Chain, Product, Quotient Rule Recap

Differentiate these.

Chain Rule

$y = (3x + 4)^3$	$y = sin(2x)$	$y = e^{2x}$
$y = (3x^3 + 1)^3$	$y = sin(x^2)$	$y = e^{(x^3)}$
$y = \dfrac{4}{4 - x^4}$	$y = \dfrac{1}{\left(\dfrac{1}{cosecx}\right)}$	$y = ln\left(\dfrac{1}{e^x}\right)$
$y = \sqrt{\sqrt{x} + 1}$	$y = sin(cos(tanx))$	$y = e^{(e^x)}$

Product Rule

$y = (3x + 4)(2x - 3)$	$y = x^2 sinx$	$y = xe^x$
$y = (3x^2 + 4)(2x^4 + 3)$	$y = sinxcosx$	$y = x^3 e^x$
$y = \sqrt{3x}\sqrt{2x}$	$y = sinxcosxtanx$	$y = xlnx$
$y = \left(\dfrac{x}{3}\right)^3 \sqrt[3]{x}$	$y = sin(x^2)cos(x^2)$	$y = e^{x^2} ln(x^2)$

Quotient Rule

$y = \dfrac{x + 1}{2x + 1}$	$y = \dfrac{3x^2}{sinx}$	$y = \dfrac{2x}{e^x}$
$y = \dfrac{x^2 + 1}{x^2 - 1}$	$y = tanx$	$y = \dfrac{e^x}{e^{-x}}$
$y = \dfrac{x^2}{\sqrt{x}}$	$y = \dfrac{x^2}{tanx}$	$y = \dfrac{e^{-4x}}{4e^{4x}}$
$y = \sqrt[3]{\dfrac{x + 1}{x - 1}}$	$y = \dfrac{cotx}{2secx}$	$y = \dfrac{ln(x^2)}{e^{x^2}}$

Chain, Product, Quotient Rule Recap - Answers

Differentiate these.

Chain Rule

$y = (3x + 4)^3$ $\frac{dy}{dx} = 9(3x + 4)^2$	$y = sin(2x)$ $\frac{dy}{dx} = 2cos(2x)$	$y = e^{2x}$ $\frac{dy}{dx} = 2e^{2x}$
$y = (3x^3 + 1)^3$ $\frac{dy}{dx} = 3(3x^3 + 1)^2(9x^2)$ $= 27x^2(3x^3 + 1)^2$	$y = sin(x^2)$ $\frac{dy}{dx} = 2xcos(x^2)$	$y = e^{(x^3)}$ $\frac{dy}{dx} = 3x^2e^{(x^3)}$
$y = \frac{4}{4 - x^4} = 4(4 - x^4)^{-1}$ $\frac{dy}{dx} = -4(4 - x^4)^{-2} \times -4x^3$ $= \frac{16x^3}{(4 - x^4)^2}$	$y = \frac{1}{\left(\frac{1}{cosecx}\right)} = \frac{1}{sinx}$ $= (sinx)^{-1}$ $\frac{dy}{dx} = -(sinx)^{-2} \times cosx$ $= \frac{-cosx}{(sinx)^2} = cotxcosecx$	$y = ln\left(\frac{1}{e^x}\right)$ $\frac{dy}{dx} = \frac{1}{\frac{1}{e^x}} \times -e^{-x}$ $= -e^x \times e^{-x}$ $= -e^0$ $= 1$ (convince yourself that this is the case)
$y = \sqrt{\sqrt{x} + 1} = \left(x^{\frac{1}{2}} + 1\right)^{\frac{1}{2}}$ $\frac{dy}{dx} = \frac{1}{2}\left(x^{\frac{1}{2}} + 1\right)^{-\frac{1}{2}} \times \frac{1}{2}x^{-\frac{1}{2}}$ $= \frac{1}{4} \times \frac{1}{\sqrt{\sqrt{x} + 1}} \times \frac{1}{\sqrt{x}}$ $= \frac{1}{4\sqrt{x}\sqrt{\sqrt{x} + 1}}$	$y = sin(cos(tanx))$ $\frac{dy}{dx} = -cos(cos(tanx))sin(tanx)sec^2x$	$y = e^{e^x}$ $\frac{dy}{dx} = e^{e^x}e^x = e^{e^x + x}$

Product Rule

$y = (3x + 4)(2x - 3)$ $\frac{dy}{dx} = 2(3x + 4) + 3(2x - 3)$ $= 12x - 1$	$y = x^2 sinx$ $\frac{dy}{dx} = x^2 cosx + 2xsinx$	$y = xe^x$ $\frac{dy}{dx} = xe^x + e^x$ $= (x + 1)e^x$
$y = (3x^2 + 4)(2x^4 + 3)$ $\frac{dy}{dx} = 8x^3(3x^2 + 4) + 6x(2x^4 + 3)$ $= 2x[4x^2(3x^2 + 4) + 3(2x^4 + 3)]$ $= 2x[12x^4 + 16x^2 + 6x^4 + 9]$ $= 2x[18x^4 + 16x^2 + 9]$	$y = sinxcosx$ $\frac{dy}{dx} = cos^2x - sin^2x$	$y = x^3 e^x$ $\frac{dy}{dx} = 3x^2 e^x + x^3 e^x$ $= x^2 e^x(x + 3)$
$y = \sqrt{3x}\sqrt{2x}$ $= (6x^2)^{\frac{1}{2}} = \sqrt{6}x$ $\frac{dy}{dx} = \sqrt{6}$ (easy method?)	$y = sinxcosxtanx$ $= sinxcosx \times \dfrac{sinx}{cosx}$ $= sin^2x$ $\frac{dy}{dx} = 2sinxcosx$	$y = xlnx$ $\frac{dy}{dx} = lnx + 1$
$y = \left(\frac{x}{3}\right)^3 \sqrt[3]{x}$ $\frac{dy}{dx} = \left(\frac{x}{3}\right)^3 \times \frac{1}{3\sqrt[3]{x}} + \sqrt[3]{x} \times \frac{x^2}{3}$ $= \frac{x^3}{9\sqrt[3]{x}} + \frac{x^2\sqrt[3]{x}}{3}$ $= \frac{x^2}{3}\left(\frac{\sqrt[3]{x^2}}{3} + \sqrt[3]{x}\right)$	$y = sin(x^2)cos(x^2)$ $\frac{dy}{dx} = 2x[cos^2(x^2) + sin^2(x^2)]$	$y = e^{x^2}ln(x^2)$ $\frac{dy}{dx} = \frac{2e^{x^2}}{x} + 2xe^{x^2}ln(x^2)$

Quotient Rule

$$y = \frac{x+1}{2x+1}$$ $$\frac{dy}{dx} = \frac{2x+1-2(x+1)}{(2x+1)^2}$$ $$= \frac{-1}{(2x+1)^2}$$	$$y = \frac{3x^2}{sinx}$$ $$\frac{dy}{dx} = \frac{6xsinx - 3x^2cosx}{sin^2x}$$ $$= \frac{3x}{sinx}\left(2 - \frac{x}{tanx}\right)$$	$$y = \frac{2x}{e^x}$$ $$\frac{dy}{dx} = \frac{2e^x - 2xe^x}{e^{2x}}$$ $$= \frac{2-2x}{e^x}$$
$$y = \frac{x^2+1}{x^2-1}$$ $$\frac{dy}{dx} = \frac{2x(x^2-1) - 2x(x^2+1)}{(x^2-1)^2}$$ $$= \frac{-4x}{(x^2-1)^2}$$	$$y = tanx = \frac{sinx}{cosx}$$ $$\frac{dy}{dx} = \frac{cos^2x + sin^2x}{cos^2x}$$ $$= 1 + sin^2x$$ $$= sec^2x$$	$$y = \frac{e^x}{e^{-x}} = e^x \times e^x = e^{2x}$$ $$\frac{dy}{dx} = 2e^{2x}$$
$$y = \frac{x^2}{\sqrt{x}}$$ $$\frac{dy}{dx} = \frac{2x\sqrt{x} - \frac{x^2}{2\sqrt{x}}}{x}$$ $$= 2\sqrt{x} - \frac{\sqrt{x}}{2}$$	$$y = \frac{x^2}{tanx}$$ $$\frac{dy}{dx} = \frac{2xtanx - x^2secx}{tan^2x}$$ $$= \frac{2x}{tanx} - \frac{x^2}{tan^2xcosx}$$ $$= \frac{x}{tanx}\left(2 - \frac{x}{tanxcosx}\right)$$	$$y = \frac{e^{-4x}}{4e^{4x}} = \frac{1}{4}(e^{-4x} \times e^{-4x})$$ $$= \frac{e^{-8x}}{4}$$ $$\frac{dy}{dx} = \frac{-2}{e^{8x}}$$
$$y = \sqrt[3]{\frac{x+1}{x-1}} = \frac{(x+1)^{\frac{1}{3}}}{(x-1)^{\frac{1}{3}}}$$ $$\frac{dy}{dx} = \frac{\frac{1}{3}(x-1)^{\frac{1}{3}}(x+1)^{\frac{-2}{3}} - \frac{1}{3}(x-1)^{-\frac{2}{3}}(x+1)^{\frac{1}{3}}}{(x-1)^{\frac{2}{3}}}$$ $$= \frac{1}{3}\left[\frac{(x-1)^{-\frac{1}{3}}}{(x+1)^{\frac{2}{3}}} - \frac{(x+1)^{\frac{1}{3}}}{(x-1)^{\frac{4}{3}}}\right]$$ $$= \frac{1}{3}\left[\frac{(x-1)-(x+1)}{(x+1)^{\frac{2}{3}}(x-1)^{\frac{4}{3}}}\right]$$ $$= \frac{-2}{3}\left[\frac{1}{\sqrt[3]{(x+1)^2(x-1)^4}}\right]$$	$$y = \frac{cotx}{2secx} = \frac{cosx}{sinx} \div \frac{2}{cosx}$$ $$= \frac{cosx}{sinx} \times \frac{cosx}{2}$$ $$= \frac{cos^2x}{2sinx}$$ $$\frac{dy}{dx} = \frac{-4sin^2xcosx - 2cos^3x}{4sin^2x}$$ $$= -xcosx - \frac{cosx}{2tan^2x}$$	$$y = \frac{ln(x^2)}{e^{x^2}}$$ $$\frac{dy}{dx} = \frac{\frac{2e^{x^2}}{x} - 2xe^{x^2}lnx^2}{e^{2x^2}}$$ $$= \frac{\frac{2}{x} - 2xlnx^2}{e^{x^2}}$$

Chain, Product, Quotient Rule Recap

Differentiate these.

Polynomials

$y = (3x + 4)^3$	$y = (3x + 4)(2x - 3)$	$y = \dfrac{x + 1}{2x + 1}$
$y = (3x^3 + 1)^3$	$y = (3x^2 + 4)(2x^4 + 3)$	$y = \dfrac{x^2 + 1}{x^2 - 1}$
$y = \dfrac{4}{4 - x^4}$	$y = \sqrt{3x}\sqrt{2x}$	$y = \dfrac{x^2}{\sqrt{x}}$
$y = \sqrt{\sqrt{x} + 1}$	$y = \left(\dfrac{x}{3}\right)^3 \sqrt[3]{x}$	$y = \sqrt[3]{\dfrac{x + 1}{x - 1}}$

Trigonometry

$y = \sin(2x)$	$y = x^2 \sin x$	$y = \dfrac{3x^2}{\sin x}$
$y = \sin(x^2)$	$y = \sin x \cos x$	$y = \tan x$
$y = \dfrac{1}{\left(\dfrac{1}{\cosec x}\right)}$	$y = \sin x \cos x \tan x$	$y = \dfrac{x^2}{\tan x}$
$y = \sin(\cos(\tan x))$	$y = \sin(x^2)\cos(x^2)$	$y = \dfrac{\cot x}{2\sec x}$

Exponentials

$y = e^{2x}$	$y = xe^x$	$y = \dfrac{2x}{e^x}$
$y = e^{(x^3)}$	$y = x^3 e^x$	$y = \dfrac{e^x}{e^{-x}}$
$y = \ln\left(\dfrac{1}{e^x}\right)$	$y = x \ln x$	$y = \dfrac{e^{-4x}}{4e^{4x}}$
$y = e^{(e^x)}$	$y = e^{x^2} \ln(x^2)$	$y = \dfrac{\ln(x^2)}{e^{x^2}}$

Build It Up Practice

$y = (2x + 3)^7$	$y = 2x(4x + 5)$	$y = \dfrac{2x}{4x + 5}$
$y = (2x^2 + 3)^7$	$y = 2x^2(4x + 5)$	$y = \dfrac{2x^2}{4x + 5}$
$y = (2x^2 + 3x)^7$	$y = (2x + 3)(4x + 5)$	$y = \dfrac{2x + 3}{4x + 5}$
$y = (2x^3 + 3x^2)^7$	$y = (2x^2 + 3x)(4x + 5)$	$y = \dfrac{2x^2 + 3x}{4x + 5}$
$y = \dfrac{1}{2x + 3}$	$y = \dfrac{1}{x}(4x + 5)$	$y = \dfrac{1}{4x + 5}$
$y = \dfrac{1}{2x^2 + 3x}$	$y = \sqrt{x}(4x + 5)$	$y = \dfrac{\sqrt{x}}{\sqrt{x} + 1}$
$y = sin(2x)$	$y = \sin x \cos x$	$y = \dfrac{1}{\sqrt{x}\sqrt{x} + 1}$
$y = sin(2x^2 + 3)$	$y = \sin 2x \cos 2x$	$y = \dfrac{\sin x}{\cos x}$
$y = \cos(2x^2 + 3)$	$y = \sin(2x + 3)\cos(4x + 5)$	$y = \dfrac{\sin 2x}{\cos 2x}$
$y = sin\left(\dfrac{1}{2x + 3}\right)$	$y = (2x + 3)^7(4x + 5)$	$y = \left(\dfrac{2x + 3}{4x + 5}\right)^7$

Build It Up Practice - Answers

$y = (2x+3)^7, \ \frac{dy}{dx} =$	$y = 2x(4x+5), \ \frac{dy}{dx} =$	$y = \frac{2x}{4x+5}, \ \frac{dy}{dx} =$
$y = (2x^2+3)^7, \ \frac{dy}{dx} =$	$y = 2x^2(4x+5), \ \frac{dy}{dx} =$	$y = \frac{2x^2}{4x+5}, \ \frac{dy}{dx} =$
$y = (2x^2+3x)^7, \ \frac{dy}{dx} =$	$y = (2x+3)(4x+5), \ \frac{dy}{dx} =$	$y = \frac{2x+3}{4x+5}, \ \frac{dy}{dx} =$
$y = (2x^3+3x^2)^7, \ \frac{dy}{dx} =$	$y = (2x^2+3x)(4x+5), \ \frac{dy}{dx} =$	$y = \frac{2x^2+3x}{4x+5}, \ \frac{dy}{dx} =$
$y = \frac{1}{2x+3}, \ \frac{dy}{dx} =$	$y = \frac{1}{x}(4x+5), \ \frac{dy}{dx} =$	$y = \frac{1}{4x+5}, \ \frac{dy}{dx} =$
$y = \frac{1}{2x^2+3x}, \ \frac{dy}{dx} =$	$y = \sqrt{x}(4x+5), \ \frac{dy}{dx} =$	$y = \frac{\sqrt{x}}{\sqrt{x+1}}, \ \frac{dy}{dx} =$
$y = sin(2x), \ \frac{dy}{dx} =$	$y = \sin x \cos x, \ \frac{dy}{dx} =$	$y = \frac{1}{\sqrt{x}\sqrt{x+1}}, \ \frac{dy}{dx} =$
$y = sin(2x^2+3), \ \frac{dy}{dx} =$	$y = \sin 2x \cos 2x, \ \frac{dy}{dx} =$	$y = \frac{\sin x}{\cos x}, \ \frac{dy}{dx} =$
$y = \cos(2x^2+3), \ \frac{dy}{dx} =$	$y = \sin(2x+3)\cos(4x+5), \ \frac{dy}{dx} =$	$y = \frac{\sin 2x}{\cos 2x}, \ \frac{dy}{dx} =$
$y = sin\left(\frac{1}{2x+3}\right), \ \frac{dy}{dx} =$	$y = (2x+3)^7(4x+5), \ \frac{dy}{dx} =$	$y = \left(\frac{2x+3}{4x+5}\right)^7, \ \frac{dy}{dx} =$