Complex Numbers
Intro Questions

(Answers follow on each subsequent slide)

$$i =$$
$$i^2 =$$
$$i^3 =$$
$$i^4 =$$

$$i^{4n} =$$
$$i^{4n+1} =$$
$$i^{4n+2} = i^{2(2n+1)} =$$
$$i^{4n+3} = i^{4n-1} =$$

$$i = i$$
$$i^2 = -1$$
$$i^3 = -i$$
$$i^4 = 1$$

$$i^{4n} = 1$$
$$i^{4n+1} = i$$
$$i^{4n+2} = i^{2(2n+1)} = -1$$
$$i^{4n+3} = i^{4n-1} = -i$$

$$8 + 6i + 3 - 4i =$$

$$8 + 6i - (3 - 4i) =$$

$$(8 + 6i)(3 - 4i) =$$

$$(8 + 6i)(8 - 6i) =$$

$$(3 + 4i)(3 - 4i) =$$

$$8 + 6i + 3 - 4i = 11 + 2i$$

$$8 + 6i - (3 - 4i) = 5 + 10i$$

$$(8 + 6i)(3 - 4i) = 48 - 14i$$

$$(8 + 6i)(8 - 6i) = 100$$

$$(3 + 4i)(3 - 4i) = 25$$

$$\frac{8+4i}{1+3i} =$$

$$\frac{8+i}{3+2i} =$$

$$\frac{4+2i}{2+3i} =$$

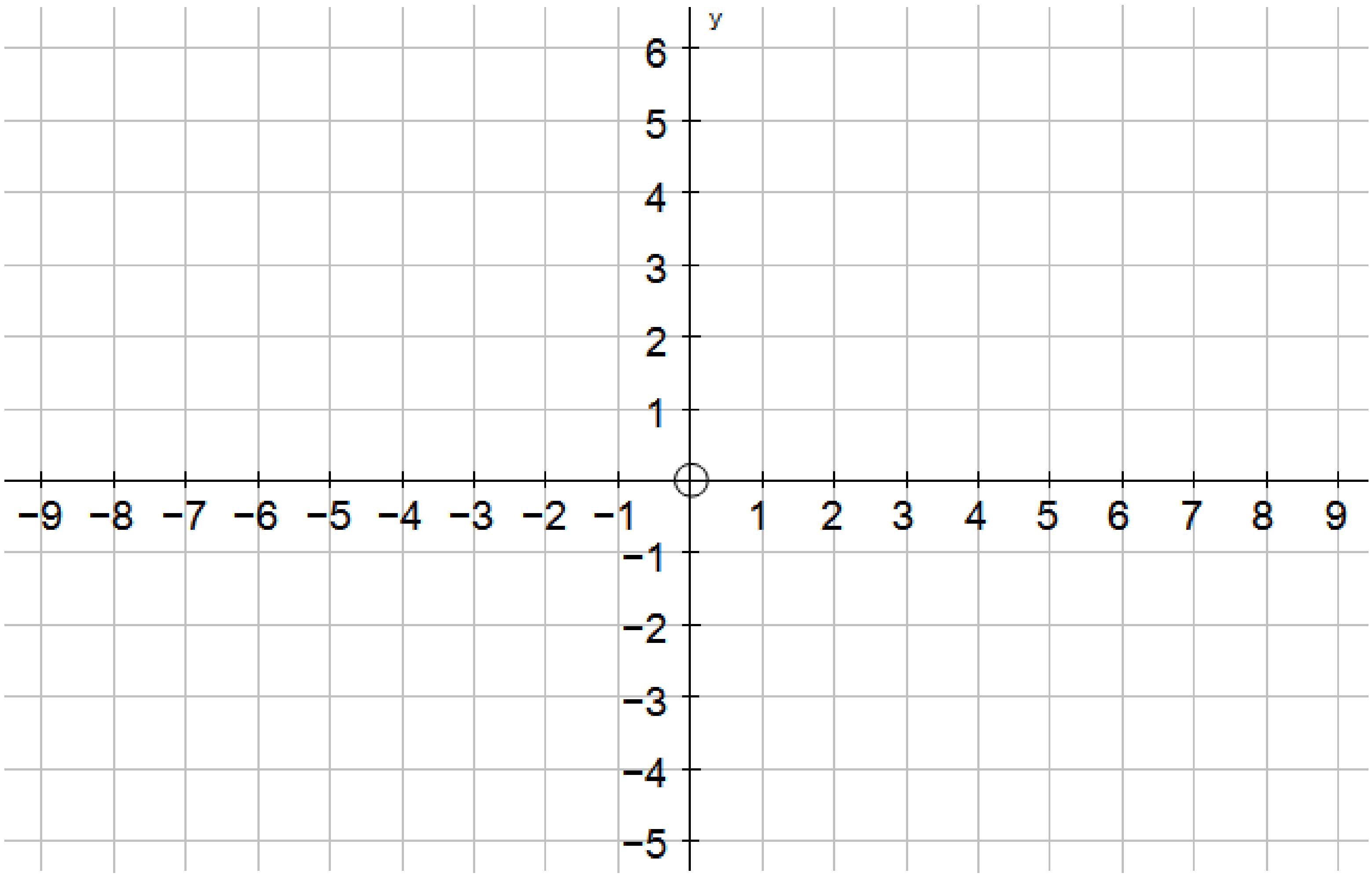
$$\frac{8+4i}{1+3i} = \frac{8+4i}{1+3i} \times \frac{1-3i}{1-3i} = 2-2i$$

$$\frac{8+i}{3+2i} = 2-i$$

$$\frac{4+2i}{2+3i} = \frac{14}{13} - \frac{8}{13}i$$

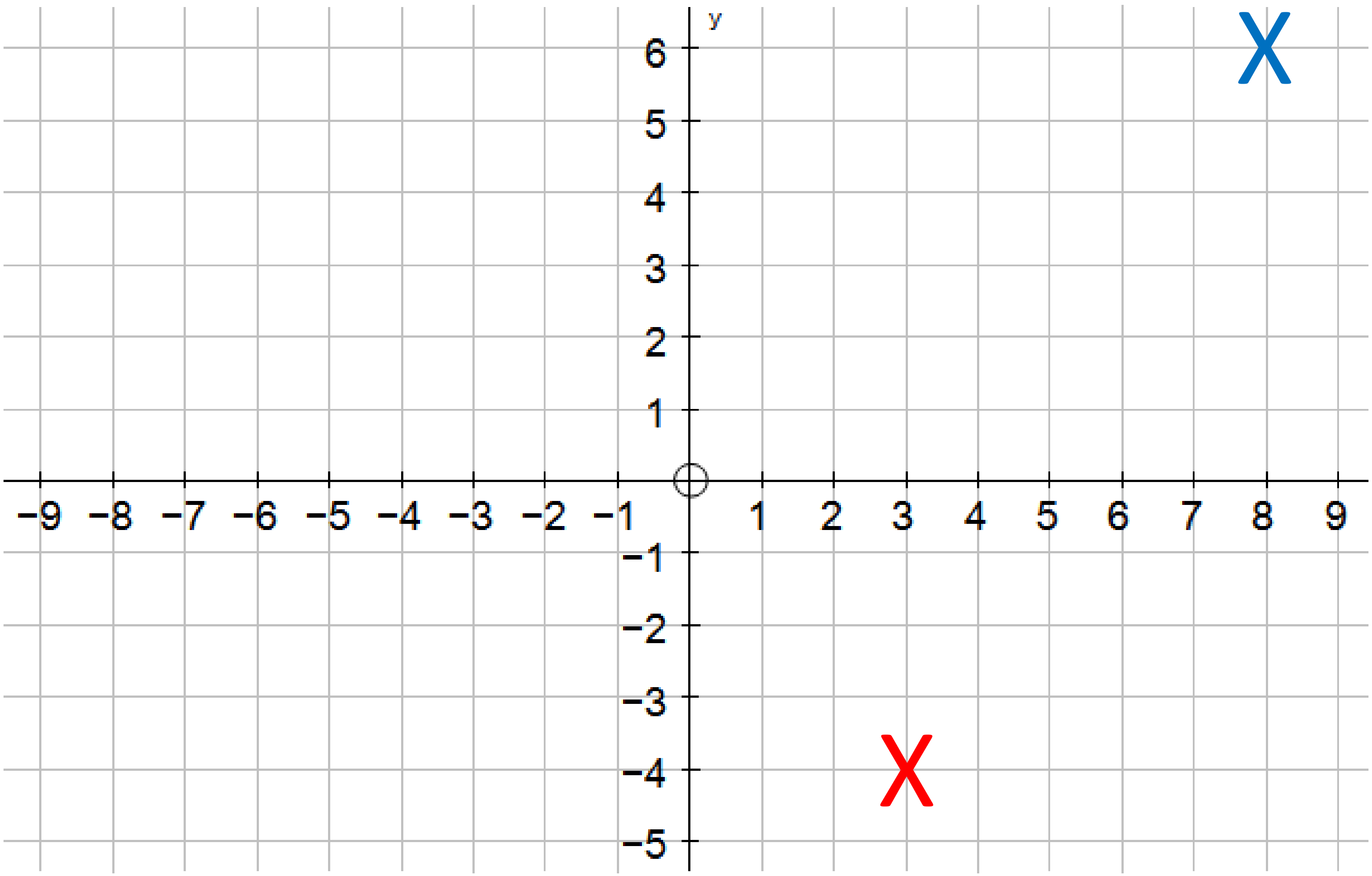
Illustrate on Argand diagram:

$8 + 6i$ and $3 - 4i$



Illustrate on Argand diagram:

$8 + 6i$ and $3 - 4i$



$a = 8 + 6i$ and $b = 3 - 4i$

Find:

$$|a|$$

$$|b|$$

$$|a + b|$$

$$|ab|$$

$a = 8 + 6i$ and $b = 3 - 4i$

Find:

$$|a| = 10$$

$$|b| = 5$$

$$|a + b| = 5\sqrt{5}$$

$$|ab| = 50$$

Find:

$$\arg(3 + 3i)$$

$$\arg(3 - 3i)$$

$$\arg\left(3 + \sqrt{3}i\right)$$

$$\arg\left(-\sqrt{3} + 3i\right)$$

$$\arg\left(-\sqrt{3} - 3i\right)$$

$$\arg(i)$$

Find:

$$\arg(3+3i) = \frac{\pi}{4}$$

$$\arg(3-3i) = \frac{5\pi}{4}$$

$$\arg\left(3+\sqrt{3}i\right) = \frac{\pi}{6}$$

$$\arg\left(-\sqrt{3}+3i\right) = \frac{7\pi}{6}$$

$$\arg\left(-\sqrt{3}-3i\right) = \frac{-7\pi}{6}$$

$$\arg(i) = \pi$$

$a = 8 + 6i$ and $b = 3 - 4i$ and $c = -3 + 4i$

Find:

$$\arg(a)$$

$$\arg(b)$$

$$\arg(ab)$$

$$\arg(c)$$

$a = 8 + 6i$ and $b = 3 - 4i$ and $c = -3 + 4i$

Find:

$$\arg(a) = tan^{-1}\left(\frac{6}{8}\right) = 0.644$$

$$\arg(b) = tan^{-1}\left(\frac{-4}{3}\right) = -0.927$$

$$\arg(ab) = tan^{-1}\left(\frac{\cdots}{\cdots}\right) = -0.283$$

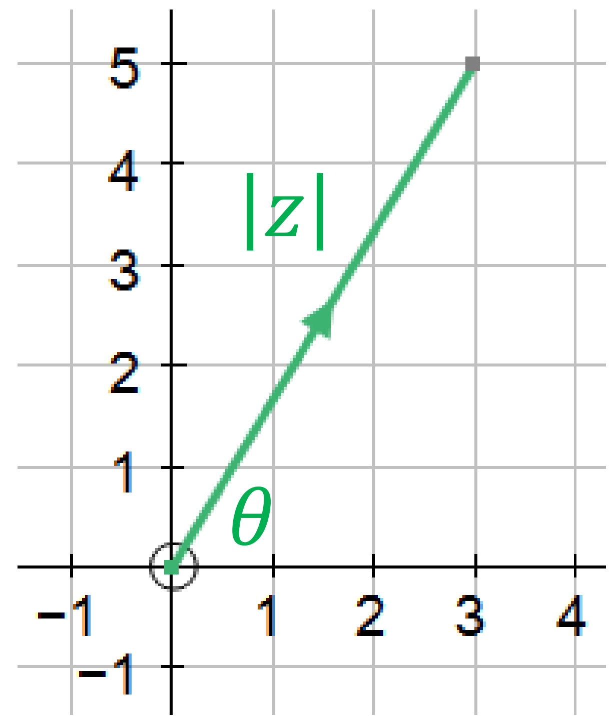
$$\arg(c) = tan^{-1}\left(\frac{4}{-3}\right) = 2\pi - 0.927$$

$$x = |z|cos\theta \text{ and } y = |z|sin\theta$$

$$x + iy = |z|cos\theta + i|z|sin\theta$$

$$x + iy = |z|(cos\theta + isin\theta)$$

$$x + iy = r(cos\theta + isin\theta)$$



$a = 8 + 6i$ and $b = 3 - 4i$

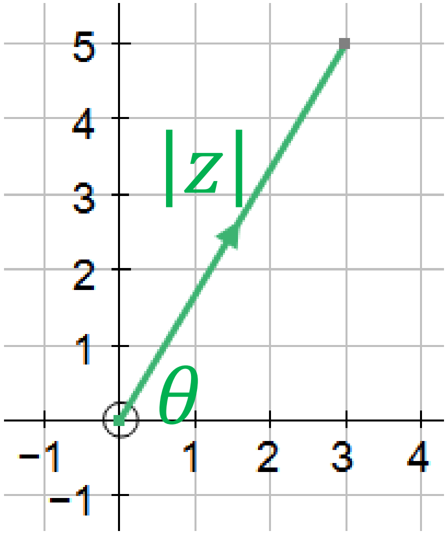
Write in modulus argument form:

a

b

$a + b$

ab



$$x + iy = r(cos\theta + isin\theta)$$

$$a = 8 + 6i \quad \text{and} \quad b = 3 - 4i$$

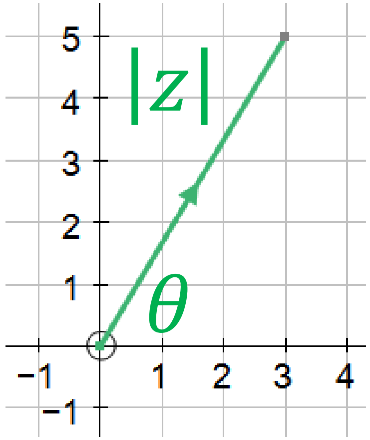
Write in modulus argument form:

$$a = 10(cos0.64 + isin0.64)$$

$$b = 5\big(cos(-0.64) + isin(-0.64)\big)$$

$$a + b = 5\sqrt{5}(cos0.18 + isin0.18)$$

$$ab = 50(cos0.28 + isin0.28)$$



$$x + iy = r(cos\theta + isin\theta)$$

$$5\left(cos\frac{\pi}{4} + isin\frac{\pi}{4}\right) \quad \text{and} \quad 2\left(cos\frac{-\pi}{6} + isin\frac{-\pi}{6}\right)$$

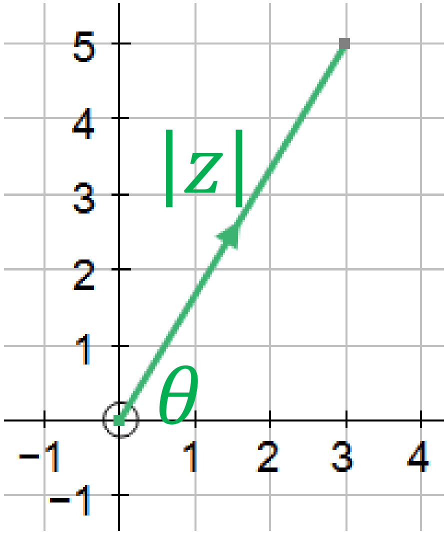
Write in Cartesian form:

$$a$$

$$b$$

$$a + b$$

$$ab$$



$$r(cos\theta + isin\theta) = x + iy$$

$5\left(cos\frac{\pi}{4} + isin\frac{\pi}{4}\right)$ and $2\left(cos\frac{-\pi}{6} + isin\frac{-\pi}{6}\right)$

Write in Cartesian form:

$$a = 4 + 4i$$

$$b = -1 - \sqrt{3}i$$

$$a + b = 3.76(cos0.65 + isin0.65)$$

$$ab = 2.42(cos\frac{-3\pi}{4} + isin\frac{-3\pi}{4})$$

$$r(cos\theta + isin\theta) = x + iy$$

$$z_1 = 5\left(cos\frac{\pi}{4} + isin\frac{\pi}{4}\right)$$

$$z_2 = 2\left(cos\frac{\pi}{3} + isin\frac{\pi}{3}\right)$$

Find $z_1 \times z_2$ and write your answer in modulus argument form

$$r(cos\theta + isin\theta) = x + iy$$

$$z_1 = 5\left(cos\frac{\pi}{4} + isin\frac{\pi}{4}\right)$$

$$z_2 = 2\left(cos\frac{\pi}{3} + isin\frac{\pi}{3}\right)$$

Find $z_1 \times z_2$ and write your answer in modulus argument form

$$z_1 \times z_2 = 10\left(cos\frac{7\pi}{12} + isin\frac{7\pi}{12}\right)$$

$$r(cos\theta + isin\theta) = x + iy$$

The Complex Conjugate

Given $z = x + iy$,

then $z^* = x - iy$

is known as the complex conjugate.

$$z = x + iy$$
$$z^* = x - iy$$

$$z = 2 + 5i$$

Find:

$$z + z^*$$

$$z - z^*$$

$$zz^*$$

$$z = 2 + 5i$$

Find:

$$z + z^* = 2 + 5i + 2 - 5i = 4$$

$$z - z^* = 2 + 5i - (2 - 5i) = 10i$$

$$zz^* = (2 + 5i)(2 - 5i) = 4 + 25 = 29$$

Solve

$$z^2 + 2z + 4 = 0$$

$$3z^2 + 6z + 9 = 0$$

$$3z^2 + 4z + 6 = 0$$

Solve

$$z^2 + 2z + 4 = 0, \qquad z = -1 \pm \sqrt{3}i$$

$$3z^2 + 6z + 9 = 0, z = -1 \pm \sqrt{2}i$$

$$3z^2 + 4z + 6 = 0, z = -\frac{2}{3} \pm \frac{\sqrt{14}}{3}i$$

$$z^2 - 4z + c = 0$$

(where c is a real number)

a) There is one root at $2 - i$. Write down the value of the other root.

b) Find the product of the roots.

c) Write down the value of c.

$$z^2 - 4z + c = 0$$

(where c is a real number)

a) There is one root at $2 - i$. Write down the value of the other root. $2 + i$

b) Find the product of the roots. 5

c) Write down the value of c. 5

For $z = x + iy$, solve

(write down an equation for z in the form above)

$$4z + 2i = 8(4 - 3i)$$

$$5z - 2z^* = 3 - 8i$$

$$2z - 4i = 3i(z^* + 2) + 5 - 8i$$

For $z = x + iy$, solve

(write down an equation for z in the form above)

$4z + 2i = 8(4 - 3i)$, $\quad z = 8 - \frac{13}{2}i$

$5z - 2z^* = 3 - 8i$, $\quad z = 1 - \frac{7}{8}i$

$$2z - 4i = 3i(z^* + 2) + 5 - 8i$$

$$z = -\frac{16}{5} - \frac{19}{5}i$$

Deduce a method for multiplying and dividing complex numbers written in modulus-argument form

$z = 3 + 4i$ $\qquad\qquad$ $z = 2 + 5i$

Sketch these on an Argand diagram
Multiply these
Sketch the result on the Argand diagram
Suggest how new angle is related to original angles
Suggest how new size is related to original size
Check and conclude