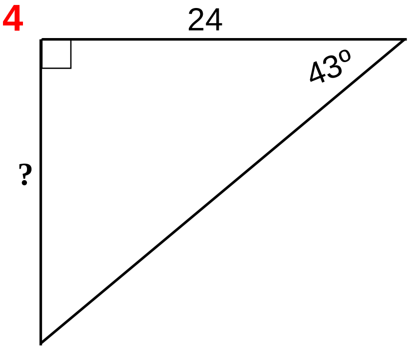

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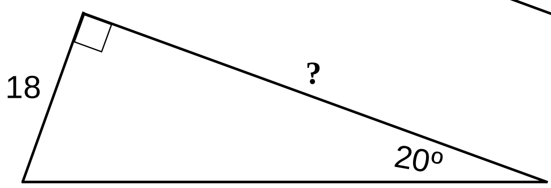

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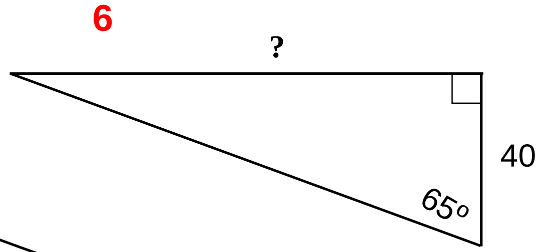

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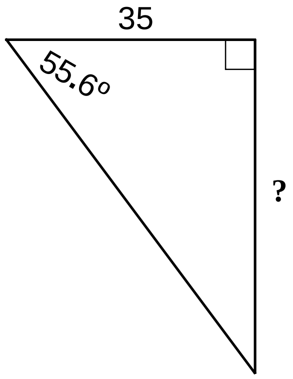

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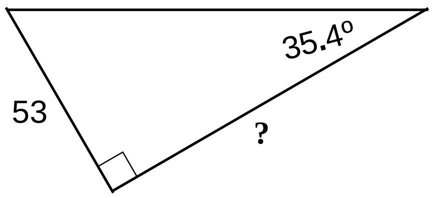

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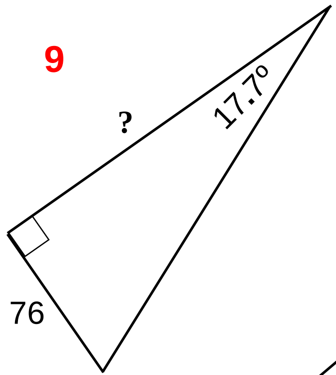

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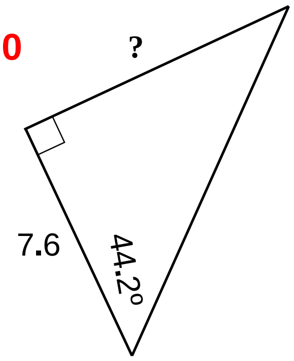

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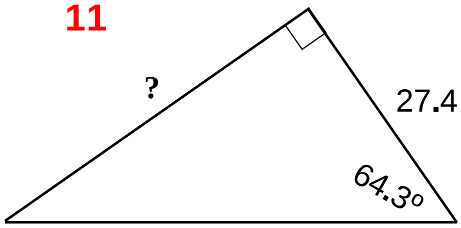

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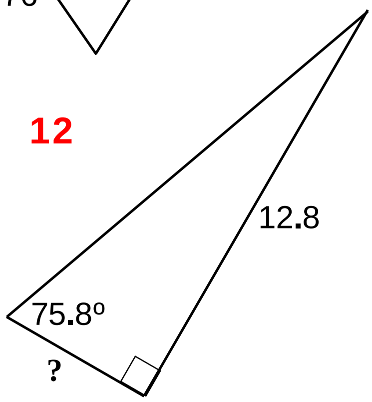

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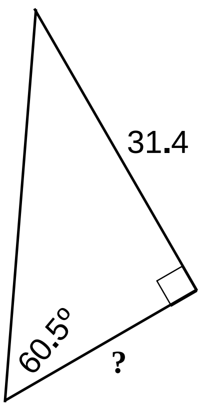

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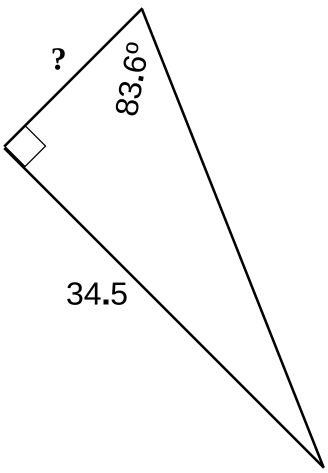

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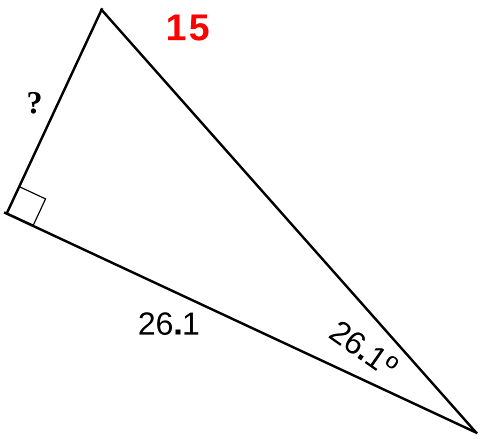

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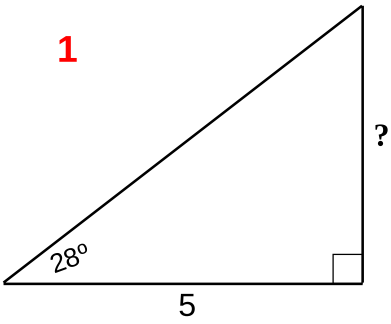
Trigonometry

Find the length of the edge, or size of the angle, marked **?** in each of these **right-angled** triangles.

Drawings are NOT to scale.

1



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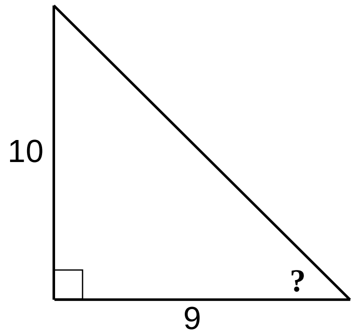

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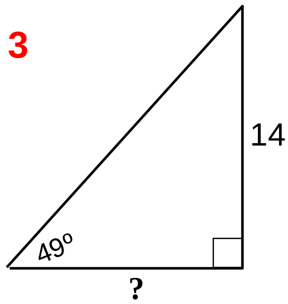

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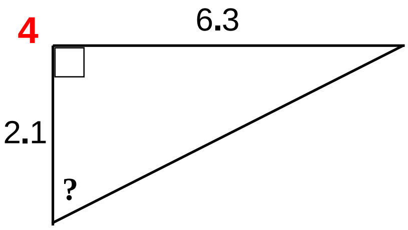

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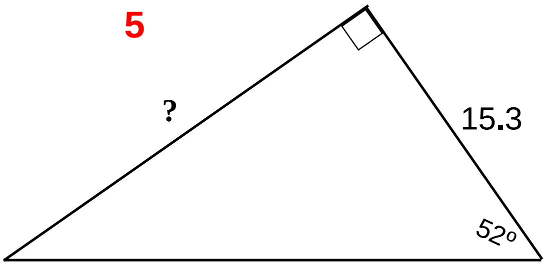

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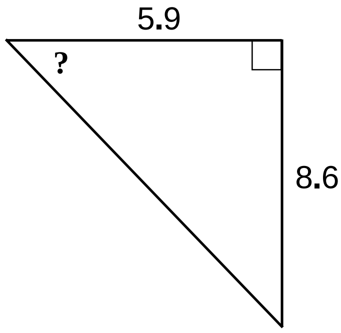

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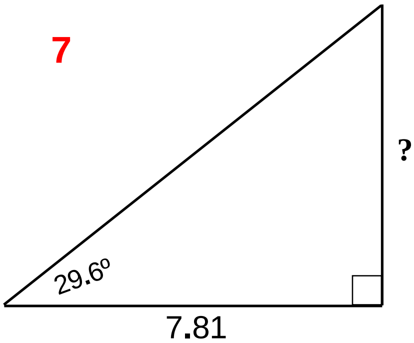

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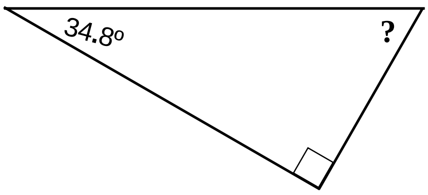

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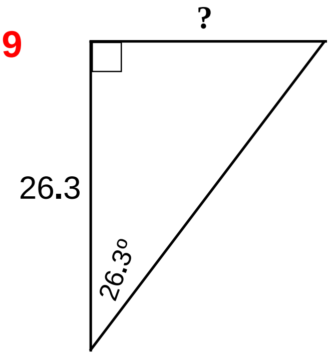

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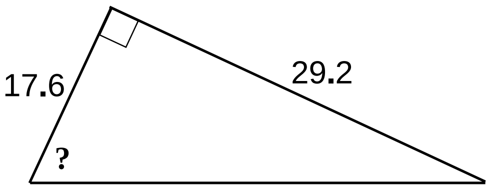

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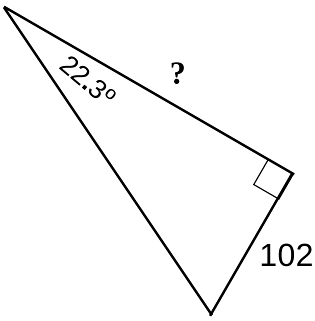

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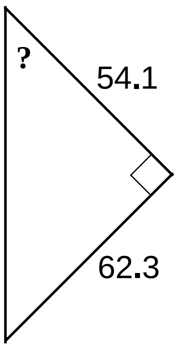

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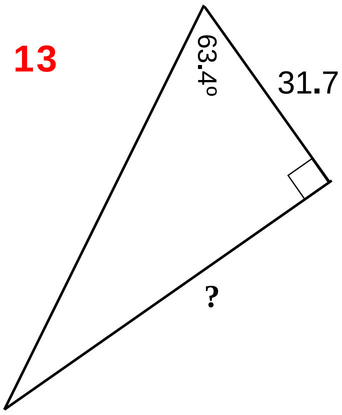

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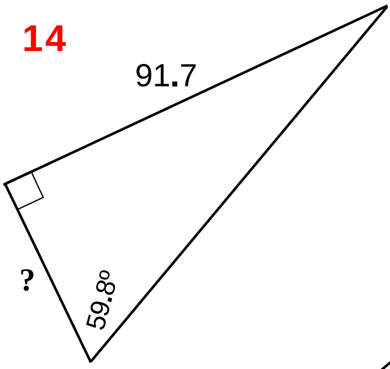

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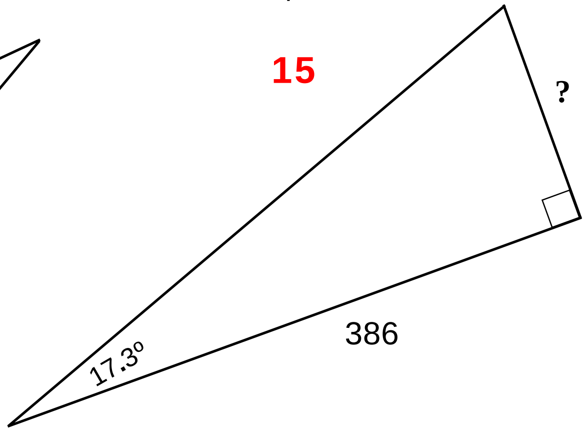

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Trigonometry

(Using tangents)

1. Susan is standing on level ground 70 metres away from a tall tower. From her position, the angle of elevation of the top of the tower is 38°. What is the height of the tower?

2. What would be the angle of elevation of the top of a church steeple, known to be 65 metres tall, to a person standing 135 metres away from it?

3. A flag-pole is 12 metres tall. An observer looks up at an angle of 25 degrees to see the top of the pole. How far is the observer from the foot of the flag-pole?

4. An oblong measures 15 cm by 8 cm. Find the angle between its diagonal and a longer edge.

5. A ladder leaning against a wall makes an angle of 63° with level ground. The foot of the ladder is 3 metres away from the wall. How far up the wall does the ladder reach?

6. A pylon carrying electricity cables has a height of 24 metres. Chris, who is standing some way away from it, measures the angle of elevation of the top of the pylon to be 51.4°. How far away from the pylon is he standing?

7. A telegraph-pole has a single wire stay fastened to its top. The stay is fixed in the ground 3.7 metres from the foot of the pole and makes an angle of 72.5° with the ground. How tall is the telegraph-pole?

8. A 45-metre tall television mast is kept upright by stay-wires which are attached to its top, and also fixed into the ground 17.5 metres away from its foot. What angle do these wires make with the gound?

9. A surveyor takes a sighting of the top of a hill which she knows, from the map, is 460 metres away from her position. From there, the angle of elevation of the top of the hill is 31.4°. What is the height of the hill?

10. From the top of lighthouse which is 50 metres above sea-level, the angle of depression of a boat is observed to be 17.5°. How far is the boat from the lighthouse?

11. A ladder leaning against a wall, reaches to a height of 12 metres when it is at an angle of 67.5 degrees to the ground. How far is the foot of the ladder away from the wall?

12. The diagonal of an oblong makes an angle of 53.8° with the shorter edge, which is 18.3 centimetres long. What is the length of the longer edge?

13. An oblong has edges of length 6.78 cm and 4.35. Find the size of the smallest angle formed where its two diagonals cross.

14. A man estimates that the angle of elevation of the top of a tree is 38 degrees. He then takes 53 paces to get to the tree. He reckons each pace to be about one metre. What will he then work out the height of the tree to be?

15. The two equal angles of an isosceles triangle (which are on its base-edge) each measure 58.4°. The perpendicular height of the triangle is 16.3 cm. What is the length of its base-edge?

16. 75 metres away from a tall post, the angle of elevation of the top of the post is 21.4°

(a) Find the height of the post.

There is a particular mark on the post which is exactly half way up. From that same observation point

(b) what is the angle of elevation of that mark?

17. An upright stick, 1 metre tall, casts a shadow which is 1.35 metres long.

(a) What is the altitude of the sun?

(The sun's altitude is the angle it makes with the horizontal)

At the same time, the shadow of a building is found to be 47 metres long

(b) What is the height of the building?

18. From the top of a cliff, 56 metres above sea-level, two buoys can be seen which lie in a straight line with the observer. The angles of depression of these buoys from the observer, are 28° and 41° respectively. Calculate the distance between the two buoys.

19. Kim sees a pylon whose angle of elevation of the top is 29°, and then walks towards it until that angle is doubled. If the height of the pylon is 27 metres, how far did Kim walk?

20. A tower is 60 metres tall. It stands between two observers, on level ground, and all three (observers and tower) are in a straight line. Each of the observers measures the angle of the elevation of the top of the tower. One sees it as 23° while the other sees it as 54°. How far are the two observers apart?

21. A man standing on the bank of a river notes a particular tree on the opposite bank. He then walks 80 metres along the river-bank in a straight line. On looking back he can see that the angle between the line of his path and a line to the tree is 36°. How wide is the river?

22. A rhombus has diagonals of lengths 6 cm and 9 cm. Calculate the sizes of its interior vertex angles *(The diagonals of a rhombus bisect each other and cross at right angles.)*

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Trigonometry

T/16

Find the size of the angle marked **?** in each of these **right-angled** triangles

Drawings are NOT to scale.

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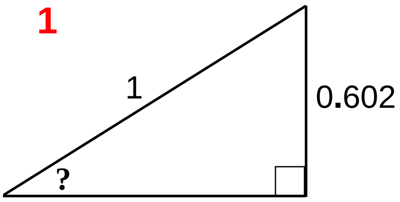

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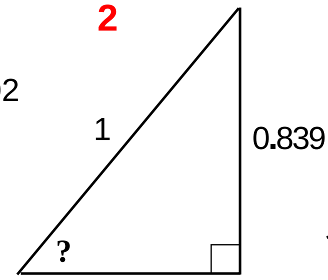

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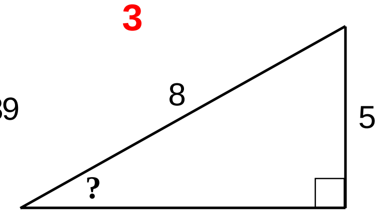

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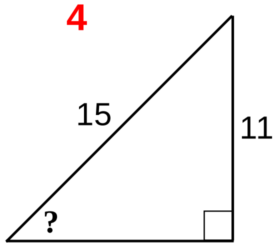

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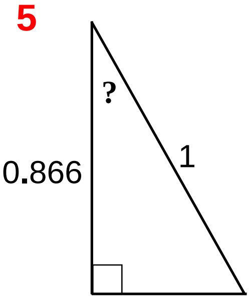

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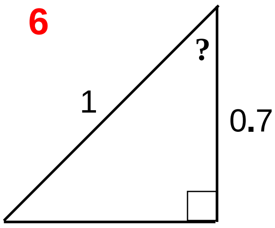

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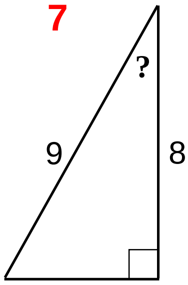

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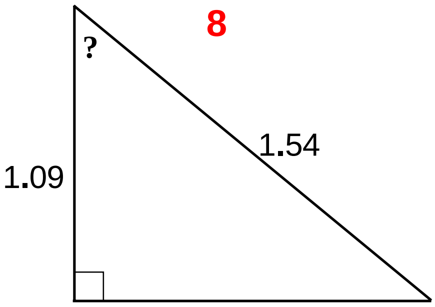

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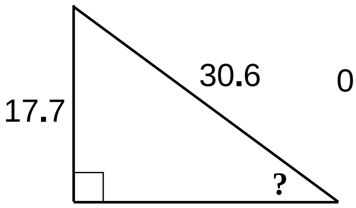

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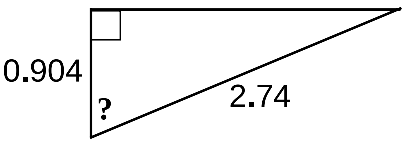

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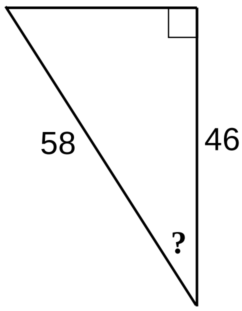

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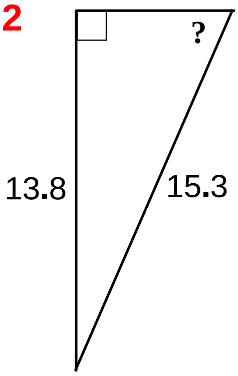

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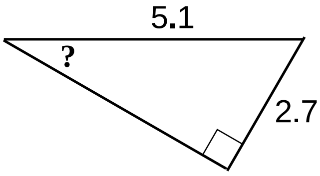

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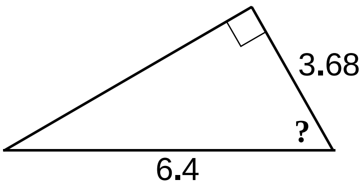

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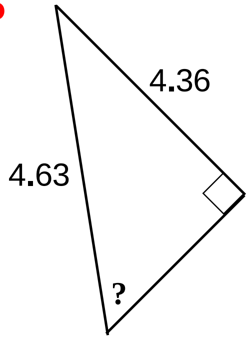

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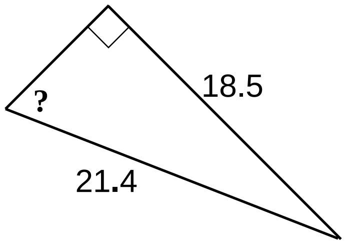

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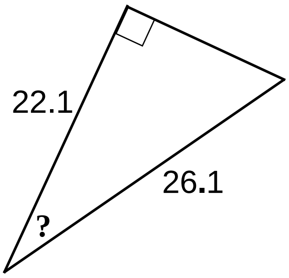

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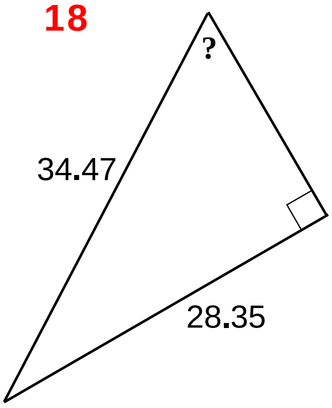

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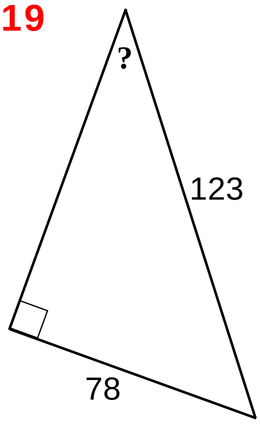

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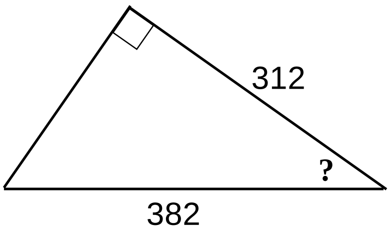

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Trigonometry

Find the length of the edge marked **?** in each of these **right-angled** triangles.

Drawings are NOT to scale.

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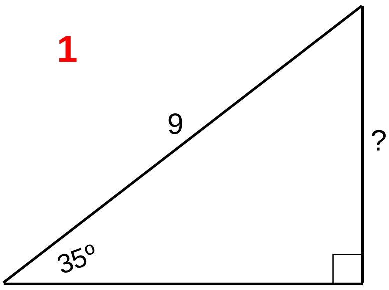

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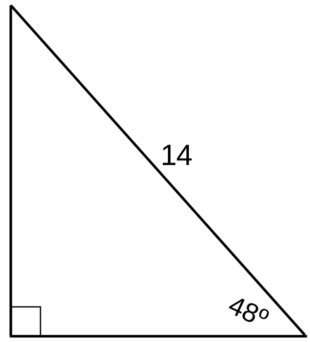

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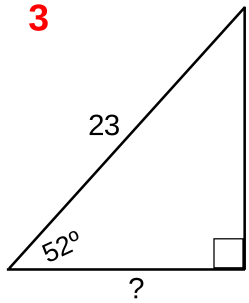

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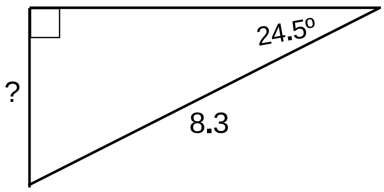

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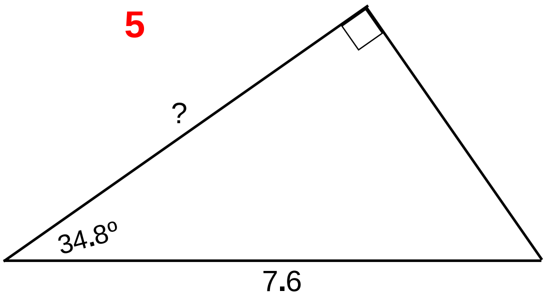

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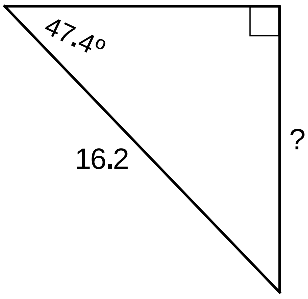

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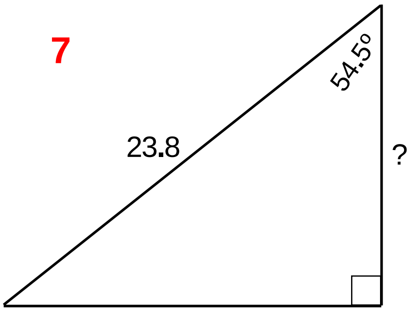

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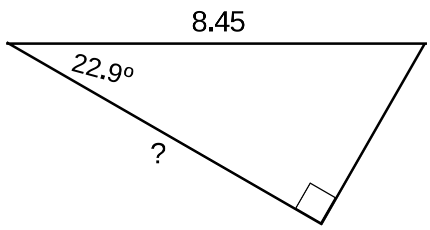

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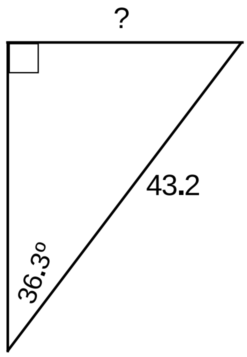

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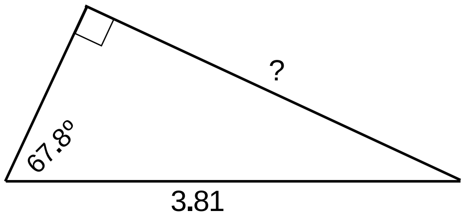

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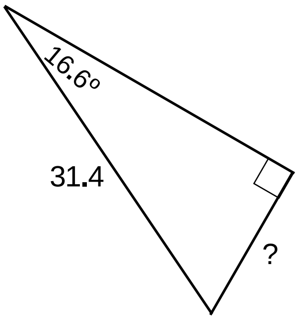

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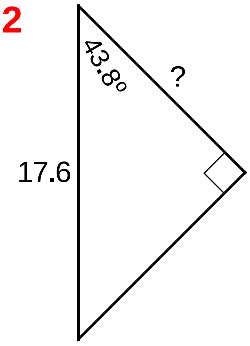

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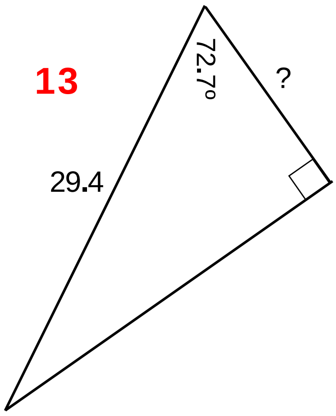

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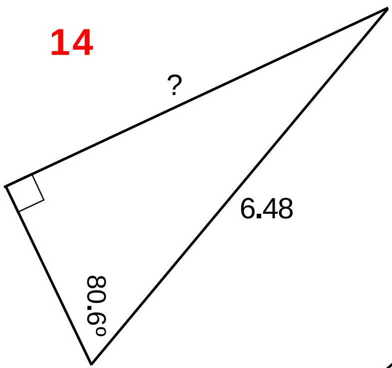

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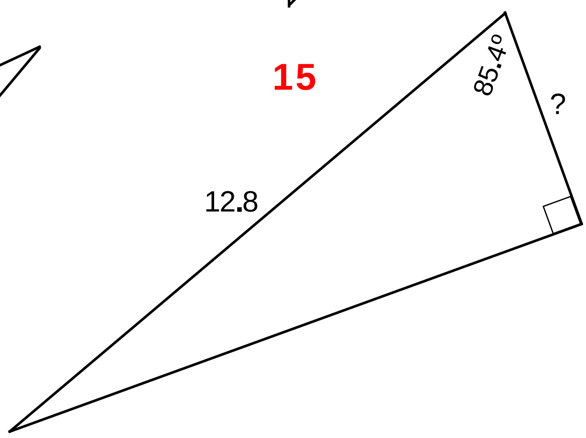

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Trigonometry

Find the length of the edge marked **?** in each of these **right-angled** triangles.

Drawings are NOT to scale.

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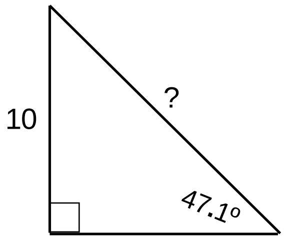

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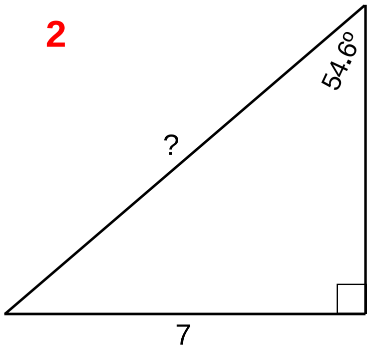

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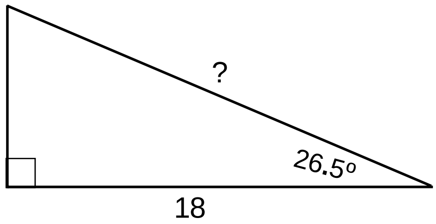

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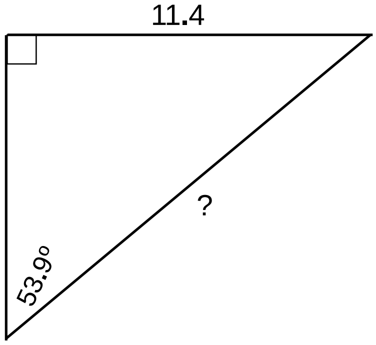

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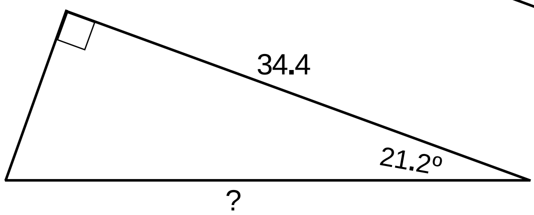

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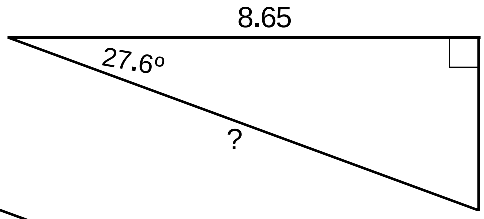

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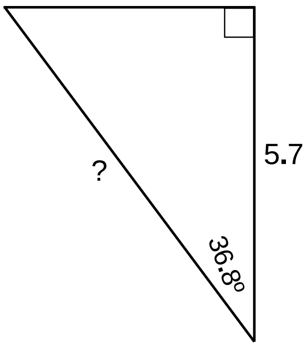

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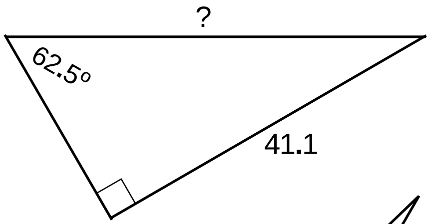

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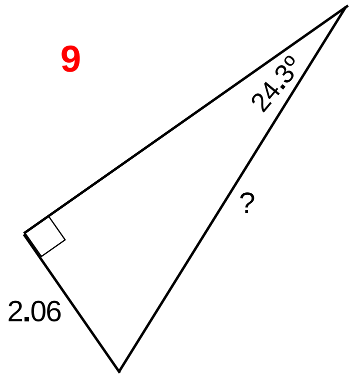

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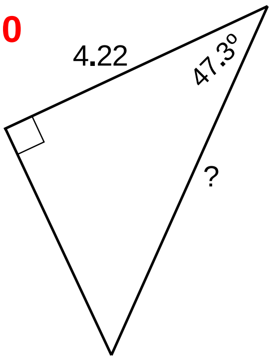

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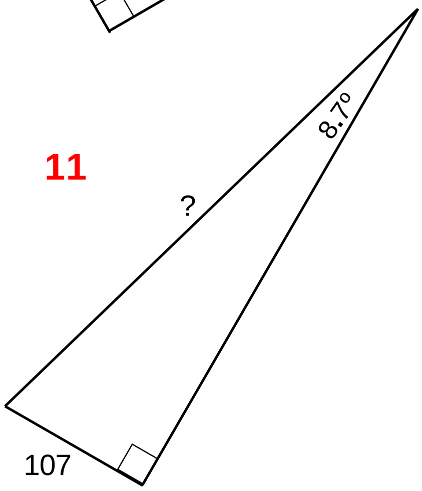

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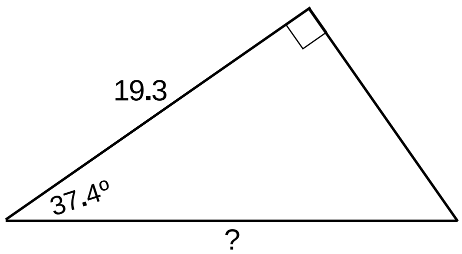

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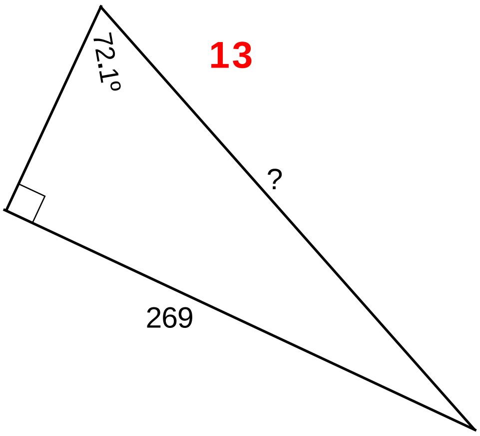

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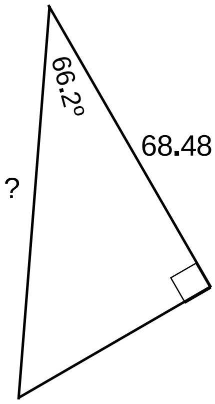

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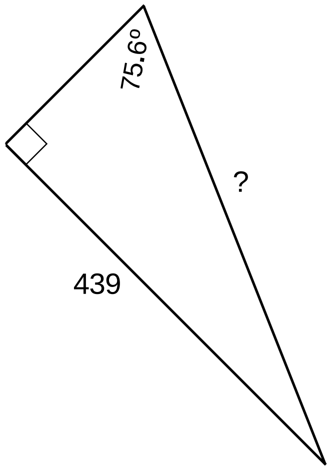

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Trigonometry

Find the length of the edge, or size of the angle, marked **?** in each of these **right-angled** triangles.

Drawings are NOT to scale.

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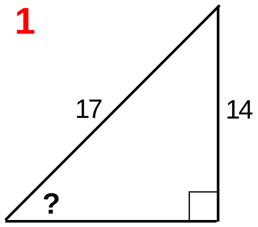

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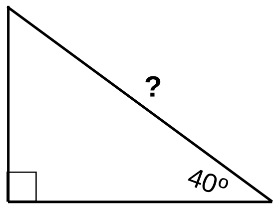

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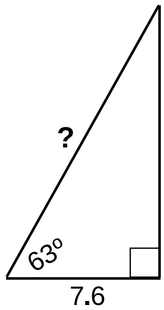

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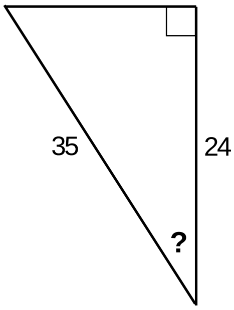

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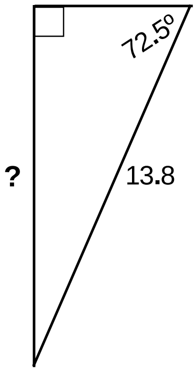

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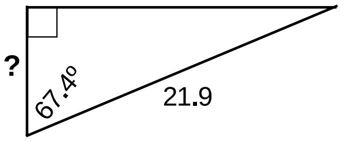

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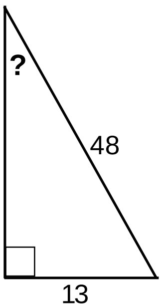

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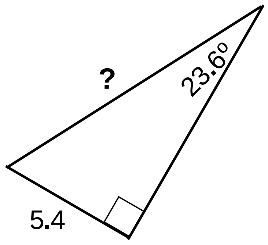

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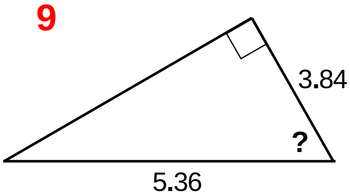

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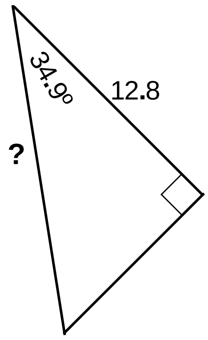

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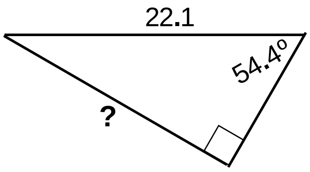

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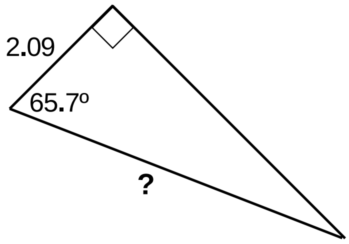

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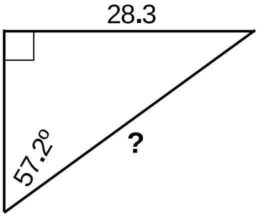

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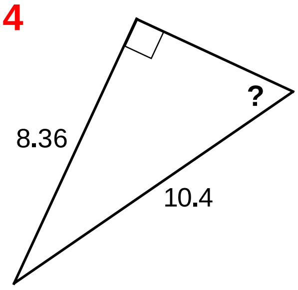

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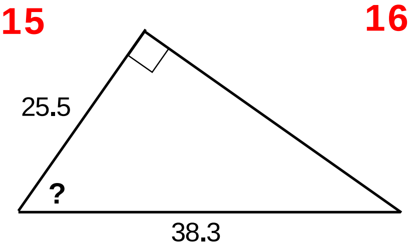

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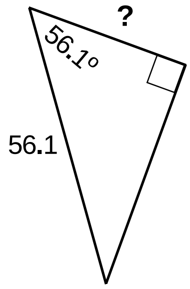

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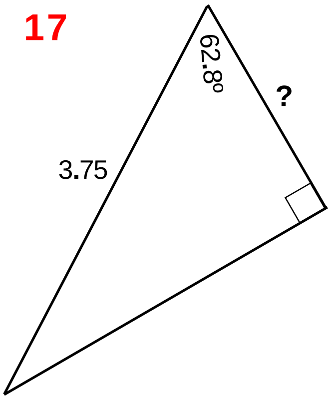

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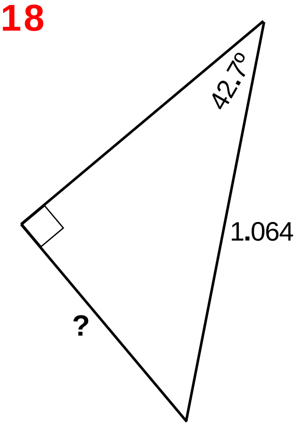

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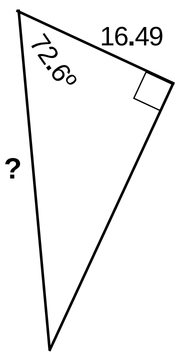

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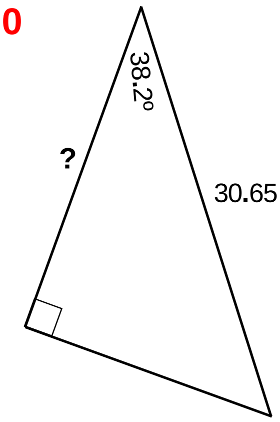

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Trigonometry

(Using sines)

1. A ladder is 8 metres long. It is leaning against the wall of a house and reaches 6 metres up the wall. Calculate the angle which the ladder must make with the level ground.
2. A kite-string has a total length of 75 metres. Calculate the height at which the kite must be flying when the string is fully out and is making an angle 37 degrees with the level ground.
3. The longest edge of an oblong measures 8 cm and its diagonal is 10 cm. What is the angle formed between the diagonal and the longest edge?
4. A television mast is stayed by a single wire cable which is 60 metres long. The stay is fastened to the top of the mast and to the ground. It makes an angle of 58° with ground. Find the height of the mast.
5. A slipway for a lifeboat is 50 metres long and is inclined at an angle of 35° to the horizontal.What is the height of the floor of the boathouse above the level of the sea?
6. The longest edge of a 30/60 set-square is 18 cm long. Calculate the length of its shortest edge.
7. The ribbons of a maypole are 10 metres long. During the course of a dance the dancers are moving in a circle around the pole and about 3.5 metres away from it. What then is the angle between the ribbons and the ground?
8. Measured on a map, the distance between the tops of two hills is 570 metres. The angle of elevation from the top of the smaller of the two hills, to the the top of the taller one is 27°. The tops of these hills are to be joined by a cable on which a ski-lift will run. What length of cable will be needed?
9. An isosceles triangle has two equal edges of length 8 cm. Its two equal angles measure 15.3°. What is the length of its other edge?
10. One section of a mountain-railway is 550 metres long. In that length is rises through a vertical height of 140 metres. Calculate the overall average angle of inclination of the track.
11. A 12 metre ladder is leaning against a wall. The foot of the ladder is 3.5 metres away from the base of the wall. What is the size of the angle which ladder makes with the ground?
12. An isosceles triangle has two edges of length 11 cm and one of length 8 cm. Find the sizes of all its angles.
13. A drilling-platform for an oil-rig is moored in the sea at a point where it is 130 metres deep. The average angle of the anchor cable with the horizontal is 28°. What length of cable is needed?
14. A house is sited just 150 metres from a straight section of road. The path to the house is straight, but is inclined at an angle of 54 degrees to the road. What is the length of the path?
15. The diagonal of an oblong is 15 cm long and makes an angle of 56° with one of its edges. Calculate the size of the oblong.
16. A vertical tent-pole is kept upright by ropes of length 4 metres tied to the top of the pole and to pegs in the ground. These ropes make an angle of 43° with the ground. What is the height of the tent-pole?
17. A telegraph-pole is supported by a stay-wire which is fastened to a point halfway up the pole. The stay-wire is 8 metres long and makes an angle of 56° with ground. Find the height of the pole.
18. A kite-string is 48 metres long. During one 'flight', when the wind strenghtened. the angle between the kite-string and the ground was seen to double in size from 27° to 54°. What was the resulting increase in the vertical height of the kite above the ground?
19. A rhombus has edge-lengths of 7 cm. Its acute interior vertex angles are both 70°. Calculate the lengths of both of its diagonals.
(The diagonals of a rhombus bisect each other and cross at right angles.)
20. A rod 1 metre long is hanging on a wall. If the bottom end of the rod is pulled out 20 centimetres from the wall, what angle will the rod make with the wall?
21. An 8 metre ladder leaning against a wall makes an angle of 63° with the ground. How far is the foot of the ladder away from the wall?
22. A parallelogram has edge-lengths of 5 cm and 9 cm respectively, and an angle of 58° between them. Calculate the perpendicular distance between the two longer edges.
23. A pair of railway-lines are 143.5 cm apart. On one particular curved section the 'banking' is formed by raising the outer rail 8 cm above the level of the inner rail. When a train is being driven around this curve, by how much does it lean out of the vertical?
24. The legs of a camera tripod are each 130 cm long. The tripod is set up on a piece of level ground so that each leg makes an angle of 70° with the gound. What is the vertical height of the top of the tripod above the ground?
25. A path going up a cliff-face from the bottom to the top is 700 metres long. Its overall average inclination to the horizontal is 15°. What is the height of the cliff-face?

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Trigonometry

Find the size of the angle marked **?** in each of these **right-angled** triangles

Drawings are NOT to scale.

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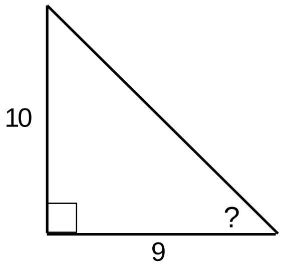

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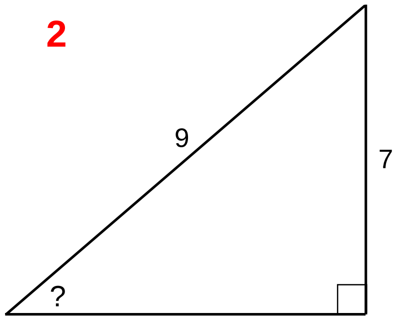

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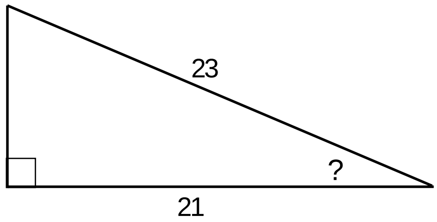

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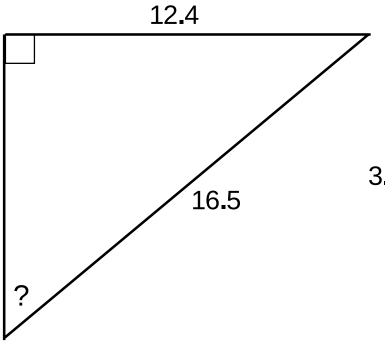

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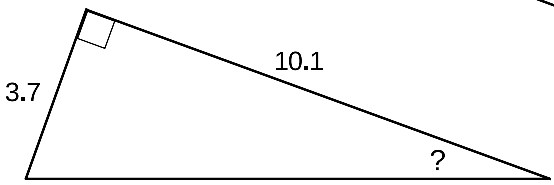

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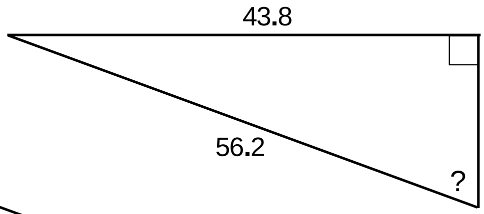

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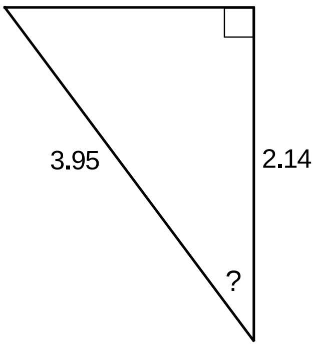

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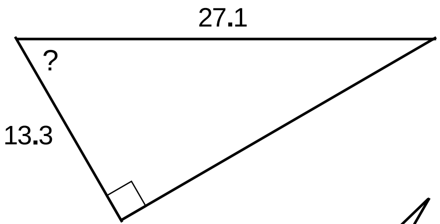

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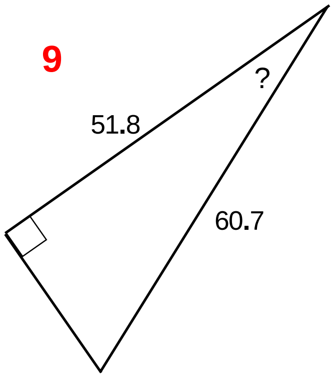

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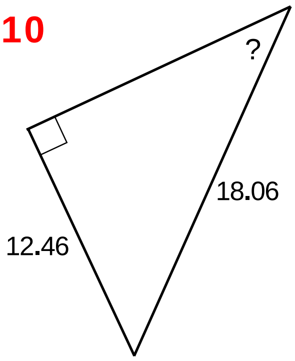

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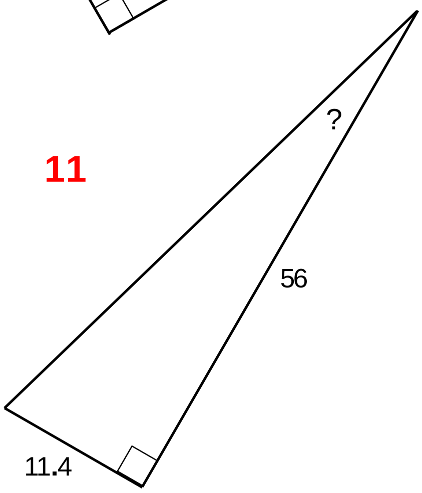

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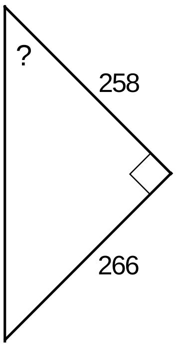

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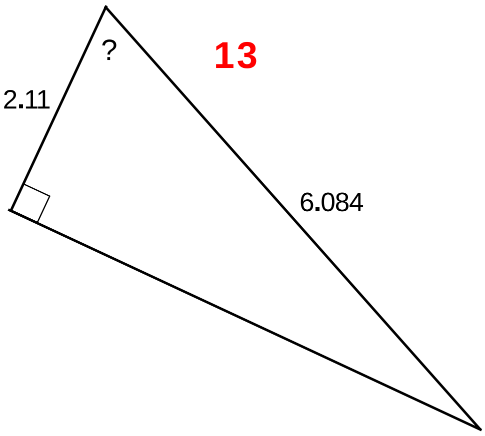

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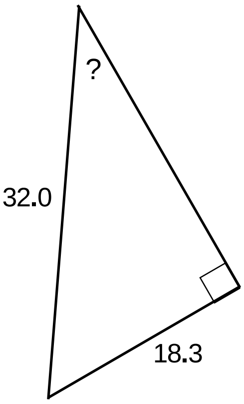

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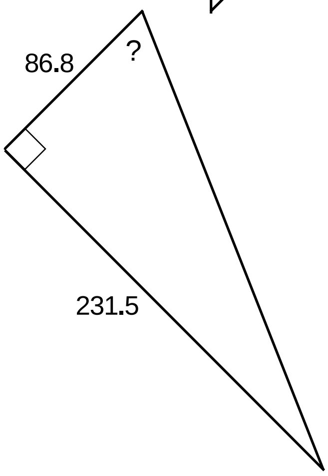

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Trigonometry

Find the length of the edge marked **?** in each of these **right-angled** triangles.

Drawings are NOT to scale.

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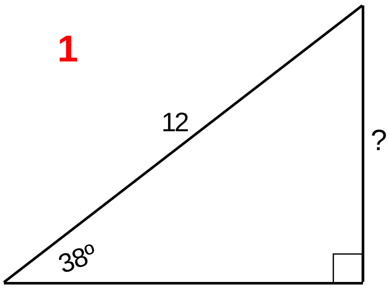

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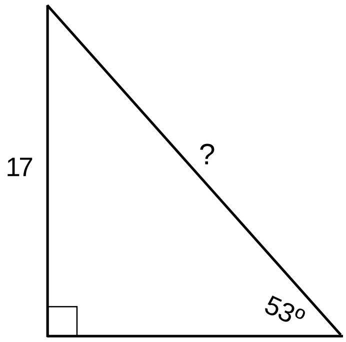

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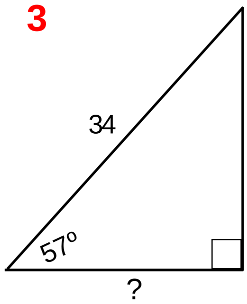

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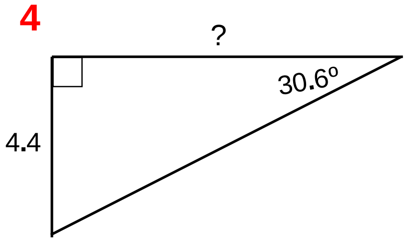

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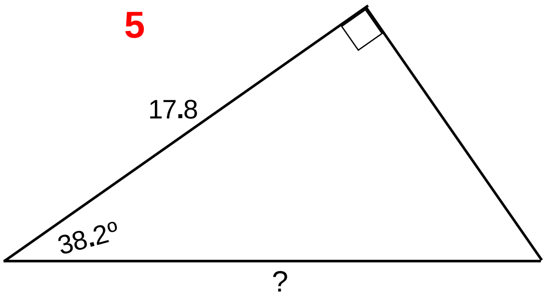

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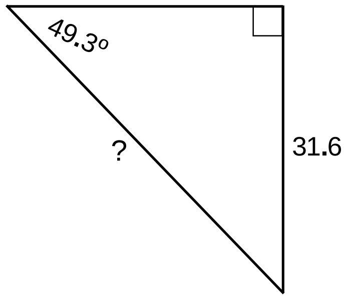

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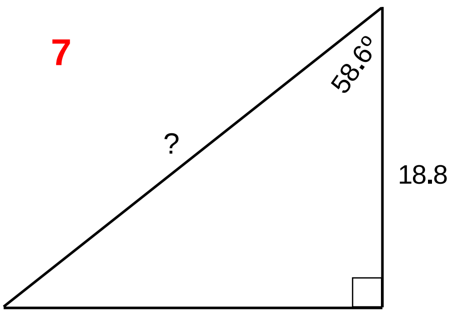

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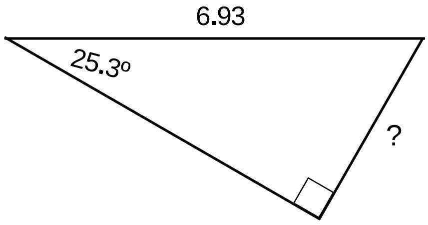

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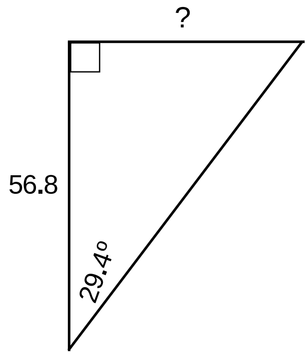

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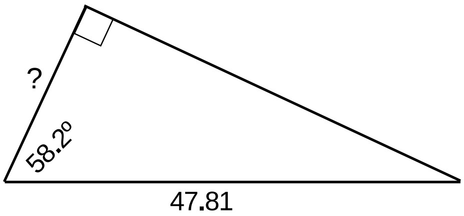

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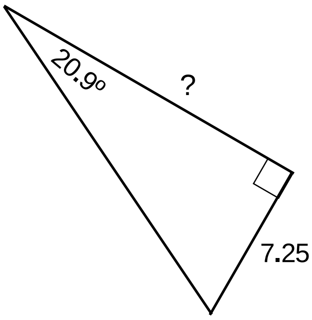

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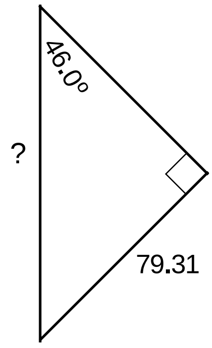

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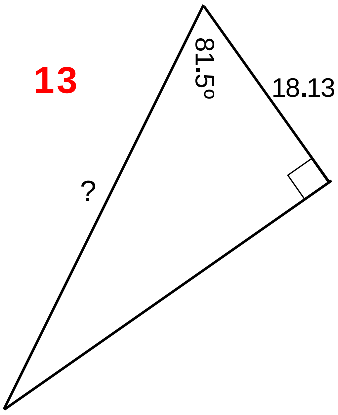

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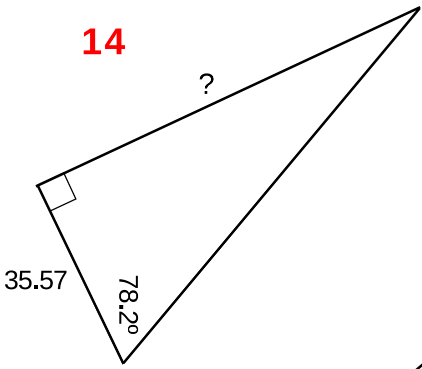

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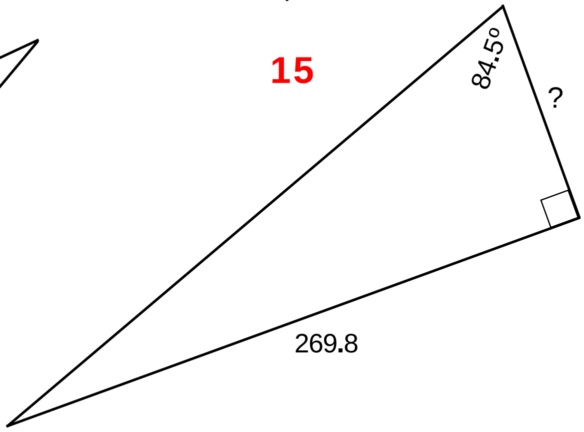

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Trigonometry

Find the length of the edge, or size of the angle, marked **?** in each of these **right-angled** triangles.

Drawings are NOT to scale.

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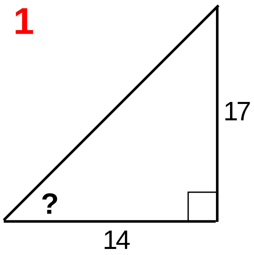

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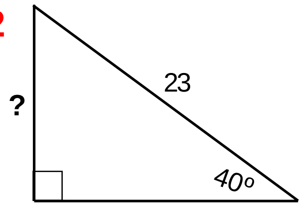

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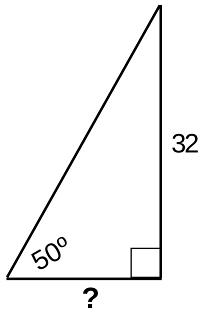

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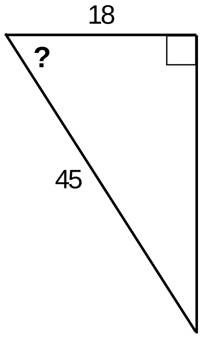

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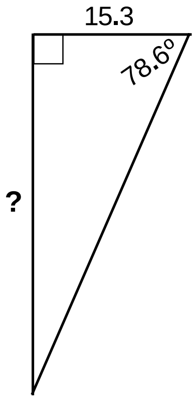

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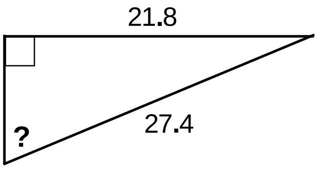

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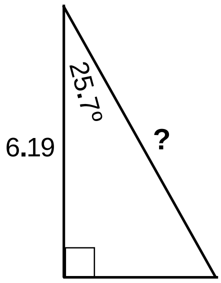

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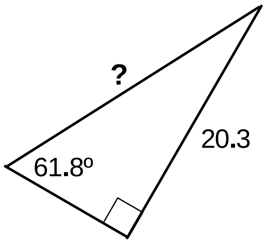

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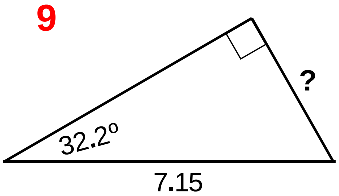

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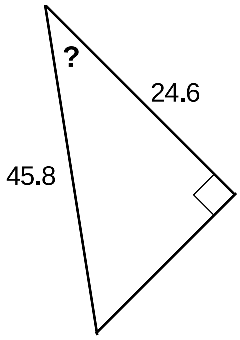

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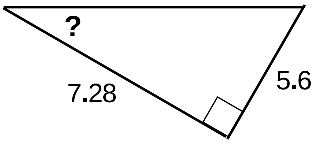

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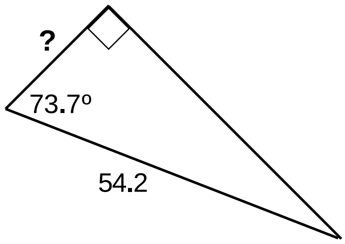

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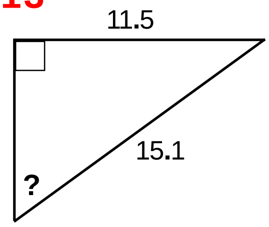

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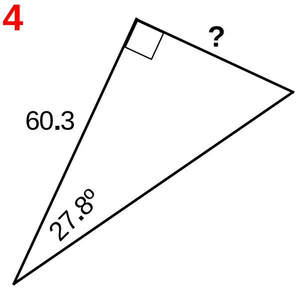

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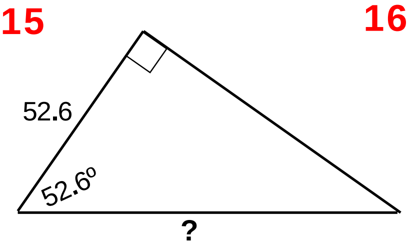

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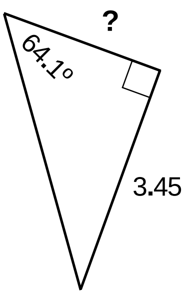

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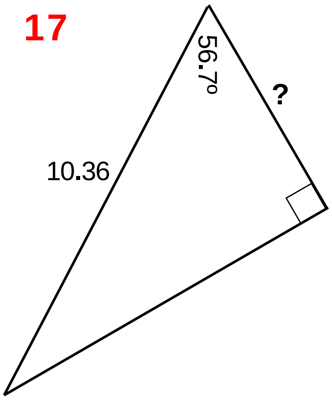

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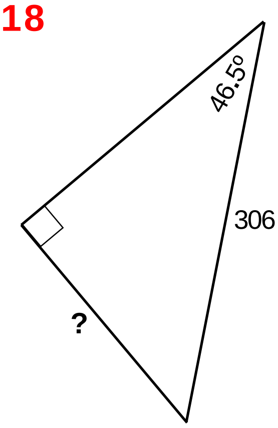

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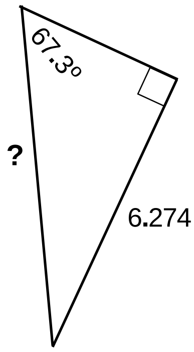

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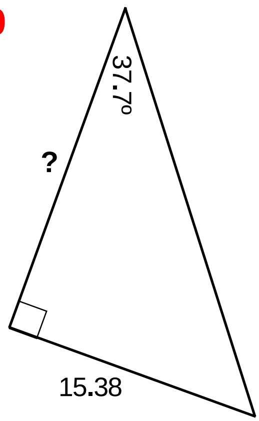

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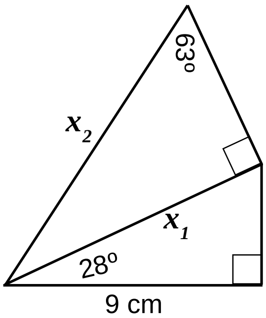
Trigonometry

Find length of edge (or size of angle) marked x in each of these diagrams. *Drawings are NOT to scale.*

1



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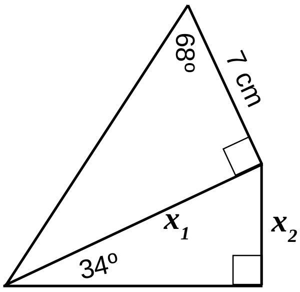

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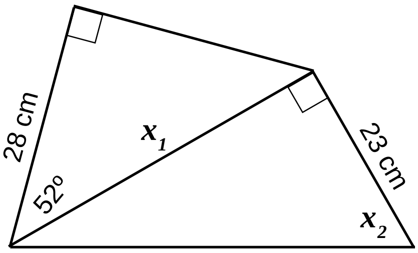

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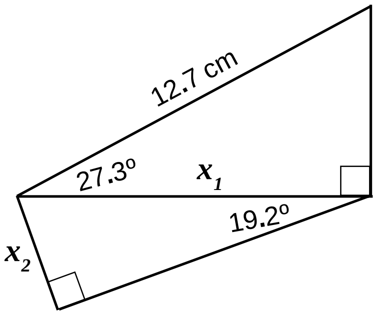

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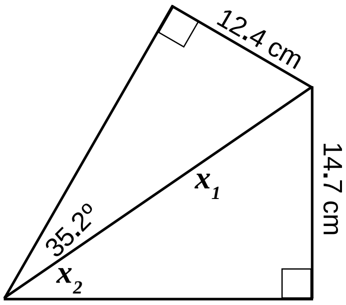

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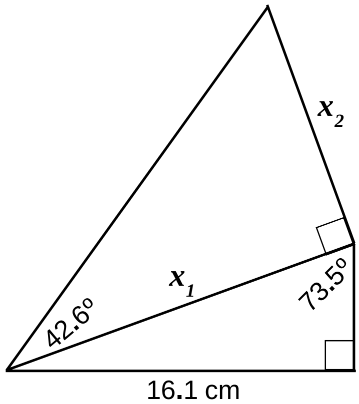

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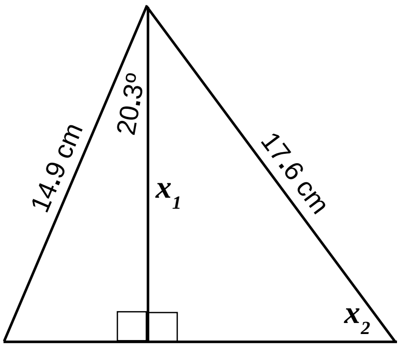

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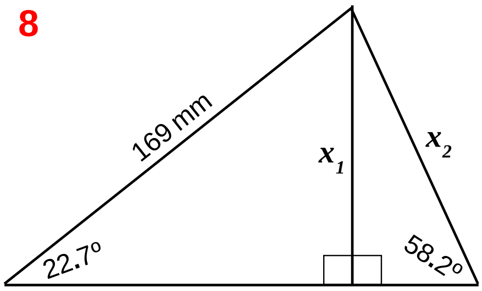

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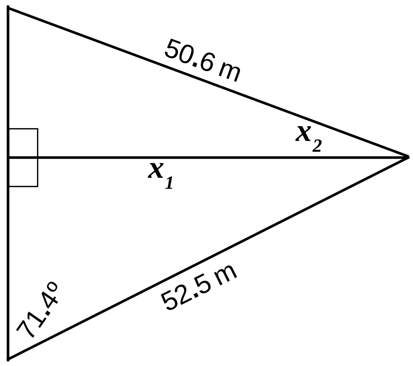

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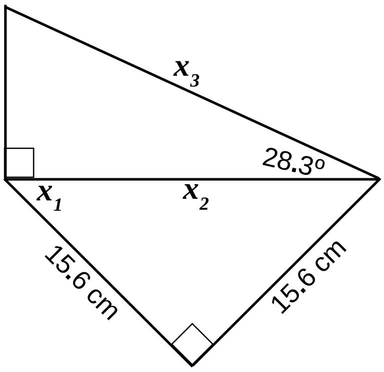

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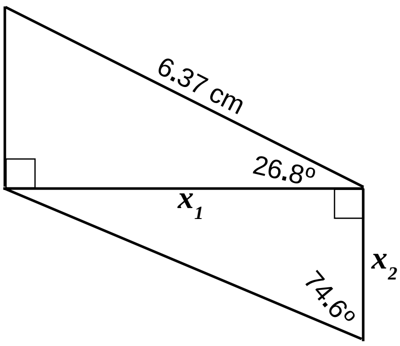

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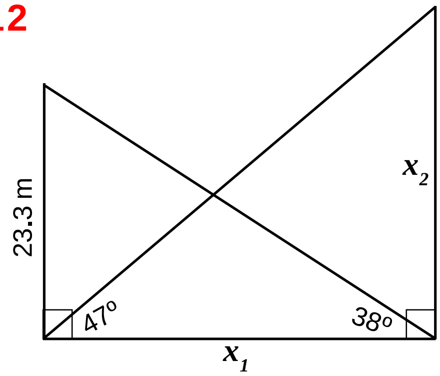

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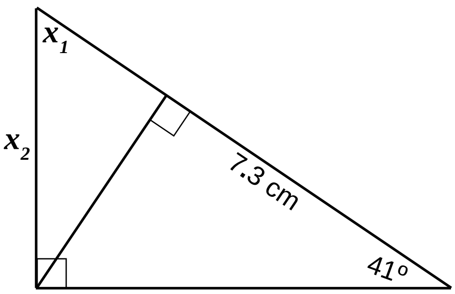

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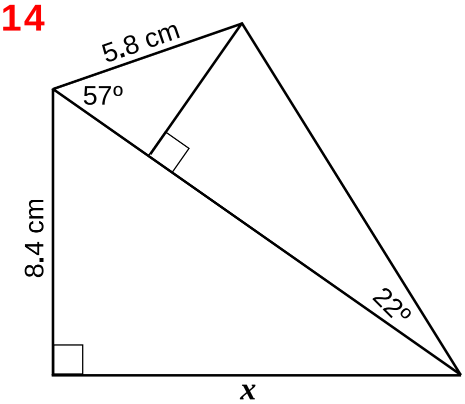

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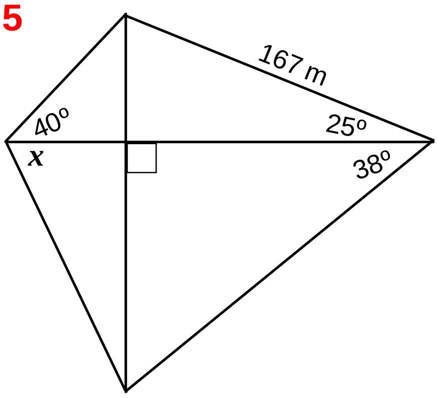

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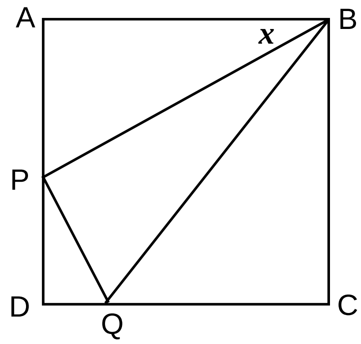
Drawings on this sheet are NOT to scale.

1. In the drawing on the right ABCD is a square.

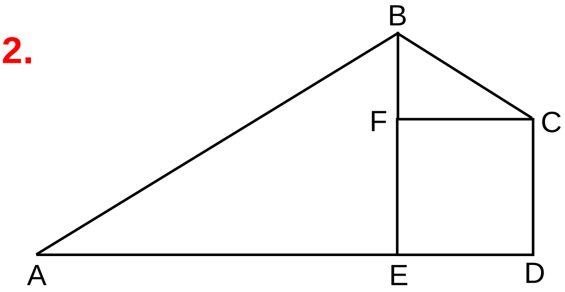
AP = 5 cm   QC = 7 cm   PB = 12 cm

Calculate

(a) the size of the angle marked x
(b) the length of AB
(c) the length of DQ
(d) the length of PD
(e) the size of $\angle$BQD



2.



In the drawing on the left CDEF is a square.

EF = 4.7 cm $\angle$BCF = 36° $\angle$BAE = 28°

Calculate the lengths of

(a) BF
(b) AB
(c) AD

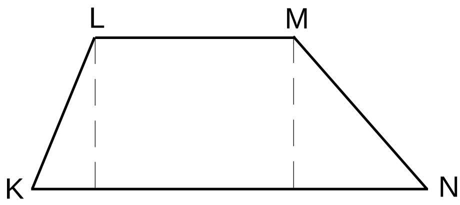
3. KLMN is a trapezium with LM parallel to KN.

LM = 8.4 cm   KN = 19.6 cm

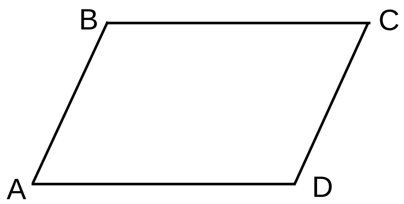
KL = 10.7 cm $\angle$LKN = 58°

Calculate

(a) the perpendicular distance between the parallel edges
(b) the area of the trapezium
(c) the perimeter of the trapezium.



4.



ABCD is a parallelogram

AB = 3.8 cm   BC = 7.9 cm $\angle$BAD = 48°

Calculate

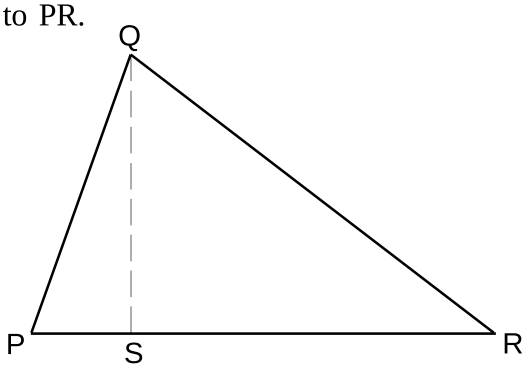
(a) the area of the parallelogram
(b) the length of the longer diagonal AC.

5. In the triangle PQR, S is the foot of the perpendicular from Q to PR.

QS = 7.5 cm $\angle$QPR = 75° $\angle$QRP = 40°

Calculate

(a) the length of SR
(b) the length of PS
(c) the length of QR
(d) the perimeter of triangle PQR
(e) the area of triangle PQR.



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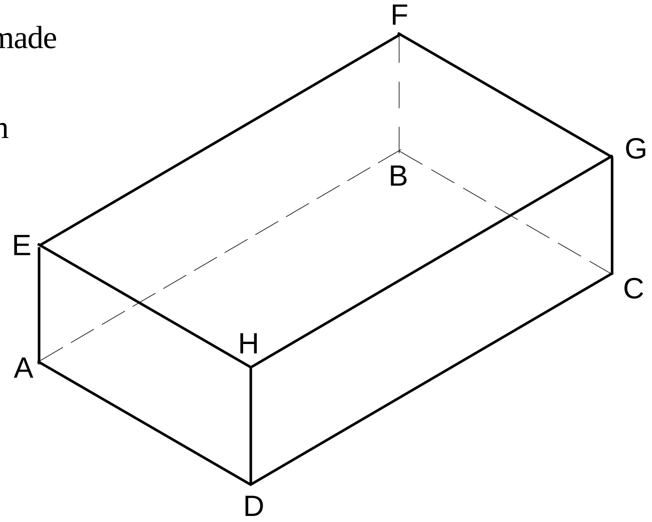
Trigonometry

Drawings on this sheet are NOT to scale.

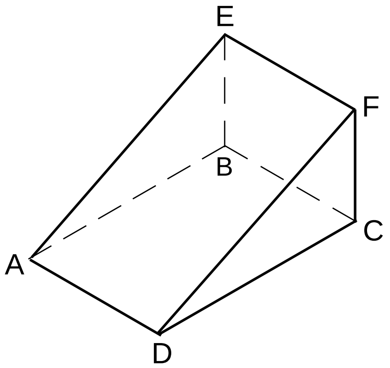
1. The drawing on the right represents a cuboidal shape made of wire with its vertices identified as ABCDEFGH.
Sizes are: AD = 15 cm AE = 10 cm CD = 25 cm
Calculate

(a) ∠GDC
(b) ∠EDA
(c) ∠EFH
(d) length of face diagonal DG
(e) length of face diagonal BD
(f) length of space diagonal DF
(g) ∠FDG
(h) ∠FDB



2. A cube has an edge-length of 10 cm.
Calculate the angle between its space diagonal, and one of its face diagonals.

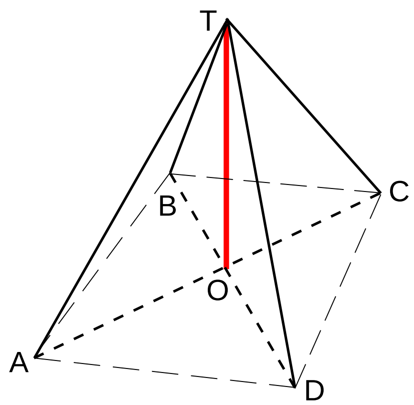
3.



The drawing on the left shows a wedge ABCDEF.
Faces ABCD and BEFC are both rectangles and are at right angles to each other.
AB = 20 cm AD = 10 cm ∠FDC = 18°
Calculate

(a) the height CF
(b) the length of edge DF
(c) ∠CAD
(d) the length of diagonal AF
(e) ∠FAC

4. The drawing on the right shows a television mast OT which has 4 stay-wires fastened to its top (at T) and to the ground at points A, B, C and D.
ABCD form a square and point O is its centre.
AD = 32 metres
The angle of elevation of T from A is 57°
Calculate (a) distance AO
(b) height of the mast OT
(c) length of one stay-wire
(d) angle of elevation of the top of the mast from a point E which midway between C and D.



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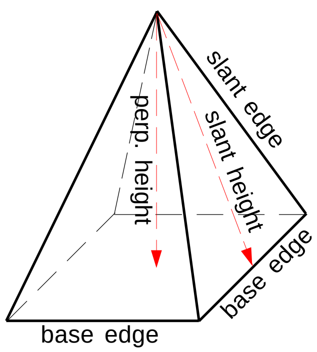

A right square-based pyramid has a base-edge of length 17 cm and a perpendicular height of 28 cm.
Calculate

(a) the length of a slant edge
(b) its slant height
(c) the angle between a slant edge and the base
(d) the area of one triangular face

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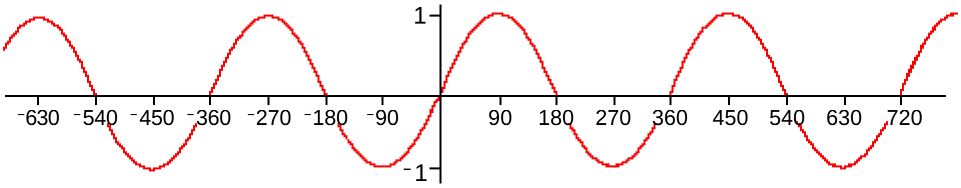
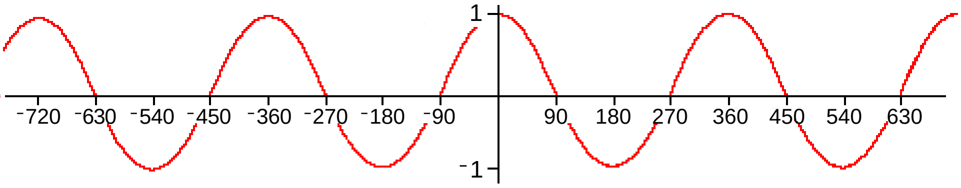
Trigonometry

(The general angle for sines and cosines)

The **sine** curve



The **cosine** curve



If sine **A** = sine x then $\mathbf{A}^\circ = 180n + (^-1)^n x^\circ$

If cosine **A** = cosine x then $\mathbf{A}^\circ = 360n \pm x^\circ$

1. Give the **sine** values of each of these angles

60° 120° 420° 480° 780°

then write down the next two angles in the sequence.

2. Give the **cosine** values of each of these angles

50° 310° 410° 670° 770°

then write down the next two angles in the sequence.

3. Give the **sine** values of each of these angles

30° 150° 210° 330° 390° 510° 570°

then write down the next two angles in the sequence.

4. Give the **cosine** values of each of these angles

70° 110° 250° 290° 430° 470° 610°

then write down the next two angles in the sequence.

5. Give the **three** smallest possible **positive** angles having these **sine** values

(i) 1 (ii) 0.734 (iii) -0.212

6. Give the **three** smallest possible **positive** angles having these **cosine** values

(i) 1 (ii) 0.406 (iii) -0.653

7. Give the **first three** angles less than zero to have these **sine** values

(i) 0 (ii) 0.318 (iii) -0.741

8. Give the **first three** angles less than zero to have these **cosine** values

(i) 0 (ii) 0.647 (iii) -0.358

9. Estimate the smallest positive angle whose sine and cosine values are equal.

10. Using the estimate made in the previous question, list four positive and four negative angles whose

(i) positive values (ii) negative values

of the sine and cosine values for those angles are equal.

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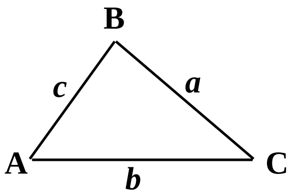
Trigonometry

T/32

The area of **any** triangle is given by one-half the product of two adjacent edges and the angle **between** them.

$$\text{Area of Triangle } \mathbf{ABC} = \tfrac{1}{2}\, \boldsymbol{ab} \sin \mathbf{C}$$



1.	Find the area of triangle ABC when	a = 8.4 cm	b = 3.7cm	C = 44°	
2.	Find the area of triangle ABC when	b = 5.9 cm	c = 7.2 cm	A = 52°	
3.	Find the area of triangle ABC when	AB = 6.7 cm	AC = 9.3 cm	$\angle$BAC = 55°	
4.	Find the area of triangle ABC when	BC = 3.1 cm	AC = 5.4cm	$\angle$ACB = 37°	
5.	Find the area of triangle ABC when	AB = 14.5 cm	BC = 9.6 cm	$\angle$ACB = 81°	$\angle$ABC = 58°
6.	Find the area of triangle ABC when	a = 2.8 cm	c = 4.1 cm	A = 40°	B = 31°
7.	Find the area of triangle XYZ when	XY = 4.3 cm	XZ = 3.2 cm	$\angle$YXZ = 55°	$\angle$YZX = 78°
8.	Find the area of triangle XYZ when	y = 8.8 cm	x = 5.7 cm	Z = 21.4°	X = 30.1°
9.	Find the area of triangle PQR when	QR = 12.4 cm	PR = 17.6 cm	$\angle$QPR = 44.7°	$\angle$PRQ = 49.2°
10.	Find the area of triangle LMN when	LN = 12.1 cm	MN = 10.6 cm	$\angle$LMN = 67.5°	$\angle$LNM = 58.5°

11. A triangle of area 21.4 cm^2 has two edges measuring 9.8 and 5.9 cm respectively.
Find the angle between those two edges.

12. A triangle ABC has an area of 17.3 cm^2. AB = 6.3 cm and $\angle$BAC = 31.4°.
What is the length of the edge AC?

13. A triangle ABC has two edges measuring 4.5 and 7.8 cm respectively.
What is the greatest area this triangle can have?
What angle between the two edges gives this greatest area?

14. A parallelogram PQRS has PQ = 11.4 cm, PS = 15.7 cm and $\angle$QPS = 53°.
Find the area of the complete parallelogram.

15. A rhombus has an edge lengths of 2.75 cm and an interior vertex angle of 65°.
What is its area?

16. A rhombus has an area of 36 cm^2 and edges of length 6.7 cm.
Give the sizes of its interior vertex angles.

17. A parallelogram has an area of 63 cm^2.
One of its edge-lengths is 7.2 cm, and one interior vertex angle is 54°.
What is its other edge-length?

18. Two triangles have the same area.
One has edges of 4.7 and 6.8 cm with an included angle of 49°.
The other has two edges measuring 7.3 and 3.9 cm respectively.
Calculate the size of one of the interior vertex angles of the second triangle.

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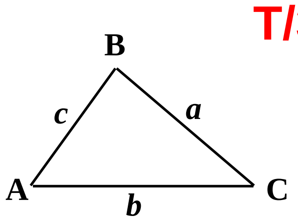
The Sine Rule

In **any** triangle the length of **any** edge is proportional to the sine of the opposite angle.

$$\frac{a}{\text{sine A}} = \frac{b}{\text{sine B}} = \frac{c}{\text{sine C}}$$



1.	Given triangle ABC has	C = 67°	B = 42°	c = 5.3 cm	find b
2.	In triangle ABC	A = 56°	C = 73°	a = 11 cm	find B, b, c
3.	Given triangle ABC has	∠BAC = 36°	AC = 4.3 cm	BC = 6.7 cm	find ∠ABC
4.	Triangle XYZ has	∠YXZ = 28°	∠YZX = 73°	YZ = 9.3 cm	find XY
5.	Triangle PQR has	P = 41°	p = 7.89 cm	r = 10.1 cm	find Q
6.	Triangle LMN has	∠LMN = 56°	∠LNM = 58°	ML = 14.8 cm	find MN

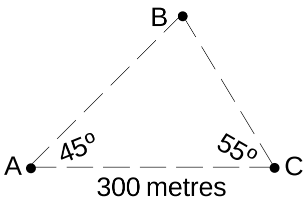
7. In the triangle ABC a = 7.4 cm b = 8.9 cm A = 47°. Calculate C.
Use a, b and C to find the area of the triangle.

8. Triangle ABC has AC = 3.24 cm, AB = 5.53 cm and ∠ABC = 32°. Find its area.

9. Triangle XYZ has ∠XYZ = 71° ∠YXZ = 33° YZ = 15.8 cm. Find its area.

10. A quadrilateral ABCD has these dimensions
AB = 4.1 cm BC = 5.2 cm BD = 3.75 cm ∠BAD = 45° ∠BCD = 35°
Find the sizes of (i) ∠BDA (ii) ∠BDC (iii) ∠ADC (iv) ∠ABC

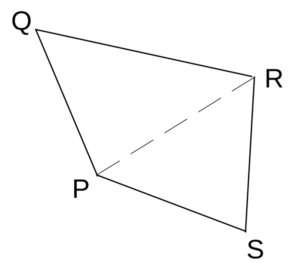
11.



In the diagram, A and C represent the positions of two boats.
They are 300 metres apart and both are racing to reach the buoy at B.
Calculate how far each has to go.

12. An isosceles triangle has two equal angles of 72° and two equal edges of 6.7 cm.
Find the length of the third edge.

13. The diagram on the right is the plan view of a field.
A survey has produced the following measurements
PQ = 650 metres ∠QPR = 83° ∠QPS = 127°
∠PQR = 56° ∠PRS = 65°
Calculate the area of the field.



Given the sizes of **A**, a, and c if $a < c$ sine **A** no triangle is possible
$a = c$ sine **A** the triangle is right-angled at **C**
$a > c$ only one triangle is possible
otherwise, two triangles are possible fitting the data given. (This is known as *'the ambiguous case'*.)
Investigate each of the following sets of data and say how many triangles are possible in each case.

14.	A = 40°	a = 6.1 cm	c = 6 cm	**20.**	A = 60°	a = 5.08 cm	c = 3.5 cm
15.	A = 70°	a = 5.25 cm	c = 5.4 cm	**21.**	A = 32.6°	a = 5.16 cm	c = 7.24 cm
16.	A = 45°	a = 15.4 cm	c = 10.6 cm	**22.**	A = 26.9°	a = 13.0 cm	c = 19.2 cm
17.	A = 30°	a = 4.5 cm	c = 9 cm	**23.**	A = 43.7°	a = 21.3 cm	c = 18.6 cm
18.	A = 20°	a = 6.79 cm	c = 8 cm	**24.**	A = 54.1°	a = 39.5 cm	c = 44.7 cm
19.	A = 50°	a = 3.09 cm	c = 4 cm	**25.**	A = 61.8°	a = 42.6 cm	c = 30.5 cm

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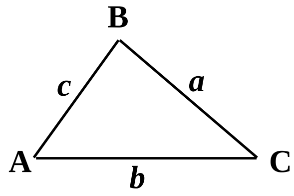
Trigonometry

The Cosine Rule: $a^2 = b^2 + c^2 - 2bc \cos \mathbf{A}$

or $\cos \mathbf{A} = \dfrac{b^2 + c^2 - a^2}{2bc}$



1.	Given	b = 2 cm	c = 3 cm	A = 60°	find ***a***
2.	Given	a = 4 cm	b = 6 cm	C = 35°	find ***c***
3.	Given	b = 7 cm	c = 9 cm	A = 50°	find ***a***
4.	Given	a = 5 cm	c = 8 cm	B = 40°	find ***b***
5.	Given	a = 6 cm	b = 7 cm	c = 8 cm	find **A**
6.	Given	a = 3 cm	b = 5 cm	c = 7 cm	find **B**
7.	Given	a = 4 cm	b = 2 cm	c = 3 cm	find **C**
8.	Given	a = 12 cm	b = 5 cm	C = 90°	find ***c***
9.	Given	b = 3.5 cm	c = 4.7 cm	A = 130°	find ***a***
10.	Given	a = 7.8 cm	c = 5.3 cm	B = 146°	find ***b***
11.	Given	a = 10.2 cm	b = 6.4 cm	c = 5.5 cm	find **A**
12.	Given	a = 7.4 cm	b = 9.6 cm	c = 16.2 cm	find **C**
13.	Given	a = 8 cm	b = 10 cm	c = 6 cm	find the size of the **smallest** angle
14.	Given	a = 4.8 cm	b = 7.1 cm	c = 5.5 cm	find the size of the **largest** angle
15.	Given	a = 8.5 cm	b = 13.2 cm	c = 14.8 cm	find the sizes of all 3 angles
16.	Given	a = 17.1 cm	b = 28.6 cm	c = 15.3 cm	find the sizes of all 3 angles

17. The goal-posts in football are 7.32 metres apart. A ball is placed on the ground 8 metres from one goal-post and 6 metres from the other. Within what angle must the ball be kicked along the ground in order to score?

18. Two people start walking from the same place at the same time. They are walking on level ground. One walks at 4 kilometres an hour going due North, and the other walks at a speed of 3.5 kilometres an hour going North-east. After 3 hours, how far apart will they be?

19. The principal 'legs' of a step-ladder are usually of two different lengths. In one case, the front leg (with the steps in it) is 2.8 metres long and the back (supporting) leg is 2.5 metres long. When in the working position, the angle between the legs is 40 degrees. What is the distance apart of the two legs on the floor?

20. A rhombus has all its edges 12.7 cm long. Its acute angle is 64°. Find the length of its shorter diagonal

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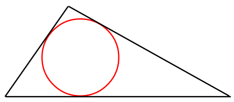
a, b, c are lengths of three edges of triangle ABC
s is length of semi-perimeter

$$s = \frac{a+b+c}{2}$$

Heron's or **Hero's** formula
for finding the area of a triangle

$$\text{Area} = \sqrt{s(s-a)(s-b)(s-c)}$$

Inscribed circle
or **Incircle**

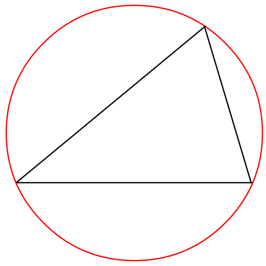


$$\text{radius } r = \frac{\text{Area}}{s}$$

Circumscribed circle
or **Circumcircle**

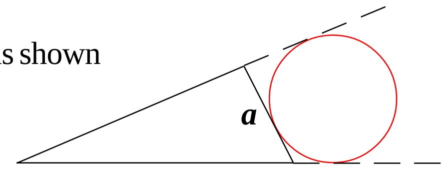
$$\text{Radius } R = \frac{abc}{4 \times \text{Area}}$$

$$\text{or} \quad R = \frac{a}{2 \sin A}$$



Escribed circle
Only 1 (of 3 possible) is shown
drawn on edge a

$$\text{radius } r_a = \frac{\text{Area}}{s-a}$$



1.	Given	a = 5 cm	b = 7 cm	c = 8 cm	find **Area** and R
2.	Given	a = 7 cm	b = 9 cm	c = 12 cm	find **Area** and r
3.	Given	a = 3 cm	b = 6 cm	c = 6 cm	find **Area** and r_a
4.	Given	a = 5 cm	b = 5 cm	c = 5 cm	find R
5.	Given	a = 9.4 cm	b = 9.4 cm	c = 9.4 cm	find r
6.	Given	a = 8.1 cm	b = 6.2 cm	c = 8.1 cm	find **Area** and r_c
7.	Given	a = 10.3 cm	b = 9.5 cm	c = 14.9 cm	find r_b
8.	Given	a = 11.4 cm	b = 13.2 cm	c = 9.7 cm	find r_a
9.	Given	a = 57.3 cm	b = 42.6 cm	c = 78.2 cm	find R
10.	Given	a = 83.2 cm	b = 71.5 cm	c = 93.6 cm	find R

11. Find the area of an equilateral triangle with an edge-length of 8.37 cm

12. An isosceles triangle has edge-lengths of 5.67 and 12.2 cm. What is its area?

13. A triangle has edges measuring 4.27 6.08 and 9.25 cm.
Give the diameter of the largest circle that can be fitted inside this triangle.

14. A rhombus has an edge-length of 6 cm and its shorter diagonal measures 3.8 cm.
Find its area.

15. A field is in the shape of a quadrilateral. Identifying its four vertices as ABCD gives
AB = 127 metres BC = 206 metres CD = 332 metres
AD = 94 metres BD = 183 metres
Calculate its area.

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1. In triangle ABC AB = 7.2 cm BC = 9.6 cm $\angle$ABC = 125°
Find its area.

2. In triangle XYZ y = 4.7 cm x = 8.3 cm X = 140°. Find Y

3. A triangle has edges measuring 3.6, 5.2 and 7.8 cm.
Find the size of its greatest angle.

4. Find the area of a triangle having two edges of 1.8 and 3.4 cm, when the angle between those two edges is
(i) 20° (ii) 160°

5. Triangle ABC has $\angle$ABC = 115° AC = 7.63 cm BC = 3.8 cm. Find $\angle$ACB

6. Triangle XYZ has $\angle$XYZ = 105° XY = 17.3 cm YZ = 26.4 cm.
Find the length of XZ

7. A triangular field has hedge-lengths of 48, 55 and 87 metres.
Find the size of the largest angle between two of the hedges.
What is the area of the field?

8. A rhombus has an edge-length of 3.5 cm and an obtuse interior vertex angle of 105°.
Find its area.

9. A parallelogram has a edges of 6.3 and 5.7 cm. Its area is 27.4 cm²
What is the size of its obtuse interior vertex angle?

10. A triangle LMN has LM = 6.3 cm LN = 5.8 cm $\angle$LMN = 32°
Calculate the two possible sizes of $\angle$LNM

11. The goal-posts in Association Fooball are 7.3 metres apart.
A player situated 5 metres from one goal-post and 6 metres from the other is about to kick the ball at the goal.
Calculate the size of the angle within which the ball must be kicked if a goal is to be scored.

12. Two ships leave the same harbour at the same time.
One steams due North for 130 miles, the other on a course of 140° for a distance of 170 miles.
How far apart are the two ships after this?

13. Two aeroplanes leave simultaneously from the same airport. One flies due East at a speed of 150 kilometres an hour; the other on a course of 250° at 170 kilometres an hour.
Calculate how far apart the two planes are, 2 hours after leaving the airport.

14. A ship leaves port at noon on a course of 070° at a speed of 20 knots.
A second ship leaves the same port at 13.30 moving at a speed of 30 knots and heading in the direction of 310°
What distance apart will the two ships be at 16.15?
(A knot is a speed of 1 nautical mile per hour.)

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1. For a triangle which has edges of 5.2 cm, 3.9 cm and 6.1 cm, calculate the size of of the largest angle.

2. Triangle ABC has AB = 8.3 cm , BC = 7.8 cm and $\angle$ABC = 56°. Find the length of AC.

3. In triangle LMN, L = 42°, m = 4.3 cm and l = 5.4 cm. Find M.

4. Find the area of triangle ABC which has AB = 12 cm , BC = 14 cm and $\angle$ABC = 36°.

5. In the triangle PQR, PQ = 3.9 cm , QR = 5.1 cm and $\angle$PQR = 131°. Calculate the length of PR.

6. For each of these, give the **three** smallest possible **positive** angles having these **sine** values
   (i) 0.150 (ii) 0.964 (iii) $^-$0.708

7. For each of these, give the **three** smallest possible **positive** angles having these **cosine** values
   (i) 0.257 (ii) 0.693 (iii) $^-$0.861

8. Given the values of the following
   (i) sine 130° (ii) sine 179° (iii) sine $^-$265°
   (iv) cosine 150° (v) cosine 108° (vi) cosine $^-$283°

9. Find the area of a triangle whose edges are 10, 12 and 18 cm.

10. Triangle XYZ has XY = 18 cm , YZ = 12.1 cm and $\angle$YXZ = 33°. Calculate the two possibilities for the size of $\angle$YZX.

11. In the triangle LMN, calculate the size of M given that l = 3.7 cm, m = 4.6 cm and n = 2.3 cm.

12. Two people start walking from the same place. One walks due South at 5 kilometres an hour, and the other walks South-west at 4 kilometres an hour. What is the distance between them after 5 hours have elapsed?

13. A regular pentagon has a circumscribed circle of radius 4.9 cm. Find the area of the pentagon.

14. A regular decagon has an inscribed circle of radius 7.3 cm. Calculate the perimeter of the decagon

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