Differential Equations

Process is...

1. Prepare for integrating by rearranging to get all x terms and y terms on separate sides
2. Integrate both sides
3. Combine the c's and rearrange into format $y = \cdots$ THIS IS THE GENERAL SOLUTION
4. Use the coordinate given to find THE PARTICULAR SOLUTION

E.G. (Ex11E Qu2i)

Find the particular solution...

$$x(x-1)\frac{dy}{dx} = y$$

$$y = 1, x = 2$$

Rearrange

$$\int \frac{1}{y}\,dy = \int \frac{1}{x(x-1)}\,dx$$

Integrate both sides

Note that

$$\frac{1}{x(x-1)} = \frac{A}{x} + \frac{B}{(x-1)}$$

$$\frac{1}{x(x-1)} = \frac{A(x-1)}{x(x-1)} + \frac{Bx}{x(x-1)}$$

$$1 = A(x-1) + Bx$$

$$x = 0 \Rightarrow A = -1$$
$$x = 1 \Rightarrow B = 1$$

$$\int \frac{1}{x(x-1)}\,dx = \int \frac{-1}{x}\,dx + \int \frac{1}{(x-1)}\,dx$$

$$\int \frac{1}{y}\,dy = \int \frac{-1}{x}\,dx + \int \frac{1}{(x-1)}\,dx$$

$$lny = -lnx + ln(x-1) + c$$

Combine and rearrange

$$y = e^{-lnx+ln(x-1)+c}$$

$$y = A \cdot e^{lnx^{-1}} \cdot e^{+ln(x-1)}$$

$$y = A \cdot e^{ln\left(\frac{1}{x}\right)} \cdot e^{ln(x-1)}$$

$$y = A \cdot \frac{1}{x} \cdot (x-1)$$

$y = \frac{A(x-1)}{x}$ THIS IS THE GENERAL SOLUTION

Use coordinate to find particular solution

$$(2, 1) \Rightarrow 1 = \frac{A(2-1)}{2}$$

$$A = 2$$

$y = \frac{2(x-1)}{x} = \frac{2x-2}{x} = 2 - \frac{2}{x}$ THIS IS THE PARTICULAR SOLUTION (any of these formats)