

## Interesting Questions

Qu 1... Edexcel unit tests, Parametric Equations -Qu 3. ([Link to markscheme](#))

The curve  $C$  has parametric equations  $x = 7 \sin t - 4$ ,  $y = 7 \cos t + 3$ ,  $-\frac{\pi}{2} \leq t \leq \frac{\pi}{3}$

- a** Show that the cartesian equation of  $C$  can be written as  $(x+a)^2 + (y+b)^2 = c$ , where  $a$ ,  $b$  and  $c$  are integers which should be stated. **(3 marks)**
- b** Sketch the curve  $C$  on the given domain, clearly stating the endpoints of the curve. **(3 marks)**
- c** Find the length of  $C$ . Leave your answer in terms of  $\pi$ . **(2 marks)**

Qu 2... AQA A2 Paper 1, June 2018 -Qu 5. ([Link to markscheme](#))

A curve is defined by the parametric equations

$$x = 4 \times 2^{-t} + 3$$

$$y = 3 \times 2^t - 5$$

Show that  $\frac{dy}{dx} = -\frac{3}{4} \times 2^{2t}$

**[3 marks]**

Find the Cartesian equation of the curve in the form  $xy + ax + by = c$ , where  $a$ ,  $b$  and  $c$  are integers.

**[3 marks]**

Qu 3... AQA A2 Paper 1, June 2018 – Qu 12. ([Link to markscheme](#))

$$p(x) = 30x^3 - 7x^2 - 7x + 2$$

Prove that  $(2x + 1)$  is a factor of  $p(x)$

**[2 marks]**

Factorise  $p(x)$  completely.

**[3 marks]**

Prove that there are no real solutions to the equation

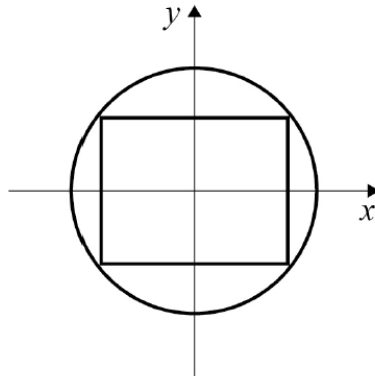
$$\frac{30 \sec^2 x + 2 \cos x}{7} = \sec x + 1$$

**[5 marks]**

Qu 4... AQA A2 Paper 1, June 2018 – Qu13. ([Link to markscheme](#))

A company is designing a logo. The logo is a circle of radius 4 inches with an inscribed rectangle. The rectangle must be as large as possible.

The company models the logo on an  $x$ - $y$  plane as shown in the diagram.



Use calculus to find the maximum area of the rectangle.

Fully justify your answer.

**[10 marks]**

Qu 5... AQA A2 Paper 2, June 2018 -Qu 8. ([Link to markscheme](#))

Determine a sequence of transformations which maps the graph of  $y = \sin x$  onto the graph of  $y = \sqrt{3} \sin x - 3 \cos x + 4$

Fully justify your answer.

**[7 marks]**

Show that the least value of  $\frac{1}{\sqrt{3} \sin x - 3 \cos x + 4}$  is  $\frac{2 - \sqrt{3}}{2}$

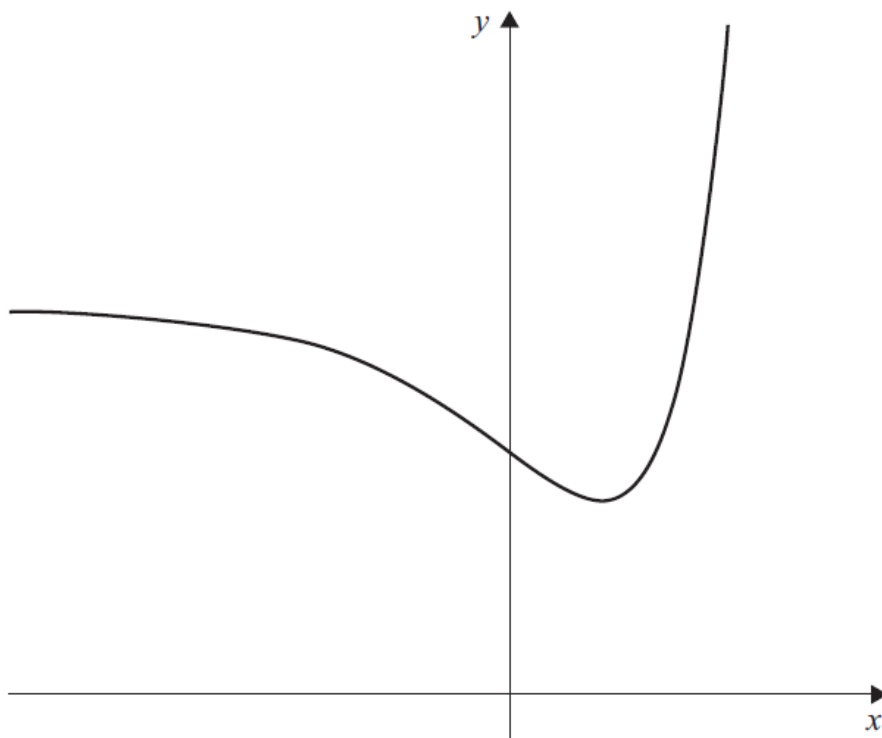
**[2 marks]**

Find the greatest value of  $\frac{1}{\sqrt{3} \sin x - 3 \cos x + 4}$

**[1 mark]**

A function  $f$  has domain  $\mathbb{R}$  and range  $\{y \in \mathbb{R} : y \geq e\}$

The graph of  $y = f(x)$  is shown.



The gradient of the curve at the point  $(x, y)$  is given by  $\frac{dy}{dx} = (x - 1)e^x$

Find an expression for  $f(x)$ .

Fully justify your answer.

**[8 marks]**

A large arch is planned for a football stadium. The parametric equations of the arch are  $x = 8(t + 10)$ ,  $y = 100 - t^2$ ,  $-10 \leq t \leq 10$  where  $x$  and  $y$  are distances in metres.

- a** Find the cartesian equation of the arch. **(3 marks)**
- b** Find the width of the arch. **(2 marks)**
- c** Find the greatest possible height of the arch. **(2 marks)**

Qu 8... Edexcel Paper 1, June 2018 - Qu7. ([Link to markscheme](#))

Given that  $k \in \mathbb{Z}^+$

(a) show that  $\int_k^{3k} \frac{2}{(3x-k)} dx$  is independent of  $k$ , (4)

(b) show that  $\int_k^{2k} \frac{2}{(2x-k)^2} dx$  is inversely proportional to  $k$ . (3)

Qu 9... AQA Paper 3, June 2018 – Qu 6. ([Link to markscheme](#))

A function  $f$  is defined by  $f(x) = \frac{x}{\sqrt{2x-2}}$

State the maximum possible domain of  $f$ .

**[2 marks]**

Qu 10... AQA Paper 3, June 2018 – Qu 8. ([Link to markscheme](#))

Prove the identity  $\frac{\sin 2x}{1 + \tan^2 x} \equiv 2 \sin x \cos^3 x$

**[3 marks]**

Hence find  $\int \frac{4 \sin 4\theta}{1 + \tan^2 2\theta} d\theta$

**[6 marks]**

Qu 11... OCR A, Paper 2, June 2018. ([Link to markscheme](#))

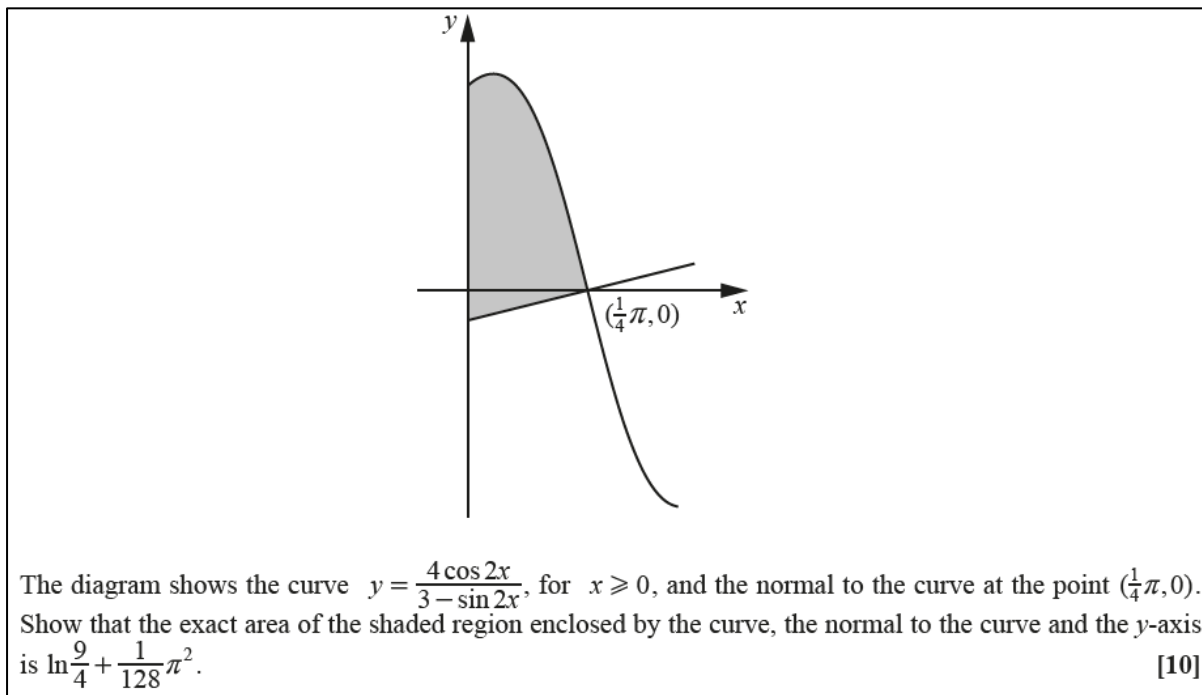
The variable  $Y$  has the distribution  $N(\mu, \frac{\mu^2}{9})$ . Find  $P(Y > 1.5\mu)$ . **[3]**

Qu 12... OCR A, Paper 2, June 2018. ([Link to markscheme](#))

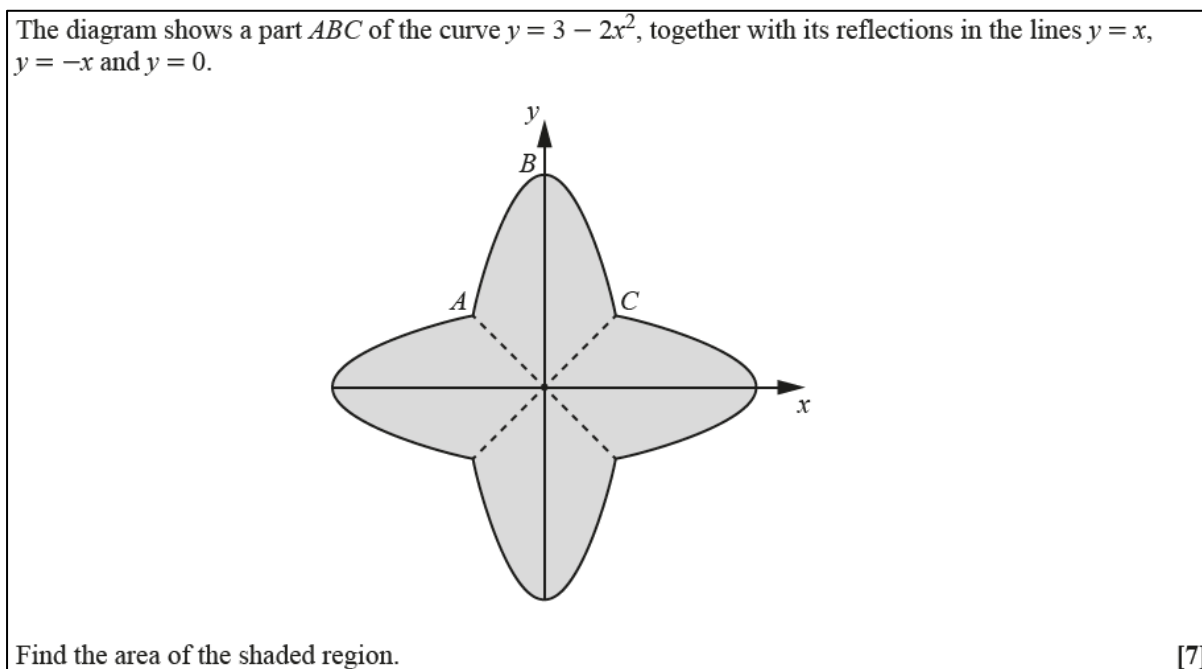
In the expansion of  $(0.15 + 0.85)^{50}$ , the terms involving  $0.15^r$  and  $0.15^{r+1}$  are denoted by  $T_r$  and  $T_{r+1}$  respectively.

Show that  $\frac{T_r}{T_{r+1}} = \frac{17(r+1)}{3(50-r)}$ . **[3]**

Qu 13... OCR, Paper 1, June 2018. ([Link to markscheme](#))



Qu 14... OCR, Paper 2, June 2018. ([Link to markscheme](#))



Qu 15... OCR Practice Papers, Set 2, Paper 3 - Qu3. ([Link to markscheme](#))

A sequence of three transformations maps the curve  $y = \ln x$  to the curve  $y = e^{3x} - 5$ . Give details of these transformations. [4]

What about  $y = e^{3x-5}$  ?

Qu 16... OCR Practice Papers, Set 4, Paper 1. ([Link to markscheme](#))

Solve the simultaneous equations

$$e^x - 2e^y = 3$$

$$e^{2x} - 4e^{2y} = 33.$$

Give your answer in an exact form. [5]

Qu 17... AQA Core 3, June 2013. ([Link to markscheme](#))

Find  $\int (\ln x)^2 dx$ . (4 marks)

Use the substitution  $u = \sqrt{x}$  to find the exact value of

$$\int_1^4 \frac{1}{x + \sqrt{x}} dx \quad (7 \text{ marks})$$

Qu 18... MEI, Paper 1, June 2018 – Qu 10. ([Link to markscheme](#))

Fig. 10 shows the graph of  $y = (k-x)\ln x$  where  $k$  is a constant ( $k > 1$ ).

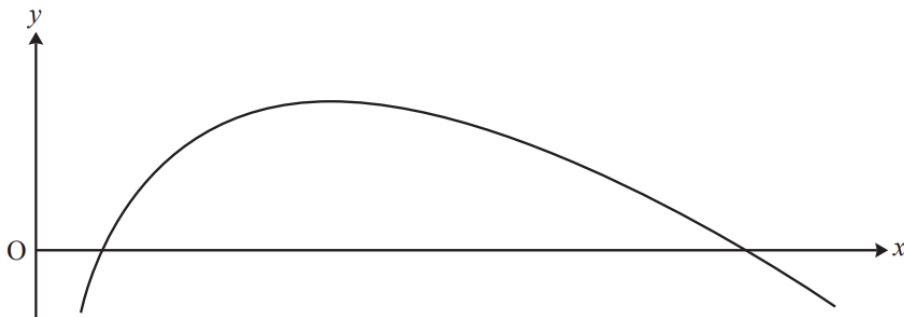


Fig. 10

Find, in terms of  $k$ , the area of the finite region between the curve and the  $x$ -axis.

[8]

Qu 19... MEI, Paper 1, June 2018 -Qu 11. ([Link to markscheme](#))

Fig. 11 shows two blocks at rest, connected by a light inextensible string which passes over a smooth pulley. Block A of mass 4.7 kg rests on a smooth plane inclined at  $60^\circ$  to the horizontal. Block B of mass 4 kg rests on a rough plane inclined at  $25^\circ$  to the horizontal. On either side of the pulley, the string is parallel to a line of greatest slope of the plane. Block B is on the point of sliding up the plane.

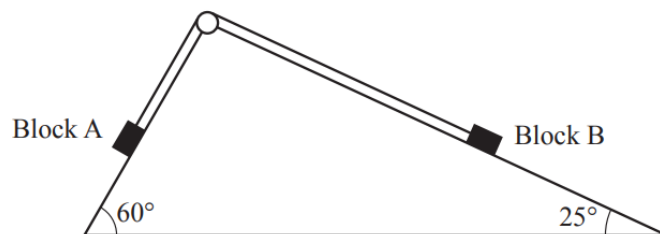


Fig. 11

- (i) Show that the tension in the string is 39.9 N correct to 3 significant figures. [2]
- (ii) Find the coefficient of friction between the rough plane and Block B. [5]

Qu 20... MEI, Paper 3, June 2018 – Qu 10. ([Link to markscheme](#))

Point A has position vector  $\begin{pmatrix} a \\ b \end{pmatrix}$  where  $a$  and  $b$  can vary, point B has position vector  $\begin{pmatrix} 4 \\ 2 \\ 0 \end{pmatrix}$  and point C has position vector  $\begin{pmatrix} 2 \\ 4 \\ 2 \end{pmatrix}$ . ABC is an isosceles triangle with  $AC = AB$ .

- (i) Show that  $a - b + 1 = 0$ . [4]
- (ii) Determine the position vector of A such that triangle ABC has minimum area. [6]

Qu 21... Edexcel Mock Papers, Paper 1 – Qu 11. ([Link to markscheme](#))

Given that

$$x = 2 \tan y \quad -\frac{\pi}{2} < y < \frac{\pi}{2}$$

show that

$$\frac{dy}{dx} = \frac{k}{4 + x^2}$$

where  $k$  is a constant to be found.

(4)

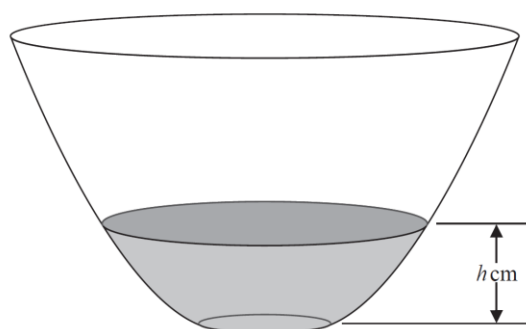


Figure 4

Figure 4 shows a bowl with a circular cross-section.

Initially the bowl is empty. Water begins to flow into the bowl.

At time  $t$  seconds after the water begins to flow into the bowl, the height of the water in the bowl is  $h$  cm.

The volume of water,  $V$  cm<sup>3</sup>, in the bowl is modelled as

$$V = 4\pi h(h + 6) \quad 0 \leq h \leq 25$$

The water flows into the bowl at a constant rate of  $80\pi$  cm<sup>3</sup> s<sup>-1</sup>

(a) Show that, according to the model, it takes 36 seconds to fill the bowl with water from empty to a height of 24 cm. (1)

(b) Find, according to the model, the rate of change of the height of the water, in cm s<sup>-1</sup>, when  $t = 8$  (8)

Given that  $a > b > 0$  and that  $a$  and  $b$  satisfy the equation

$$\log a - \log b = \log(a - b)$$

(a) show that

$$a = \frac{b^2}{b - 1} \quad (3)$$

(b) Write down the full restriction on the value of  $b$ , explaining the reason for this restriction. (2)



Qu 24... Edexcel Mock Papers, Paper 1 – Qu 13. ([Link to markscheme](#))

Given that  $p$  is a positive constant,

(a) show that

$$\sum_{n=1}^{11} \ln(p^n) = k \ln p$$

where  $k$  is a constant to be found,

(2)

(b) show that

$$\sum_{n=1}^{11} \ln(8p^n) = 33 \ln(2p^2)$$

(2)

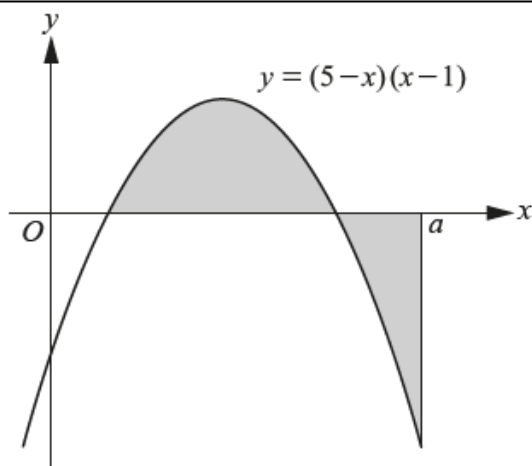
(c) Hence find the set of values of  $p$  for which

$$\sum_{n=1}^{11} \ln(8p^n) < 0$$

giving your answer in set notation.

(2)

Qu 25... OCR AS Paper 2, 2019. ([Link to markscheme](#))



The diagram shows part of the curve  $y = (5-x)(x-1)$  and the line  $x = a$ .

Given that the total area of the regions shaded in the diagram is 19 units<sup>2</sup>, determine the exact value of  $a$ .

[8]

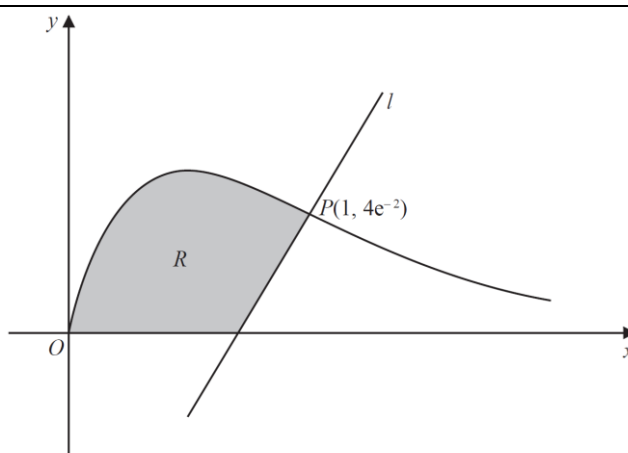


Figure 7

Figure 7 shows a sketch of the curve with equation

$$y = 4xe^{-2x} \quad x \geq 0$$

The line  $l$  is the normal to the curve at the point  $P(1, 4e^{-2})$

The finite region  $R$ , shown shaded in Figure 7, is bounded by the curve, the line  $l$ , and the  $x$ -axis.

Find the exact value of the area of  $R$ .

*(Solutions based entirely on graphical or numerical methods are not acceptable.)*

**(10)**

(i) Show that  $\sum_{r=1}^{16} (3 + 5r + 2^r) = 131\,798$

**(4)**

(ii) A sequence  $u_1, u_2, u_3, \dots$  is defined by

$$u_{n+1} = \frac{1}{u_n}, \quad u_1 = \frac{2}{3}$$

Find the exact value of  $\sum_{r=1}^{100} u_r$

**(3)**

Qu 28... AQA, A2 Paper 2, 2019. ([Link to markscheme](#))

Solve the differential equation

$$\frac{dt}{dx} = \frac{\ln x}{x^2 t} \quad \text{for } x > 0$$

given  $x = 1$  when  $t = 2$

Write your answer in the form  $t^2 = f(x)$

[7 marks]

Qu 29... Edexcel, A2 Paper 1, June 2019 – Qu 5. ([Link to markscheme](#))

$$f(x) = 2x^2 + 4x + 9 \quad x \in \mathbb{R}$$

- (i) Describe fully the transformation that maps the curve with equation  $y = f(x)$  onto the curve with equation  $y = g(x)$  where

$$g(x) = 2(x - 2)^2 + 4x - 3 \quad x \in \mathbb{R}$$

- (ii) Find the range of the function

$$h(x) = \frac{21}{2x^2 + 4x + 9} \quad x \in \mathbb{R}$$

(4)

Qu 30... Edexcel unit tests, Integration – Qu 8. ([Link to markscheme](#))

Use the substitution  $x = 4\sin^2\theta$  to find

$$\int_0^3 \sqrt{\left(\frac{x}{4-x}\right)} dx,$$

giving your answer in the form  $a\pi + b$ , where  $a$  and  $b$  are exact constants.

(9)

The binomial expansion of

$$\frac{1}{\sqrt{4-x}}$$

Can be used to find an approximation to  $\sqrt{2}$ .

Possible values of  $x$  that could be substituted into this expansion are:

- $x = -14$  because  $\frac{1}{\sqrt{4-x}} = \frac{1}{\sqrt{18}} = \frac{\sqrt{2}}{6}$
- $x = 2$  because  $\frac{1}{\sqrt{4-x}} = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$
- $x = -\frac{1}{2}$  because  $\frac{1}{\sqrt{4-x}} = \frac{1}{\sqrt{\frac{9}{2}}} = \frac{\sqrt{2}}{3}$

Without evaluating your expansion,

- (i) state, giving a reason, which of the three values of  $x$  should not be used (1)
- (ii) state, giving a reason, which of the three values of  $x$  would lead to the most accurate approximation to  $\sqrt{2}$  (1)

The curve  $C$ , in the standard Cartesian plane, is defined by the equation

$$x = 4 \sin 2y \quad -\frac{\pi}{4} < y < \frac{\pi}{4}$$

The curve  $C$  passes through the origin  $O$

- (a) Find the value of  $\frac{dy}{dx}$  at the origin. (2)
- (b) Show that, for all points  $(x, y)$  lying on  $C$ ,

$$\frac{dy}{dx} = \frac{1}{a\sqrt{b-x^2}}$$

where  $a$  and  $b$  are constants to be found.

(3)

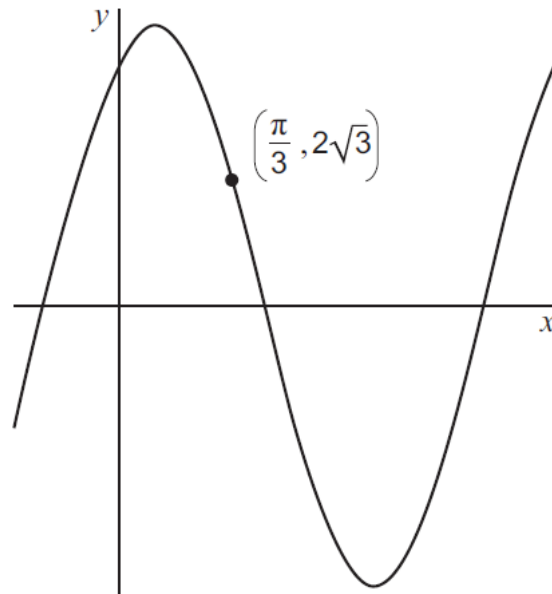
Qu 33... AQA, A2 Paper 2, 2019. ([Link to markscheme](#))

A curve has equation

$$y = a \sin x + b \cos x$$

where  $a$  and  $b$  are constants.

The maximum value of  $y$  is 4 and the curve passes through the point  $(\frac{\pi}{3}, 2\sqrt{3})$  as shown in the diagram.



Find the exact values of  $a$  and  $b$ .

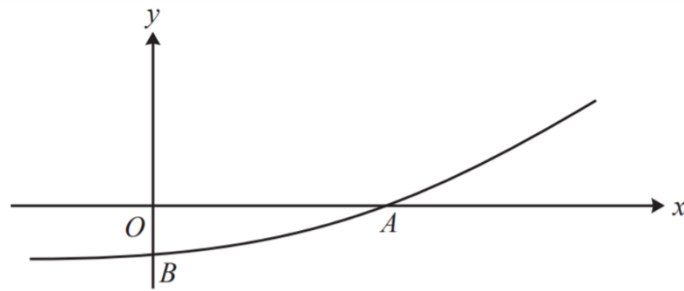
[6 marks]

Qu 34... Edexcel Unit Tests, A2 Stats, Topic 2, Hypothesis Testing. ([Link to markscheme](#))

A random sample of size  $n$  is to be taken from a population that is normally distributed with mean 40 and standard deviation 3. Find the minimum sample size such that the probability of the sample mean being greater than 42 is less than 5%.

(Total 5 marks)

Qu 35... OCR A Core 3 June 2013, Differentiation. ([Link to markscheme](#))



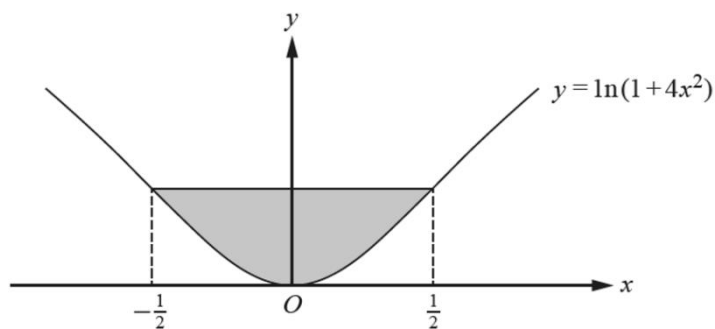
The diagram shows the curve with equation

$$x = (y + 4) \ln(2y + 3).$$

Find the gradient of the curve at each of the points  $A$  and  $B$ , giving each answer correct to 2 decimal places.

[8]

Qu 36... OCR A Practice Papers Set 1, Paper 3, Question 6. ([Link to markscheme](#))



The diagram shows the curve  $y = \ln(1 + 4x^2)$ . The shaded region is bounded by the curve and a line parallel to the  $x$ -axis which meets the curve where  $x = \frac{1}{2}$  and  $x = -\frac{1}{2}$ .

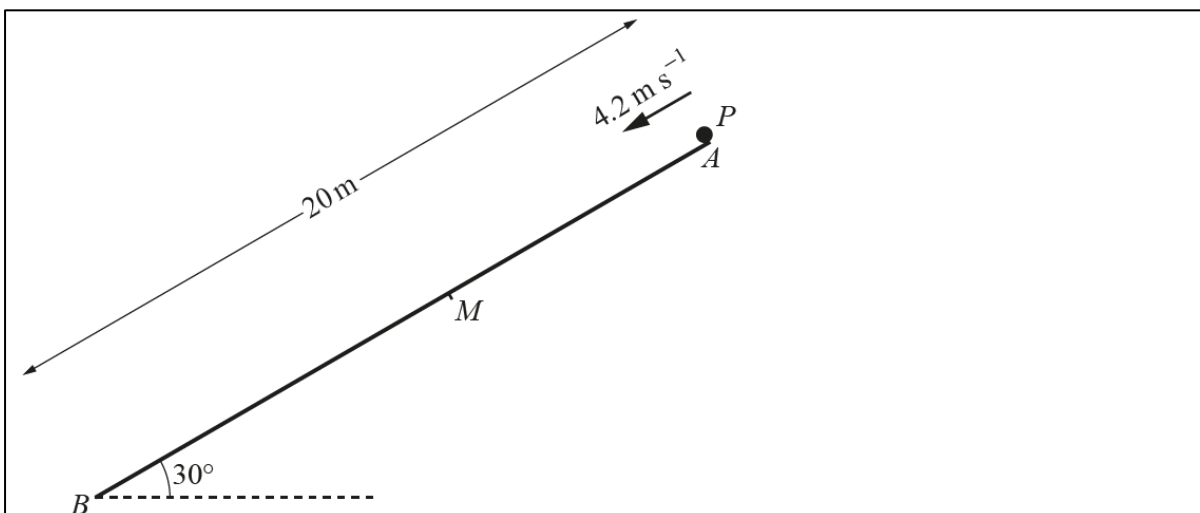
(i) Show that the area of the shaded region is given by

$$\int_0^{\ln 2} \sqrt{e^y - 1} \, dy. \quad [3]$$

(ii) Show that the substitution  $e^y = \sec^2 \theta$  transforms the integral in part (i) to  $\int_0^{\frac{1}{4}\pi} 2 \tan^2 \theta \, d\theta$ . [2]

(iii) Hence find the exact area of the shaded region. [3]

Qu 37... OCR A Practice Papers Set 4, Paper 3, Question 10. ([Link to markscheme](#))



*A* and *B* are points at the upper and lower ends, respectively, of a line of greatest slope on a plane inclined at  $30^\circ$  to the horizontal. The distance  $AB$  is 20 m. *M* is a point on the plane between *A* and *B*. The surface of the plane is smooth between *A* and *M*, and rough between *M* and *B*.

A particle *P* is projected with speed  $4.2 \text{ m s}^{-1}$  from *A* down the line of greatest slope (see diagram). *P* moves down the plane and reaches *B* with speed  $12.6 \text{ m s}^{-1}$ . The coefficient of friction between *P* and the rough part of the plane is  $\frac{\sqrt{3}}{6}$ .

(a) Find the distance  $AM$ .

[8]

Qu 38... OCR A Practice Papers Set 2, Paper 2, Question 6. ([Link to markscheme](#))

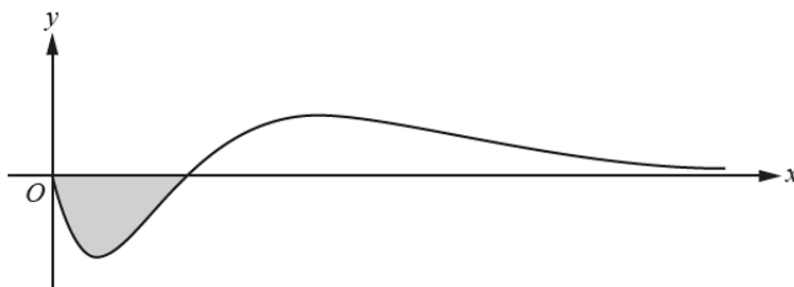
**In this question you must show detailed reasoning.**

(i) Use the formula for  $\tan(A - B)$  to show that  $\tan \frac{\pi}{12} = 2 - \sqrt{3}$ . [4]

(ii) Solve the equation  $2\sqrt{3} \sin 3A - 2 \cos 3A = 1$  for  $0^\circ \leq A < 180^\circ$ . [7]

Qu 39... OCR A Practice Papers Set 2, Paper 3, Question 5. ([Link to markscheme](#))

In this question you must show detailed reasoning.



The function  $f$  is defined for the domain  $x \geq 0$  by

$$f(x) = (2x^2 - 3x)e^{-x}.$$

The diagram shows the curve  $y = f(x)$ .

- (i) Find the range of  $f$ . [6]
- (ii) Find the exact area of the shaded region enclosed by the curve and the  $x$ -axis. [7]

Qu 40... OCR A Sample Assessment Paper, Maths & Statistics, Question 12. ([Link to markscheme](#))

The table shows information for England and Wales, taken from the UK 2011 census.

Total population	Number of children aged 5-17
56 075 912	8 473 617

A random sample of 10 000 people in another country was chosen in 2011, and the number,  $m$ , of children aged 5-17 was noted.

It was found that there was evidence at the 2.5% level that the proportion of children aged 5-17 in the same year was higher than in the UK.

Unfortunately, when the results were recorded the value of  $m$  was omitted.

Use an appropriate normal distribution to find an estimate of the smallest possible value of  $m$ . [5]

Qu 41... OCR, A2 Paper 2, 2018, Question 5 ([Link to markscheme](#))

Given that 853 is a prime number, find the square number  $S$  such that  $S + 853$  is also a square number.



Qu 43... OCR Practice Papers, Set 1, Paper 1, Question 12. ([Link to markscheme](#))

**In this question you must show detailed reasoning.**

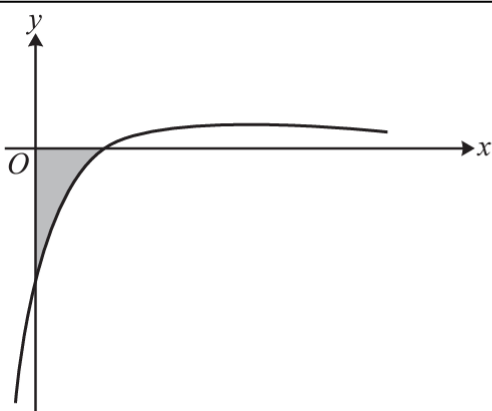
A curve has equation

$$x \sin y + \cos 2y = \frac{5}{2}$$

for  $x \geq 0$  and  $0 \leq y < 2\pi$ .

Determine the exact coordinates of each point on the curve at which the tangent to the curve is parallel to the  $y$ -axis. [9]

Qu 43... OCR, A2 Paper 3, 2019, Question 6 ([Link to markscheme](#))



The diagram shows part of the curve  $y = \frac{2x-1}{(2x+3)(x+1)^2}$ .

Find the exact area of the shaded region, giving your answer in the form  $p + q \ln r$ , where  $p$  and  $q$  are positive integers and  $r$  is a positive rational number. [10]

Qu 44... OCR Practice papers Set 2, Paper 2, Question 7 ([Link to markscheme](#))

A tank is shaped as a cuboid. The base has dimensions 10 cm by 10 cm. Initially the tank is empty. Water flows into the tank at  $25 \text{ cm}^3$  per second. Water also leaks out of the tank at  $4h^2 \text{ cm}^3$  per second, where  $h$  cm is the depth of the water after  $t$  seconds. Find the time taken for the water to reach a depth of 2 cm. [9]

Qu 45... OCR A2 Paper 1, 2019, Question 2 ([link to markscheme](#))

The point  $A$  is such that the magnitude of  $\vec{OA}$  is 8 and the direction of  $\vec{OA}$  is  $240^\circ$ .

(a) (i) Show the point  $A$  on the axes provided in the Printed Answer Booklet. [1]

(ii) Find the position vector of point  $A$ .  
Give your answer in terms of  $\mathbf{i}$  and  $\mathbf{j}$ . [3]

The point  $B$  has position vector  $6\mathbf{i}$ .

(b) Find the exact area of triangle  $AOB$ . [2]

The point  $C$  is such that  $OABC$  is a parallelogram.

(c) Find the position vector of  $C$ .  
Give your answer in terms of  $\mathbf{i}$  and  $\mathbf{j}$ . [2]

Qu 46... OCR A2 Paper 2, 2019, Question 9 ([link to markscheme](#))

(a) The masses, in grams, of plums of a certain kind have the distribution  $N(55, 18)$ .

(i) Find the probability that a plum chosen at random has a mass between 50.0 and 60.0 grams. [1]

(ii) The heaviest 5% of plums are classified as extra large.

Find the minimum mass of extra large plums. [1]

(iii) The plums are packed in bags, each containing 10 randomly selected plums.

Find the probability that a bag chosen at random has a total mass of less than 530 g. [4]

(b) The masses, in grams, of apples of a certain kind have the distribution  $N(67, \sigma^2)$ . It is given that half of the apples have masses between 62 g and 72 g.

Determine  $\sigma$ . [5]

Qu 47... AQA Level 2 Certificate in Further Maths, Paper 2, 2017, Question 24 ([Link to markscheme](#))

Write  $12x^2 - 60x + 5$  in the form  $a(bx + c)^2 + d$  where  $a, b, c$  and  $d$  are integers.

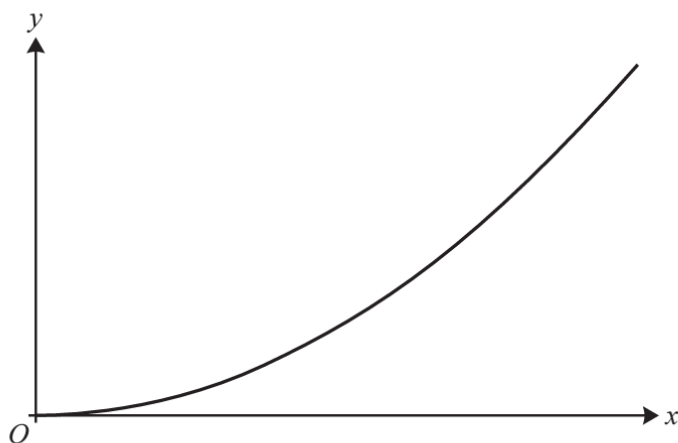
[5 marks]

In this question you should assume that  $-1 < x < 1$ .

- (a) For the binomial expansion of  $(1-x)^{-2}$
- (i) find and simplify the first four terms, [2]
  - (ii) write down the term in  $x^n$ . [1]
- (b) Write down the sum to infinity of the series  $1+x+x^2+x^3+\dots$ . [1]
- (c) Hence or otherwise find and simplify an expression for  $2+3x+4x^2+5x^3+\dots$  in the form  $\frac{a-x}{(b-x)^2}$  where  $a$  and  $b$  are constants to be determined. [3]

- (a) Use the substitution  $u^2 = x^2 + 3$  to show that  $\int \frac{4x^3}{\sqrt{x^2+3}} dx = \frac{4}{3}(x^2-6)\sqrt{x^2+3} + c$ . [5]

- (b) In this question you must show detailed reasoning.



The graph shows part of the curve  $y = \frac{4x^3}{\sqrt{x^2+3}}$ .

Find the exact area enclosed by the curve  $y = \frac{4x^3}{\sqrt{x^2+3}}$ , the normal to this curve at the point  $(1, 2)$  and the  $x$ -axis. [7]

Qu 50... Edexcel Specimen Paper 3, Question 3 ([Link to markscheme](#))

For a particular type of bulb, 36% grow into plants with blue flowers and the remainder grow into plants with white flowers. Bulbs are sold in mixed bags of 40.

Russell selects a random sample of 5 bags of bulbs.

- (a) Find the probability that fewer than 2 of these bags will contain more bulbs that grow into plants with blue flowers than grow into plants with white flowers (4)

Maggie takes a random sample of  $n$  bulbs.

Using a normal approximation, the probability that more than 244 of these  $n$  bulbs will grow into blue flowers is 0.0521 to 4 decimal places.

- (b) Find the value of  $n$ . (6)

Qu 51... OCR A2 Paper 1, 2021, Question 7 ([Link to markscheme](#))

The curve  $y = (x^2 - 2)\ln x$  has one stationary point which is close to  $x = 1$ .

- (a) Show that the  $x$ -coordinate of this stationary point satisfies the equation  $2x^2 \ln x + x^2 - 2 = 0$ . [2]

- (b) Show that the Newton-Raphson iterative formula for finding the root of the equation in part (a) can be written in the form  $x_{n+1} = \frac{2x_n^2 \ln x_n + 3x_n^2 + 2}{4x_n (\ln x_n + 1)}$ . [4]

Qu 52... OCR A2 Paper 1, 2019, Question 7 ([Link to markscheme](#))

**In this question you must show detailed reasoning.**

A sequence  $u_1, u_2, u_3 \dots$  is defined by  $u_n = 25 \times 0.6^n$ .

Use an algebraic method to find the smallest value of  $N$  such that  $\sum_{n=1}^{\infty} u_n - \sum_{n=1}^N u_n < 10^{-4}$ . [8]

Qu 53... OCR A2 Paper 2, 2021, Question 5 ([Link to markscheme](#))

**In this question you must show detailed reasoning.**

Points  $A$ ,  $B$  and  $C$  have coordinates  $(0, 6)$ ,  $(7, 5)$  and  $(6, -2)$  respectively.

- (a) Find an equation of the perpendicular bisector of  $AB$ . [3]
- (b) Hence, or otherwise, find an equation of the circle that passes through points  $A$ ,  $B$  and  $C$ . [5]

Qu 54... OCR A2 Paper 1, 2019, Question 12 ([Link to markscheme](#))

A curve has equation  $y = a^{3x^2}$ , where  $a$  is a constant greater than 1.

- (a) Show that  $\frac{dy}{dx} = 6xa^{3x^2} \ln a$ . [3]
- (b) The tangent at the point  $(1, a^3)$  passes through the point  $(\frac{1}{2}, 0)$ .  
Find the value of  $a$ , giving your answer in an exact form. [4]
- (c) By considering  $\frac{d^2y}{dx^2}$  show that the curve is convex for all values of  $x$ . [5]

Qu 55... OCR A2 Paper 2, 2020, Question 15 ([Link to markscheme](#))

**In this question you must show detailed reasoning.**

The random variable  $X$  has probability distribution defined as follows.

$$P(X = x) = \begin{cases} \frac{15}{64} \times \frac{2^x}{x!} & x = 2, 3, 4, 5, \\ 0 & \text{otherwise.} \end{cases}$$

- (a) Show that  $P(X = 2) = \frac{15}{32}$ . [1]

The values of three independent observations of  $X$  are denoted by  $X_1$ ,  $X_2$  and  $X_3$ .

- (b) Given that  $X_1 + X_2 + X_3 = 9$ , determine the probability that at least one of these three values is equal to 2. [6]

Freda chooses values of  $X$  at random until she has obtained  $X = 2$  exactly three times. She then stops.

- (c) Determine the probability that she chooses exactly 10 values of  $X$ . [3]

$A$  and  $B$  are fixed points in the  $x$ - $y$  plane. The position vectors of  $A$  and  $B$  are  $\mathbf{a}$  and  $\mathbf{b}$  respectively.

State, with reference to points  $A$  and  $B$ , the geometrical significance of

(a) the quantity  $|\mathbf{a} - \mathbf{b}|$ , [1]

(b) the vector  $\frac{1}{2}(\mathbf{a} + \mathbf{b})$ . [1]

The circle  $P$  is the set of points with position vector  $\mathbf{p}$  in the  $x$ - $y$  plane which satisfy

$$\left| \mathbf{p} - \frac{1}{2}(\mathbf{a} + \mathbf{b}) \right| = \frac{1}{2}|\mathbf{a} - \mathbf{b}|.$$

(c) State, in terms of  $\mathbf{a}$  and  $\mathbf{b}$ ,

(i) the position vector of the centre of  $P$ , [1]

(ii) the radius of  $P$ . [1]

It is now given that  $\mathbf{a} = \begin{pmatrix} 2 \\ -1 \end{pmatrix}$ ,  $\mathbf{b} = \begin{pmatrix} 4 \\ 5 \end{pmatrix}$  and  $\mathbf{p} = \begin{pmatrix} x \\ y \end{pmatrix}$ .

(d) Find a cartesian equation of  $P$ . [4]

The circle  $C_1$  has Cartesian equation

$$x^2 + y^2 = 10x + k \quad x \in \mathbb{R} \quad y \in \mathbb{R}$$

where  $k$  is a constant.

The curve  $C_2$  has parametric equations

$$x = t^2 \quad y = 2t \quad t \in \mathbb{R}$$

The curves  $C_1$  and  $C_2$  intersect at 4 distinct points.

Find the range of possible values for  $k$ , giving your answer in set notation.

(6)

The day length,  $Y$  hours, is defined as the difference between the time the sun rises and the time the sun sets on a particular day. For Manchester, England, the following model is proposed for years which are not leap years.

$$Y = a \sin\left(\frac{2\pi}{365}t + b\right) + c,$$

where  $t$  is the time in days since the start of the year and  $a$ ,  $b$  and  $c$  are constants.

The maximum value of  $Y$ , which is 17.03, occurs on June 21st, when  $t = 172$ . The minimum value of  $Y$ , which is 7.47, occurs on December 21st, when  $t = 355$ .

(a) Show that  $a = 4.78$  and  $c = 12.25$ . [2]

(b) Determine the value of  $b$  correct to 3 significant figures. [2]

On September 1st, when  $t = 244$ , the day length is recorded as 13.76 hours.

(c) Show that the model is a good fit for this value. [2]

In Reykjavik, Iceland, on June 21st the maximum day length was 21.14 hours and on December 21st the minimum day length was 4.12 hours.

(d) Use this information to refine the model for Manchester to produce a possible model for the day length in Reykjavik. [1]

On September 1st the day length in Reykjavik is recorded as 14.56 hours.

(e) Determine whether your possible model for Reykjavik is a good fit for this value. [1]

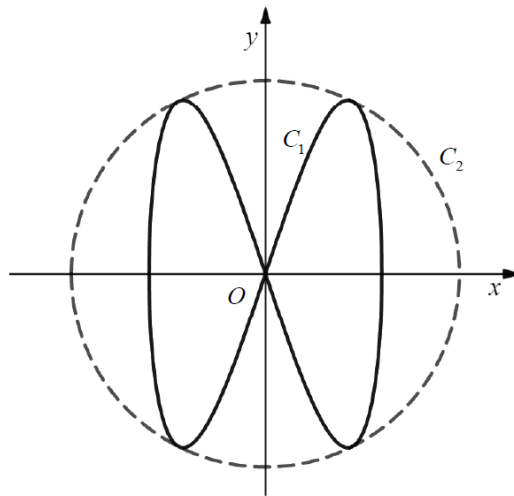
The table shows information about three geometric series. The three geometric series have different common ratios.

	First term	Common ratio	Number of terms	Last term
Series 1	1	2	$n_1$	1024
Series 2	1	$r_2$	$n_2$	1024
Series 3	1	$r_3$	$n_3$	1024

(a) Find  $n_1$ . [2]

(b) Given that  $r_2$  is an integer less than 10, find the value of  $r_2$  and the value of  $n_2$ . [2]

(c) Given that  $r_3$  is **not** an integer, find a possible value for the sum of all the terms in Series 3. [4]



**Figure 5**

Figure 5 shows a sketch of the curve  $C_1$  with parametric equations

$$x = 2 \sin t, \quad y = 3 \sin 2t \quad 0 \leq t < 2\pi$$

(a) Show that the Cartesian equation of  $C_1$  can be expressed in the form

$$y^2 = kx^2(4 - x^2)$$

where  $k$  is a constant to be found.

**(4)**

The circle  $C_2$  with centre  $O$  touches  $C_1$  at four points as shown in Figure 5.

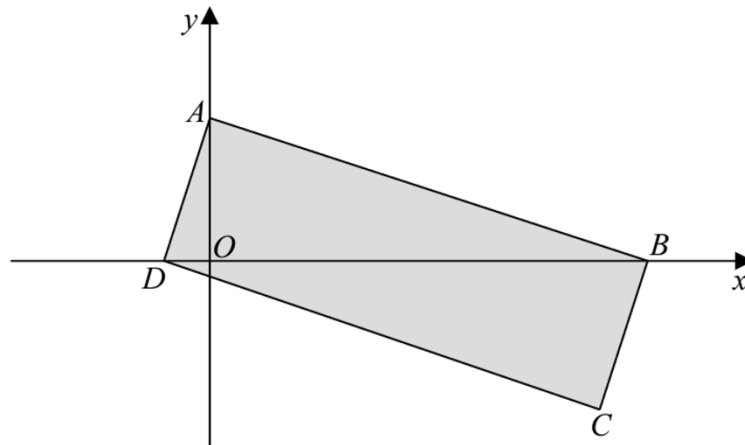
(b) Find the radius of this circle.

**(5)**

Given that  $x = 3 \tan 2y$  find  $\frac{dy}{dx}$  in terms of  $x$  without involving any trigonometrical functions.

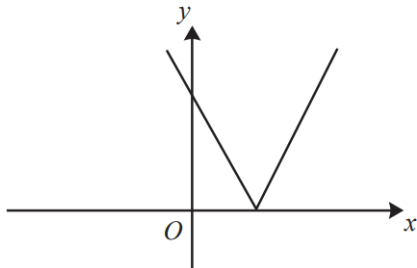


Qu 62... adapted from Edexcel Sample Paper 2 June 2012, Question 8 ([Link to markscheme](#))



Given that the straight line through the points  $A$  and  $B$  has equation  $5y + 2x = 10$   
find the area of the rectangle  $ABCD$ . (7)

Qu 63... OCR A2 Paper 1 June 2020 - Question 9 ([Link to markscheme](#))

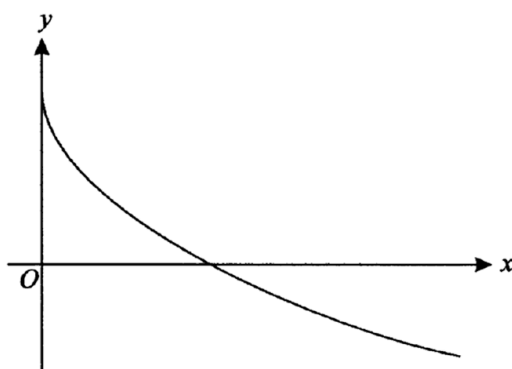


The diagram shows the graph of  $y = |2x - 3|$ .

Given that the graphs of  $y = |2x - 3|$  and  $y = ax + 2$  have two distinct points of intersection, determine

- (a) the set of possible values of  $a$ , [4]
- (b) the  $x$ -coordinates of the points of intersection of these graphs, giving your answers in terms of  $a$ . [3]

Qu 64... OCR A2 Paper 1 June 2020 - Question 9 ([Link to markscheme](#))



The function  $f$  is defined by  $f(x) = 2 - \sqrt{x}$  for  $x \geq 0$ . The graph of  $y = f(x)$  is shown above.

- (i) State the range of  $f$ . [1]
- (ii) Find the value of  $ff(4)$ . [2]
- (iii) Given that the equation  $|f(x)| = k$  has two distinct roots, determine the possible values of the constant  $k$ . [2]

Qu 65... Edexcel A2 Paper 3 Statistics June 2021 - Question 6 ([Link to markscheme](#))

The discrete random variable  $X$  has the following probability distribution

$x$	$a$	$b$	$c$
$P(X = x)$	$\log_{36} a$	$\log_{36} b$	$\log_{36} c$

where

- $a, b$  and  $c$  are distinct integers ( $a < b < c$ )
- all the probabilities are greater than zero

(a) Find

- (i) the value of  $a$
- (ii) the value of  $b$
- (iii) the value of  $c$

Show your working clearly.

(5)

Qu 66... OCR AS Paper 2 June 2023 - Question 8 ([Link to markscheme](#))

**In this question you must show detailed reasoning.**

Given that  $\int_4^a \left( \frac{4}{\sqrt{x}} + 3 \right) dx = 7$ , find the value of  $a$ .

[7]

Qu 67... OCR A2 Paper 1 June 2022 - Question 12 ([Link to markscheme](#))

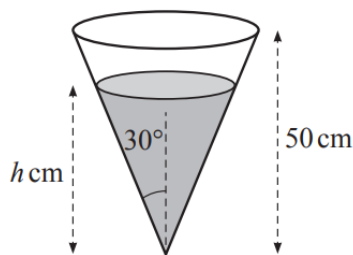
A curve has parametric equations  $x = \frac{1}{t}$ ,  $y = 2t$ . The point  $P$  is  $\left( \frac{1}{p}, 2p \right)$ .

The tangent to this curve at  $P$  crosses the  $x$ -axis at the point  $A$  and the normal to this curve at  $P$  crosses the  $x$ -axis at the point  $B$ .

Show that the ratio  $PA : PB$  is  $1 : 2p^2$ .

[12]

Qu 68... OCR A2 Paper 2 June 2022 - Question 8 ([Link to markscheme](#))



The diagram shows a water tank which is shaped as an inverted cone with semi-vertical angle  $30^\circ$  and height 50 cm. Initially the tank is full, and the depth of the water is 50 cm.

Water flows out of a small hole at the bottom of the tank. The rate at which the water flows out is modelled by  $\frac{dV}{dt} = -2h$ , where  $V \text{ cm}^3$  is the volume of water remaining and  $h \text{ cm}$  is the depth of water in the tank  $t$  seconds after the water begins to flow out.

Determine the time taken for the tank to become empty.

[For a cone with base radius  $r$  and height  $h$  the volume  $V$  is given by  $\frac{1}{3}\pi r^2 h$ .]

[7]

Qu 69... MEI A2 Paper 2 June 2023 - Question 17 ([Link to markscheme](#))

**In this question you must show detailed reasoning.**

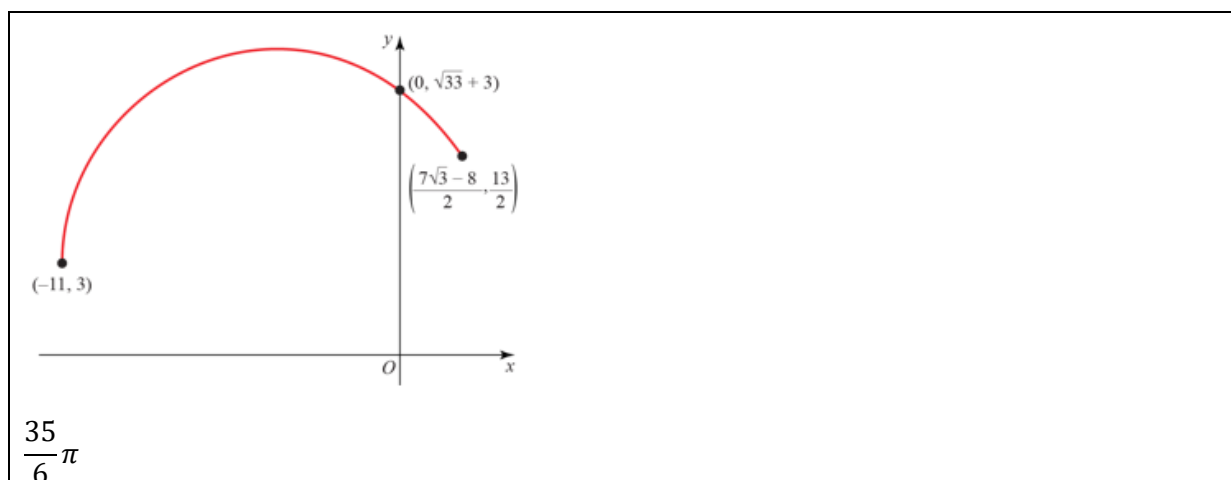
Solve the equation  $2 \sin x + \sec x = 4 \cos x$ , where  $-\pi < x < \pi$ .

[6]



## Interesting Questions - Answers

Qu 1... Edexcel unit tests, Parametric Equations - Qu 3. ([Link back to question](#))



Qu 2... AQA A2 Paper 1, June 2018 -Qu 5. ([Link back to question](#))

$$xy + 5x - 3y = 27$$

Qu 3... AQA A2 Paper 1, June 2018 – Qu 12. ([Link back to question](#))

$$(2x + 1)(5x - 2)(3x - 1)$$

Qu 4... AQA A2 Paper 1, June 2018 – Qu13. ([Link back to question](#))

32 sq in

Qu 5... AQA A2 Paper 2, June 2018 -Qu 8. ([Link back to question](#))

Translation  $\begin{pmatrix} \frac{\pi}{3} \\ 0 \end{pmatrix}$

Stretch in the y-direction scale  
factor  $2\sqrt{3}$

Translation  $\begin{pmatrix} 0 \\ 4 \end{pmatrix}$

$$\text{Greatest value} = \frac{2 + \sqrt{3}}{2}$$

Qu 6... AQA A2 Paper 2, June 2018 - Qu 7. ([Link back to question](#))

$$f(x) = (x-2)e^x + 2e$$

Qu 7... Edexcel unit tests, parametric Equations -Qu 6. ([Link back to question](#))

$$y = -\frac{1}{64}x^2 + \frac{5}{2}x$$

Width = 160m

Height = 100m

Qu 8... Edexcel Paper 1, June 2018 - Qu7. ([Link back to question](#))

-

Qu 9... AQA Paper 3, June 2018 – Qu 6. ([Link back to question](#))

$$\{x \in \mathbb{R} : x > 1\}$$

Qu 10... AQA Paper 3, June 2018 – Qu 8. ([Link back to question](#))

$$= -\cos^4 2\theta + c$$

Qu 11... OCR A, Paper 2, June 2018. ([Link back to question](#))

0.0668 or 0.067

Qu 12... OCR A, Paper 2, June 2018. ([Link back to question](#))

-

Qu 13... OCR, Paper 1, June 2018. ([Link back to question](#))

-

Qu 14... OCR, Paper 2, June 2018. ([Link back to question](#))

$$\frac{44}{3}$$

Qu 15... OCR Practice Papers, Set 2, Paper 3 - Qu3. ([Link back to question](#))

(reflection) in the line  $y = x$

(stretch) scale factor  $\frac{1}{3}$  parallel to the  $x$ -axis

(translation)  $\begin{pmatrix} 0 \\ -5 \end{pmatrix}$

Qu 16... OCR Practice Papers, Set 4, Paper 1. ([Link back to question](#))

$(\ln 7, \ln 2)$

Qu 17... AQA Core 3, June 2013. ([Link back to question](#))

$$= 2 \ln \frac{3}{2} \text{ or } \ln \frac{9}{4} \text{ or } 2 \ln 3 - 2 \ln 2$$

Qu 18... MEI, Paper 1, June 2018 – Qu 10. ([Link back to question](#))

$$= \frac{1}{2} k^2 \ln k - \frac{3}{4} k^2 + k - \frac{1}{4}$$

Qu 19... MEI, Paper 1, June 2018 -Qu 11. ([Link back to question](#))

$$\mu = 0.656$$

Qu 20... MEI, Paper 3, June 2018 – Qu 10. ([Link back to question](#))

Position vector  $\begin{pmatrix} 2.5 \\ 3.5 \\ 0 \end{pmatrix}$

Qu 21... Edexcel Mock Papers, Paper 1 – Qu 11. ([Link back to question](#))

$$\frac{dy}{dx} = \frac{2}{4 + x^2}$$

Qu22... Edexcel Mock Papers, Paper 1 - Qu 10. ([Link back to question](#))

$$\frac{10}{13} \text{ (cm s}^{-1}\text{) or awrt } 0.769 \text{ (cm s}^{-1}\text{)}$$

Qu 23... Edexcel, A2 Paper 1, June 2019 – Qu 9. ([Link back to question](#))

$$\text{as } a > 0 \Rightarrow \frac{b^2}{b-1} > 0 \Rightarrow b > 1$$

Qu 24... Edexcel Mock Papers, Paper 1 – Qu 13. ([Link back to question](#))

66lnp

In set notation, e.g.  $\left\{ p : 0 < p < \frac{1}{\sqrt{2}} \right\}$

Qu 25... OCR AS Paper 2, 2019 – Qu7. ([Link back to question](#))

$$a = \frac{9 + \sqrt{21}}{2} \text{ only}$$

Qu 26... Edexcel Mock Papers, Paper 1 -Qu 14. ([Link back to question](#))

$$\text{Area}(R) = 1 - 3e^{-2} - 32e^{-6} \text{ or } \frac{e^6 - 3e^4 - 32}{e^6}$$

Qu 27... Edexcel A2 Paper 2, 2018 – Qu 4. ([Link back to question](#))

$$\frac{325}{3} \left( \text{or } 108\frac{1}{3} \text{ or } 108.\dot{3} \text{ or } \frac{1300}{12} \text{ or } \frac{650}{6} \right)$$

Qu 28... AQA, A2 Paper 2, 2019. ([Link back to question](#))

$$t^2 = 6 - 2 \left( \frac{1 + \ln x}{x} \right)$$

Qu 29... Edexcel, A2 Paper 1, June 2019 – Qu 5. ([Link back to question](#))

Translate  $\begin{pmatrix} 2 \\ -4 \end{pmatrix}$

$$0 < h(x) \leq 3$$



Qu 30... Edexcel unit tests, Integration – Qu 8. ([Link back to question](#))

$$= \frac{4}{3}\pi - \sqrt{3}$$

Qu 31... Edexcel, A2 Paper 1, June 2019 – Qu 4. ([Link back to question](#))

- |             |   |
|-------------|---|
| <b>(i)</b>  | States $x = -14$ and gives a valid reason.<br>Eg explains that the expansion is not valid for $ x  > 4$ |
| <b>(ii)</b> | States $x = -\frac{1}{2}$ and gives a valid reason.<br>Eg. explains that it is closest to zero          |

Qu 32... AQA, A2 Paper 2, 2019. ([Link back to question](#))

$$\text{At } (0,0) \frac{dy}{dx} = \frac{1}{8}$$
$$\frac{dy}{dx} = \frac{1}{2\sqrt{16-x^2}}$$

Qu 33... Edexcel, A2 Paper 1, June 2019 – Qu 14. ([Link back to question](#))

$$a = 2, b = 2\sqrt{3}$$

Qu 34... Edexcel Unit Tests, A2 Stats, Topic 2, Hypothesis Testing. ([Link to question](#))

$$n = 7$$

Qu 35... OCR A Core 3 June 2013, Differentiation. ([Link to question](#))

$$0.27 \text{ for gradient at } A, \quad 0.17 \text{ or } \frac{1}{6} \text{ for gradient at } B$$

Qu 36... OCR A Practice Papers Set 1, Paper 3, Question 6. ([Link to question](#))

$$= 2\left(1 - \frac{1}{4}\pi\right)$$

Qu 37... OCR A Practice Papers Set 4, Paper 3, Question 10. ([Link to question](#))

the distance  $AM$  is 8.8 m

Qu 38... OCR A Practice Papers Set 2, Paper 2, Question 6. ([Link to question](#))

- a) ("Show that")  
b)  $A = 14.8, 65.2, 134.8$

Qu 39... OCR A Practice Papers Set 2, Paper 2, Question 6. ([Link to question](#))

$$-e^{-\frac{1}{2}} \leq y \leq 9e^{-3} \quad \text{Integral is } 1 - 7e^{-\frac{3}{2}} < 0 \text{ so area is } 7e^{-\frac{3}{2}} - 1$$

Qu 40... OCR A Sample Assessment Paper, Maths & Statistics, Question 12. ([Link to question](#))

Minimum  $m$  is 1581

Qu 41... OCR, A2 Paper 2, 2018, Question 5. ([Link to question](#))

Start with  $n^2 - 853 = m^2$  then use difference of two squares and 853 is prime to obtain

$$n + m = 853 \quad \text{and} \quad n - m = 1$$

Qu 42... OCR Practice Papers, Set 1, Paper 1, Question 12. ([Link to question](#))

$$\left(\frac{7}{2}, \frac{\pi}{2}\right), \left(2\sqrt{3}, \frac{\pi}{3}\right), \left(2\sqrt{3}, \frac{2\pi}{3}\right) \text{ but be sure to discount } \sin y = -\frac{\sqrt{3}}{2} \text{ as } x < 0 \text{ isn't allowed}$$

Qu 43... OCR 2019, Paper 3, Question 6. ([Link to question](#))

$$\text{Area} = 1 + 8\ln\left(\frac{8}{9}\right)$$

Qu44... OCR Practice papers Set 2, Paper 2, Question 7 ([Link to question](#))

Time when depth is 2 cm, and vol is 200cm<sup>3</sup>, is 11.0 seconds (10.9861)

Qu 45... OCR A2 Paper 1, 2019, Question 12 ([Link to question](#))

- a i) Length 8, angle 240 measured anti-clockwise from +ve x axis.  
a ii)  $-4i - 4\sqrt{3}j$   
b)  $12\sqrt{3}$   
c)  $10i + 4\sqrt{3}j$

Qu 46... OCR A2 Paper 2, 2019, Question 9 ([Link to question](#))

- a i) 0.761, 0.762 or 0.76  
a ii) 62(.0)  
a iii) 0.068(0)  
b)  $\sigma = 7.41$  or  $\sigma = 7.42$

Ans 47... AQA Level 2 Certificate in Further Maths, Paper 2, 2017, Question 24 ([Link to question](#))

$$3(2x - 5)^2 - 70$$

i.e.  $a = 3$ ,  $b = 2$ ,  $c = -5$ ,  $d = 70$

Ans 48... OCR A2 Paper 2, 2020, Question 3 ([Link to question](#))

$$\equiv 1 + 2x + 3x^2 + 4x^3 \quad (n+1)x^n \quad \frac{1}{1-x} \text{ oe} \quad = \frac{2-x}{(1-x)^2}$$

Ans 49... OCR A2 Paper 1, 2021, Question 11 ([Link to question](#))

Area under curve =  $\frac{4}{3}(6\sqrt{3} - 10) = 0.523$   
Area under triangle = 11  
Total area =  $8\sqrt{3} - \frac{7}{3} = 11.523$

Ans 50... Edexcel Paper 3, 2022, Question 3 ([Link to question](#))

- a) So  $P(C \leq 1) = 0.9945\dots$  awrt 0.995  
b)  $0.36n + 0.78\sqrt{n} - 244.5 = 0$   
 $n = 625$

Ans 51... OCR A2 Paper 1, 2021, Question 7 ([Link to question](#))

(Both parts of the question are show that)

Ans 52... OCR A2 Paper 1, 2019, Question 7 ([Link to question](#))

$$N > 25.125 \text{ i.e. } N = 26$$

Ans 53... OCR A2 Paper 2, 2021, Question 5 ([Link to question](#))

(a)  $y = 7x - 19$

(b)  $(x - 3)^2 + (y - 2)^2 = 25$

Ans 54... OCR A2 Paper 1, 2019, Question 12 ([Link to question](#))

b)  $a = e^{\frac{1}{3}}$

c)  $\frac{d^2y}{dx^2} = 2e^{x^2} + 4x^2e^{x^2}$  or  $\frac{d^2y}{dx^2} = a^{3x^2}(6\ln a)(1 + 6x^2\ln a)$ , then show why this is always positive

Ans 55... OCR A2 Paper 2, 2020, Question 15 ([Link to question](#))

a) "Show that"

b) 0.854 or  $\frac{117}{137}$

c) 0.0433

Ans 56... OCR A2 Paper 2, 2020, Question 7 ([Link to question](#))

a) Length of AB

b) Midpoint of AB

c) i)  $\frac{1}{2}(\mathbf{a} + \mathbf{b})$ ,      ii)  $\frac{1}{2}|\mathbf{a} - \mathbf{b}|$

d)  $(x - 3)^2 + (y - 2)^2 = 10$

Ans 57... Edexcel Mock Set 4, Paper 2, Question 10 ([Link to question](#))

$$-9 < k < 0, \text{ or in set notation}$$

$$\{k: k < 0\} \cap \{k: k > -9\}$$

Ans 58... Edexcel Mock Set 4, Paper 2, Question 10 ([Link to question](#))

a)  $a = 4.78$  and  $c = 12.25$

- b)  $b = -1.39$
- c) Use  $t = 244$  and then  $Y = 13.81$ , which is fairly close to 13.75
- d)  $a = 8.51$  and  $c = 12.63$
- e) New model is not a good fit as it gives 15.40 hrs

Ans 59... Edexcel Mock Set 4, Paper 2, Question 10 ([Link to question](#))

- a)  $n_1 = 11$
- b)  $r_2 = 4$   
 $n_2 = 6$
- c)  $r_3 = \sqrt{2}$  (or any other power of  $\sqrt{2}$ ). Using  $\sqrt{2}$  gives  $n_3 = 21$   
 $S_{21} = 2047 + 1023\sqrt{2}$  (or 2490).  
Other versions of  $r_3$  based on  $\sqrt[n]{2}$  work here

Ans 60... Edexcel Mock Papers Set 2 (2020), Paper 2, Question 14 ([Link to question](#))

- a)  $y^2 = \frac{9}{4}x^2(4 - x^2)$
- b)  $r = \frac{10}{3}$

Ans 61... adapted from Edexcel Core 3 June 2012, Question 7b ([Link to question](#))

$$\frac{dy}{dx} = \frac{3}{18 + 2x^2}$$

Ans 62... adapted from Edexcel Sample Paper 2 June 2012, Question 8 ([Link to question](#))

$$\text{Area } ABCD = 11.6$$

Qu 63... OCR A2 Paper 1 June 2020 - Question 9 ([Link to Question](#))

- a)  $-\frac{4}{3} < a < 2$
- b)  $x = \frac{1}{2+a}$

Qu 64... A great question from a long time ago ([Link to Question](#))

- i.  $f(x) \leq 2$
- ii.  $ff(4) = 2$
- iii.  $0 < k \leq 2$

Qu 65... Edexcel A2 Paper 3 Statistics June 2021 - Question 6 ([Link to Question](#))

$$a = 2, b = 3, c = 6$$

Qu 66... OCR AS Paper 2 June 2023 - Question 8 ([Link to Question](#))

$a^{\frac{1}{2}} \neq -5$  as  $a^{\frac{1}{2}}$  can't be negative

$$a^{\frac{1}{2}} = \frac{7}{3} \Rightarrow a = \frac{49}{9}$$

Qu 67... OCR A2 Paper 2 June 2023 - Question 12 ([Link to Question](#))

$$\begin{aligned} PA : PB &= \frac{1}{p} \sqrt{4p^4 + 1} : 2p \sqrt{4p^4 + 1} \\ &= \frac{1}{p} : 2p \\ &= 1 : 2p^2 \quad \text{A.G.} \end{aligned}$$

Qu 68... OCR A2 Paper 2 June 2022 - Question 8 ([Link to Question](#))

$$\text{Time} = \frac{625\pi}{3} \text{ secs or } 654 \text{ secs (3 sf) oe}$$

Qu 69... MEI A2 Paper 2 June 2023 - Question 17 ([Link to Question](#))

$$x = -1.24905 \quad x = 1.8925 \quad x = \frac{\pi}{4} \quad x = -\frac{3\pi}{4}$$

All four correct and no extras!