A Harder Differential Equations Question

(Core 4 Page 237 Exercise 11F Qu 3)

Mould is spreading through a piece of cheese on mass 1 kg in such a way that at a time t days, x kg of the cheese has become mouldy where…

$$\frac{dx}{dt} = 2x(1-x)$$

Initially 0.01 kg of the cheese is mouldy.

Show that…

$$x = \frac{e^{2t}}{(99+e^{2t})}$$

And calculate after how long 0.99 kg of the cheese will be mouldy, giving your answer to the nearest hour.

$$\frac{dx}{x(1-x)} = 2dt$$

$$\frac{1}{x(1-x)} = \frac{A}{x} + \frac{B}{1-x}$$

$$1 = A(1-x) + Bx$$

$$x = 1 \Rightarrow B = 1$$

$$x = 0 \Rightarrow A = 1$$

$$\frac{1}{x(1-x)} = \frac{1}{x} + \frac{1}{(1-x)}$$

$$\int \frac{dx}{x(1-x)} = \int \frac{1}{x} + \frac{1}{1-x} dx = 2t + c$$

$$lnx - ln(1-x) = 2t + c$$

$$ln\frac{x}{(1-x)} = 2t + c$$

$$\frac{x}{(1-x)} = e^{2t+c}$$

$$\frac{x}{(1-x)} = Ae^{2t}$$

$$t = 0, x = 0.01 \Rightarrow$$

$$A = \frac{0.01}{0.99} = \frac{1}{99}$$

$$\frac{x}{(1-x)} = \frac{e^{2t}}{99}$$

$$99x = e^{2t}(1-x)$$

$$99x = e^{2t} - xe^{2t}$$

$$99x + xe^{2t} = e^{2t}$$

$$x(99 + e^{2t}) = e^{2t}$$

$$x = \frac{e^{2t}}{(99+e^{2t})}$$