Improper Integrals
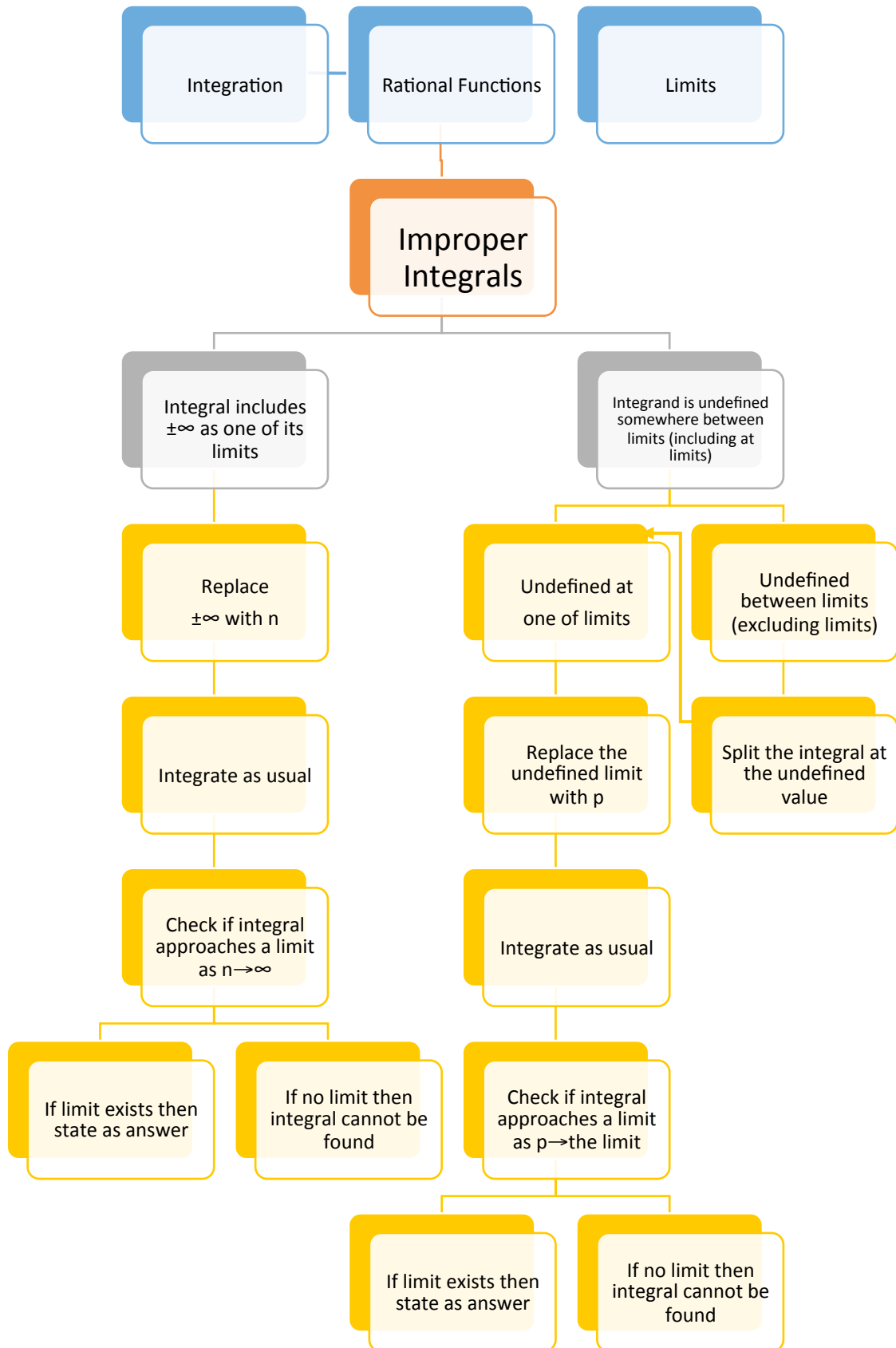
An integral is improper if either the interval is not finite or the function to integrate is not continuous within the interval.

Integration

Rational Functions

Limits

Improper Integrals

Integral includes ±∞ as one of its limits

Replace ±∞ with n

Integrate as usual

Check if integral approaches a limit as n→∞

If limit exists then state as answer

If no limit then integral cannot be found

Integrand is undefined somewhere between limits (including at limits)

Undefined at one of limits

Undefined between limits (excluding limits)

Replace the undefined limit with p

Split the integral at the undefined value

Integrate as usual

Check if integral approaches a limit as p→the limit

If limit exists then state as answer

If no limit then integral cannot be found

Improper Integrals

- The integrand = the expression to be integrated
- The integral = the expression after it has been integrated

Type 1 – the integral included $\pm\infty$ as one of its limits

1. Substitute $\infty = n$.

2. Integrate as normal.

3. Evaluate the integral using the limit n and the other limit.

4. Determine if, as $n \to \infty$, the integral approaches a finite value.

5. If yes then the improper integral can be found and the answer is this finite value.

6. If no then the improper integral cannot be found and does not have a finite value (or area).

Type 2 – the integral is undefined at somewhere between the limits (including at limits)

1. Determine where the integral is undefined. If undefined between the limits then split the single integral into two integrals at this point.

2. Substitute the x value where the integral is undefined for p.

3. Integrate as normal.

4. Evaluate the integral(s) using the limit p and the other limit.

5. Determine is, as $p \to the\ limit\ which\ was\ replaced$, the integral approaches a finite value.

6. If yes then the improper integral can be found and the answer is this finite value. If the integral was split into two, remember to add both parts together to obtain the final answer.

7. If no then the improper integral cannot be found and does not have a finite value (or area).