Binomial Theorem

$$(1+x)^n = 1 + \frac{n}{1}x + \frac{n(n-1)}{2!}x^2 + \frac{n(n-1)(n-2)}{3!}x^3 \ldots$$

n must be rational and not positive integer

$|x| < 1$

Questions

Rewrite as approximations using the binomial expansion as far as the x^4 term:

1) $(1+x)^{-5}$
2) $(1+2x)^{-5}$
3) $(4+x)^{-5}$
4) $(4+12x)^{-5}$

Rewrite as approximations using the binomial expansion as far as the x^3 term:

5) $(1+x)^{1/2}$
6) $(1+2x)^{1/2}$
7) $(4+x)^{1/2}$
8) $(4+12x)^{1/2}$

9) $(1+x)^{-3} + (2+3x)^{-3}$

Extra Practice

As far as the x^4 term

10) $(1+x)^{-6}$
11) $(1+2x)^{-6}$
12) $(4+3x)^{-5}$

As far as the x^3 term

13) $(1+x)^{1/4}$
14) $(1+2x)^{1/4}$
15) $(4+12x)^{1/4}$
16) $(4+x)^{1/4}$

Application

Show that $\left(1+\frac{x}{25}\right)^{\frac{1}{2}} = 1 + \frac{x}{50} - \frac{x^2}{5000} + \frac{x^3}{250000} - \ldots$

By substituting $x = 1$ into the expression above, deduce that $\sqrt{26} \approx 5.09902$

Find an approximation for $\sqrt{28}$.

Why can't this expansion be used to find an approximation for $\sqrt{35}$?

Answers

Rewrite as approximations using the binomial expansion as far as the x^4 term:

1) $(1+x)^{-5} = 1-5x+15x^2-35x^3+70x^4$

2) $(1+2x)^{-5} = 1-10x+60x^2-280x^3+1120x^4$

3) $(4+x)^{-5} = 4^{-5}(1+\frac{x}{4})^{-5} = \frac{1}{1024}-\frac{5}{4096}x+\frac{15}{16384}x^2-\frac{35}{65536}x^3+\frac{35}{131072}x^4$

4) $(4+12x)^{-5} = 4^{-5}(1+3x)^{-5} = \frac{1}{1024}-\frac{15}{1024}x+\frac{135}{1024}x^2-\frac{945}{1024}x^3+\frac{2835}{512}x^4$

Rewrite as approximations using the binomial expansion as far as the x^3 term:

5) $(1+x)^{1/2} = 1+\frac{1}{2}x-\frac{1}{8}x^2+\frac{1}{16}x^3$

6) $(1+2x)^{1/2} = 1+x-\frac{1}{2}x^2+\frac{1}{2}x^3$

7) $(4+x)^{1/2} = 4^{1/2}\left[(1+\frac{x}{4})^{1/2}\right] = 2+\frac{1}{4}x-\frac{1}{64}x^2+\frac{1}{512}x^3$

8) $(4+12x)^{1/2} = 4^{1/2}\left[(1+3x)^{1/2}\right] = 2+3x-\frac{9}{4}x^2+\frac{27}{8}x^3$

9) $(1+x)^{-3}+(2+3x)^{-3} = \frac{9}{8}-\frac{57}{16}x+\frac{123}{16}x^2-\frac{455}{32}x^3$

Extra practice:

10) $(1+x)^{-6} = 1-6x+21x^2-56x^3+126x^4$

11) $(1+2x)^{-6} = 1-12x+84x^2-448x^3+2016x^4$

12) $(4+3x)^{-5} = 4^{-5}\left[(1+\frac{3}{4}x)^{-5}\right] = \frac{1}{1024}-\frac{15}{4096}x+\frac{135}{16384}x^2-\frac{945}{65536}x^3+\frac{2835}{262144}x^4$

13) $(1+x)^{1/4} = 1+\frac{1}{4}x-\frac{3}{32}x^2+\frac{7}{128}x^3$

14) $(1+2x)^{1/4} = 1+\frac{1}{2}x-\frac{3}{8}x^2+\frac{7}{16}x^3$

15) $(4+12x)^{1/4} = \sqrt{2}+\frac{3\sqrt{2}}{4}x-\frac{27\sqrt{2}}{32}x^2+\frac{189\sqrt{2}}{128}x^3$

16) $(4+x)^{1/4} = \sqrt{2}+\frac{\sqrt{2}}{16}x-\frac{3\sqrt{2}}{512}x^2+\frac{7\sqrt{2}}{8192}x^3$

$$\begin{array}{r} 1.\;2\;\;3\;\;4\;\;5\;\;6\;\;7\;\;9\;\;0\;\;1\;\;2\ldots \\ 81\overline{\big)\,100.^{19}0^{28}0^{37}0^{46}0^{55}0^{64}0^{73}0^{1}0^{10}0^{19}0\ldots.} \end{array}$$

Activity…

$$(1+x)^n = 1 + \frac{n}{1}x + \frac{n(n-1)}{2!}x^2 + \frac{n(n-1)(n-2)}{3!}x^3\ldots$$

Check for…

$$n = -1\,, x = -x$$

Compare LHS vs RHS

Compare Graphs (between limits)

Compare the geometric series

Substitute values of x to check ($x = 1, x = {}^{1}\!/_{2}$)

$$n = -2, \qquad x = -x$$

Compare LHS vs RHS

Compare Graphs (between limits)

Compare the geometric series

Substitute values of x to check ($x = 1, x = {}^{1}\!/_{2}\,, x = 0.1$)

Relate to 100/81 (and consider the $10x^9$ term)

$$n = {}^{1}\!/_{2}\,, \qquad x = x$$

Compare LHS vs RHS

Compare Graphs (between limits)

Compare the geometric series

Substitute values of x to check

Square both sides

Coefficient	x^n	Term	Sum
1	1	1	1
2	0.1	0.2	1.2
3	0.01	0.03	1.23
4	0.001	0.004	1.234
5	0.0001	0.0005	1.2345
6	0.00001	0.00006	1.23456
7	0.000001	0.000007	1.234567
8	0.0000001	0.0000008	1.2345678
9	0.00000001	0.00000009	1.23456789
10	0.000000001	0.00000001	1.2345679
		1.2345679	