Differentiating $y = a^{kx}$

$$y = a^x$$

$$\frac{dy}{dx} = lna \times a^x$$

$$y = a^{kx}$$

$$\frac{dy}{dx} = klna \times a^{kx}$$

Proof by implicit differentiation

$$y = a^x$$

$$lny = ln(a^x) = xlna$$

$$\frac{1}{y}\frac{dy}{dx} = lna$$

$$\frac{dy}{dx} = lna \times y = lna \times a^x$$

$$y = a^{kx}$$

$$lny = ln(a^{kx}) = kxlna$$

$$\frac{1}{y}\frac{dy}{dx} = klna$$

$$\frac{dy}{dx} = klna \times y = klna \times a^{kx}$$

Differentiate these…

1. $y = 2^x$
2. $y = 7^x$
3. $y = \pi^x$
4. $y = e^x$
5. $y = 2^{5x}$
6. $y = 7^{3x}$
7. $y = e^{3x}$
8. $y = \sqrt{3}^{\sqrt{2}x}$
9. $y = 3 \times 7^x$
10. $y = x2^x$
11. $y = \frac{2^x}{2x}$
12. $y = 5^{3x}(x^2 + 4x)$
13. $y = \frac{5^x}{5^{3x}}$
14. $y = x^x$
15. $v = (4 \times 3^t)i + (7 \times 5^{2t} + 6t)j$
16. $y = 2^{(3^x)}$

AQA A2 Paper 1, June 2018…

A curve is defined by the parametric equations

$$x = 4 \times 2^{-t} + 3$$

$$y = 3 \times 2^t - 5$$

Show that $\frac{dy}{dx} = -\frac{3}{4} \times 2^{2t}$

[3 marks]

Answers…

1. $\frac{dy}{dx} = ln2 \times 2^x$
2. $\frac{dy}{dx} = ln7 \times 7^x$
3. $\frac{dy}{dx} = ln\pi \times \pi^x$
4. $\frac{dy}{dx} = lne \times e^x = e^x$
5. $\frac{dy}{dx} = 5ln2 \times 2^{5x}$
6. $\frac{dy}{dx} = 3ln7 \times 7^{3x}$
7. $\frac{dy}{dx} = 3lne \times e^{3x} = 3e^{3x}$
8. $\frac{dy}{dx} = \sqrt{2}ln(\sqrt{3}) \times \sqrt{3}^{\sqrt{2}x}$
9. $\frac{dy}{dx} = 3ln7 \times 7^x$
10. $\frac{dy}{dx} = xln2 \times 2^x + 2x$
11. $\frac{dy}{dx} = \frac{2^x(2xln2-2)}{(2x)^2}$
12. $\frac{dy}{dx} = 5^{3x}(2x+4) + 3ln5 \times 5^{3x}(x^2+4x)$
13. $\frac{dy}{dx} = -2ln5 \times 5^{-2x}$
14. $\frac{dy}{dx} = x^x(1+lnx)$
15. $\frac{dy}{dx} = (4ln3 \times 3^x)i + (14ln5 \times 5^{2t} + 6)j$
16. $\frac{dy}{dx} = 2^{3^x} \times 3^x ln2 ln3$

Integrating $y = a^{kx}$

$$\int a^x \, dx = \frac{a^x}{lna}$$

$$\int a^{kx} \, dx = \frac{a^{kx}}{klna}$$