Intersecting Planes

How do the following sets of planes intersect each other?

In each case, give as much information as possible about the intersection

$x - y + z + 5 = 0$ $x - y + z + 2 = 0$ $x - y + 7z + 2 = 0$	$x - y - z + 7 = 0$ $x - y + z + 2 = 0$ $x - y + 7z + 2 = 0$	$x - y - z + 2 = 0$ $x - y + z + 2 = 0$ $x - y + 7z + 2 = 0$
$-x - y - z + 2 = 0$ $x - y + z + 2 = 0$ $x - y + 7z + 2 = 0$	$-x - y - z + 7 = 0$ $x - y + z + 2 = 0$ $x - y + 7z + 2 = 0$	$x - y + z + 5 = 0$ $x - y + z + 2 = 0$ $x - y + z - 5 = 0$

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$x - y + z + 5 = 0$ $r = \begin{pmatrix} 0 \\ 0 \\ -2 \end{pmatrix} + \mu \begin{pmatrix} -2 \\ 0 \\ 2 \end{pmatrix} + \lambda \begin{pmatrix} 0 \\ 2 \\ 2 \end{pmatrix}$ $r \cdot \begin{pmatrix} 1 \\ -1 \\ 7 \end{pmatrix} = -2$	$x - y - z + 7 = 0$ $r = \begin{pmatrix} 0 \\ 0 \\ -2 \end{pmatrix} + \mu \begin{pmatrix} -2 \\ 0 \\ 2 \end{pmatrix} + \lambda \begin{pmatrix} 0 \\ 2 \\ 2 \end{pmatrix}$ $r \cdot \begin{pmatrix} 1 \\ -1 \\ 7 \end{pmatrix} = -2$	$x - y - z + 2 = 0$ $r = \begin{pmatrix} 0 \\ 0 \\ -2 \end{pmatrix} + \mu \begin{pmatrix} -2 \\ 0 \\ 2 \end{pmatrix} + \lambda \begin{pmatrix} 0 \\ 2 \\ 2 \end{pmatrix}$ $r \cdot \begin{pmatrix} 1 \\ -1 \\ 7 \end{pmatrix} = -2$
$-x - y - z + 2 = 0$ $r = \begin{pmatrix} 0 \\ 0 \\ -2 \end{pmatrix} + \mu \begin{pmatrix} -2 \\ 0 \\ 2 \end{pmatrix} + \lambda \begin{pmatrix} 0 \\ 2 \\ 2 \end{pmatrix}$ $r \cdot \begin{pmatrix} 1 \\ -1 \\ 7 \end{pmatrix} = -2$	$-x - y - z + 7 = 0$ $r = \begin{pmatrix} 0 \\ 0 \\ -2 \end{pmatrix} + \mu \begin{pmatrix} -2 \\ 0 \\ 2 \end{pmatrix} + \lambda \begin{pmatrix} 0 \\ 2 \\ 2 \end{pmatrix}$ $r \cdot \begin{pmatrix} 1 \\ -1 \\ 7 \end{pmatrix} = -2$	$x - y + z + 5 = 0$ $r = \begin{pmatrix} 0 \\ 0 \\ -2 \end{pmatrix} + \mu \begin{pmatrix} -2 \\ 0 \\ 2 \end{pmatrix} + \lambda \begin{pmatrix} 0 \\ 2 \\ 2 \end{pmatrix}$ $r \cdot \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix} = 5$

Transforming Planes

Start with three identical planes such as the following...

$x + y + z = 1$	$x + y + z = 1$	$x + y + z = 1$

Perform the following changes and write down each of your new equations of planes as you go.

a) Rotate two of them to create a sheaf.

b) Translate one plane of your sheaf to create a prism

c) Rotate your translated plane to create two parallel planes

d) Change your third plane to create three parallel planes

e) Go back to your sheaf and change one of the planes to create an intersection at a single point

Identifying Relationships between Planes

$x + y + z = 4$	$x + 4y + z = 0$	$-x - y - z = 4$	$x + y + z = 0$
$4x + y + z = 0$	$x + 4y + z = 4$	$x + 4y + 4z = 4$	$x - y + z = 0$

From the eight planes above, choose 3 that are…

a) All Parallel

b) Two Parallel but the third not

c) Prism

d) Sheaf

e) Single point

Create your own set of eight planes to fulfil each of the criteria above. Is it possible to do it with just seven planes? Or six?

Intersecting Planes - Answers

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$x - y + z + 5 = 0$ $x - y + z + 2 = 0$ $x - y + 7z + 2 = 0$ Two Parallel Planes	$x - y - z + 7 = 0$ $x - y + z + 2 = 0$ $x - y + 7z + 2 = 0$ Prism	$x - y - z + 2 = 0$ $x - y + z + 2 = 0$ $x - y + 7z + 2 = 0$ Sheaf
$-x - y - z + 2 = 0$ $x - y + z + 2 = 0$ $x - y + 7z + 2 = 0$ Intersect at a single point $(0,2,0)$	$-x - y - z + 7 = 0$ $x - y + z + 2 = 0$ $x - y + 7z + 2 = 0$ Intersect at a single point $\left(\frac{5}{2}, \frac{9}{2}, 0\right)$	$x - y + z + 5 = 0$ $x - y + z + 2 = 0$ $x - y + z - 5 = 0$ Parallel Planes

Intersecting Planes - Answers

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$x - y + z + 5 = 0$ $r = \begin{pmatrix} 0 \\ 0 \\ -2 \end{pmatrix} + \mu \begin{pmatrix} -2 \\ 0 \\ 2 \end{pmatrix} + \lambda \begin{pmatrix} 0 \\ 2 \\ 2 \end{pmatrix}$ $r \cdot \begin{pmatrix} 1 \\ -1 \\ 7 \end{pmatrix} = -2$ Two Parallel Planes	$x - y - z + 7 = 0$ $r = \begin{pmatrix} 0 \\ 0 \\ -2 \end{pmatrix} + \mu \begin{pmatrix} -2 \\ 0 \\ 2 \end{pmatrix} + \lambda \begin{pmatrix} 0 \\ 2 \\ 2 \end{pmatrix}$ $r \cdot \begin{pmatrix} 1 \\ -1 \\ 7 \end{pmatrix} = -2$ Prism	$x - y - z + 2 = 0$ $r = \begin{pmatrix} 0 \\ 0 \\ -2 \end{pmatrix} + \mu \begin{pmatrix} -2 \\ 0 \\ 2 \end{pmatrix} + \lambda \begin{pmatrix} 0 \\ 2 \\ 2 \end{pmatrix}$ $r \cdot \begin{pmatrix} 1 \\ -1 \\ 7 \end{pmatrix} = -2$ Sheaf
$-x - y - z + 2 = 0$ $r = \begin{pmatrix} 0 \\ 0 \\ -2 \end{pmatrix} + \mu \begin{pmatrix} -2 \\ 0 \\ 2 \end{pmatrix} + \lambda \begin{pmatrix} 0 \\ 2 \\ 2 \end{pmatrix}$ $r \cdot \begin{pmatrix} 1 \\ -1 \\ 7 \end{pmatrix} = -2$ Intersect at a single point	$-x - y - z + 7 = 0$ $r = \begin{pmatrix} 0 \\ 0 \\ -2 \end{pmatrix} + \mu \begin{pmatrix} -2 \\ 0 \\ 2 \end{pmatrix} + \lambda \begin{pmatrix} 0 \\ 2 \\ 2 \end{pmatrix}$ $r \cdot \begin{pmatrix} 1 \\ -1 \\ 7 \end{pmatrix} = -2$ Intersect at a single point	$x - y + z + 5 = 0$ $r = \begin{pmatrix} 0 \\ 0 \\ -2 \end{pmatrix} + \mu \begin{pmatrix} -2 \\ 0 \\ 2 \end{pmatrix} + \lambda \begin{pmatrix} 0 \\ 2 \\ 2 \end{pmatrix}$ $r \cdot \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix} = 5$ Parallel Planes