

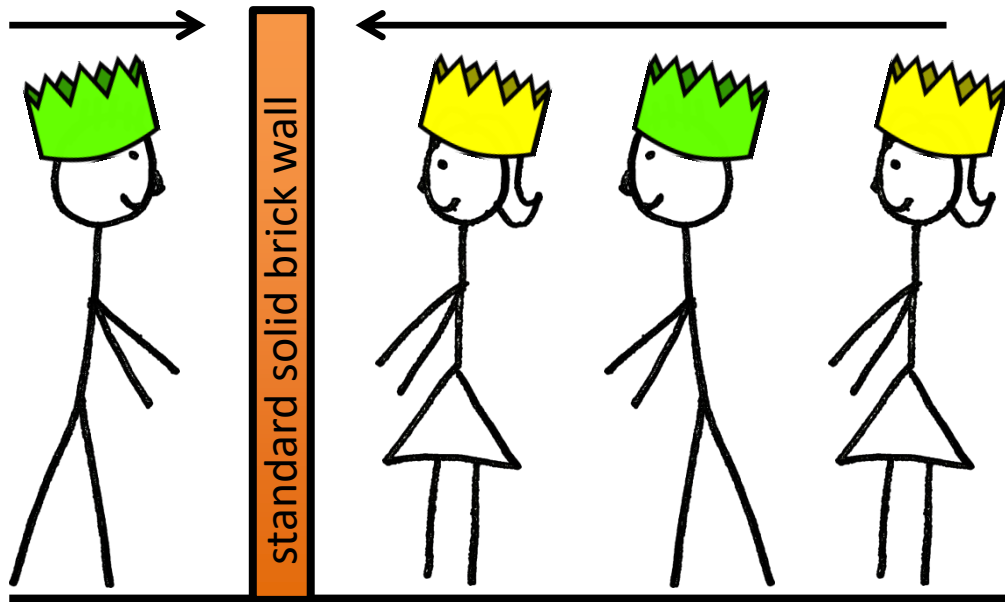
1

1089

What to do: Place this card face down somewhere away from you. Ask a student to tell you a non-palindromic three digit number. Write it down, then reverse it and write this down. Find the difference, then reverse this and add these two numbers together.

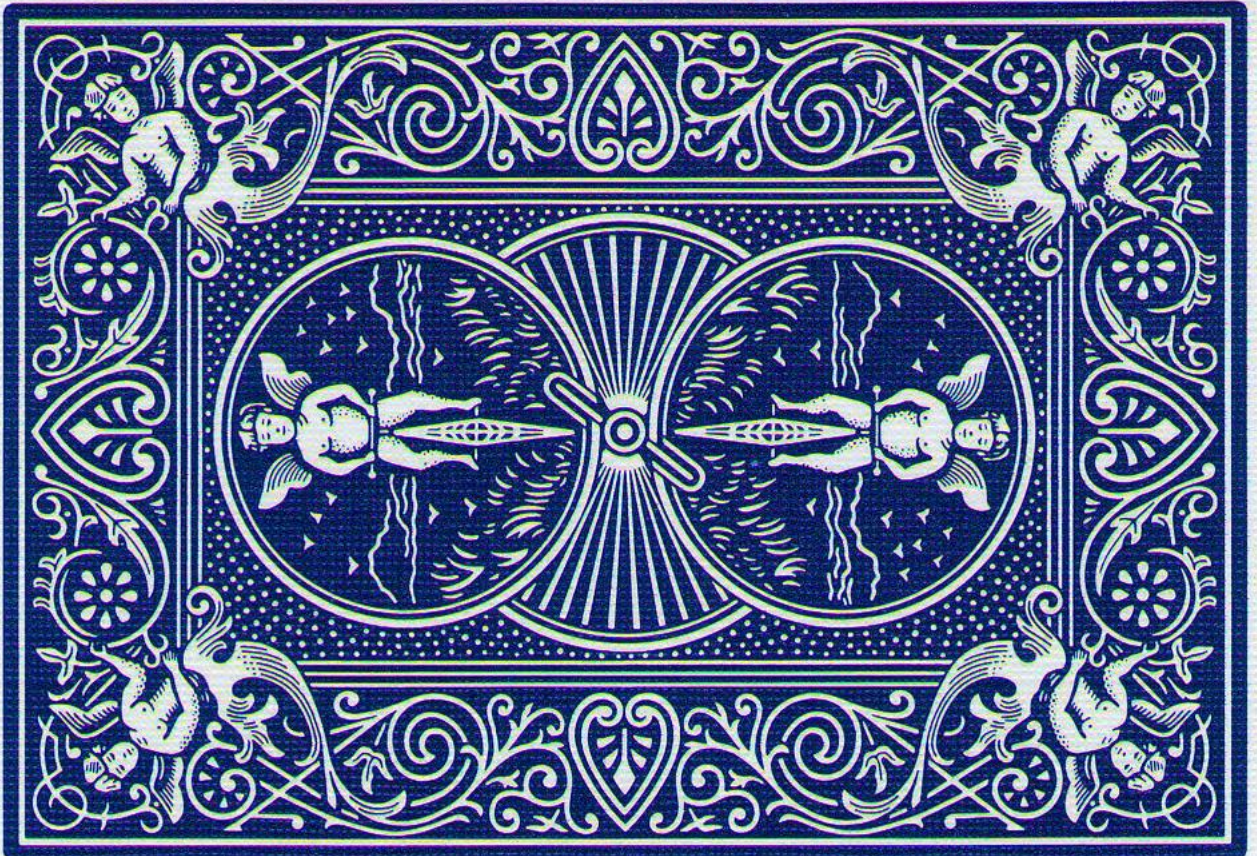
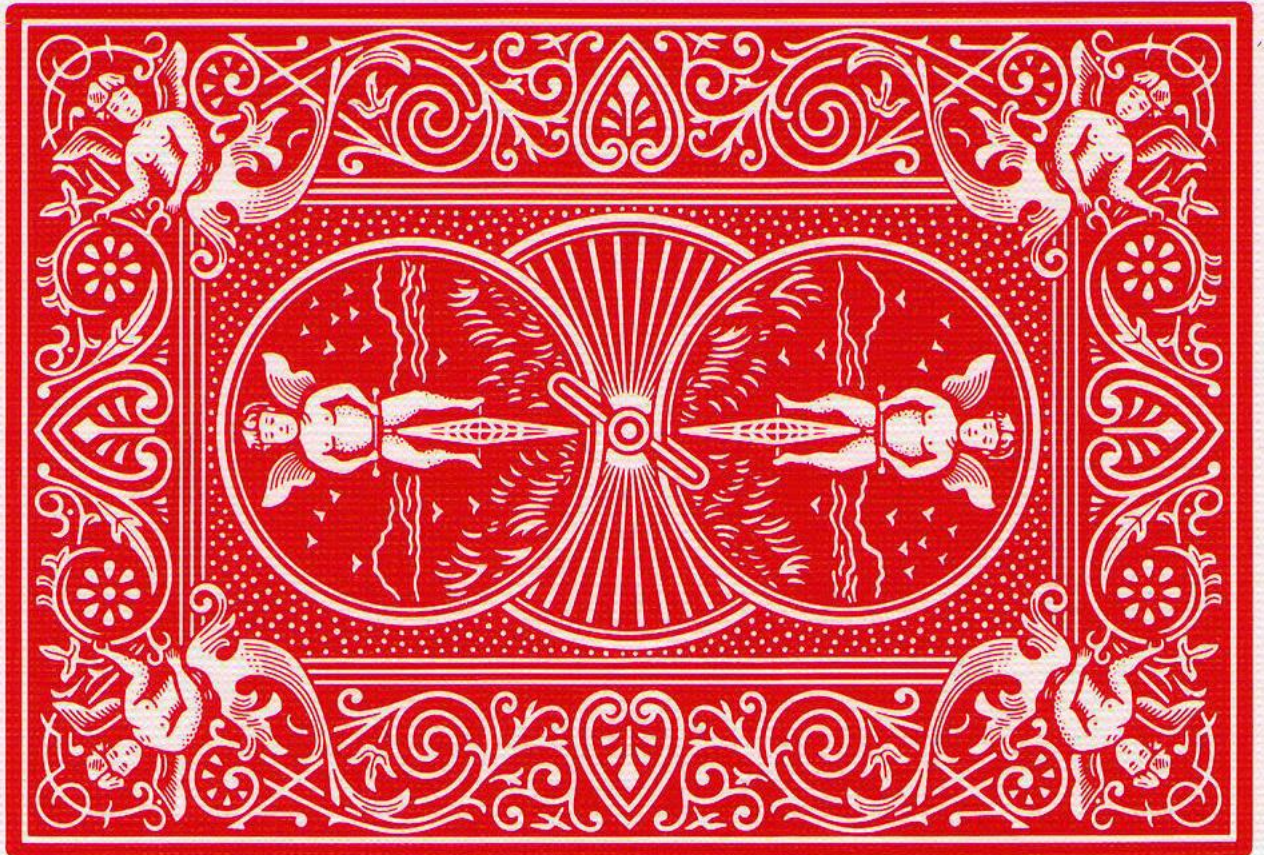
Now ask a student to turn this card over.

2



These four people know that there are two of each colour hat but they don't know what colour they are wearing. They can only see what's in front of them, they aren't allowed to talk to each other and (rather obviously) can't see through the wall. After a minute or two, one of them indicates that they know the colour of their hat and are completely sure of it.

How come?



3

Use all of

3

3

8

8

and any of to make

+	-
×	÷

24

$$\begin{array}{r} \varepsilon \\ 8 \\ \hline 8 \end{array}$$

4

What's the link?

1

$11^1 = 11$

1 1

$11^2 = 121$

1 2 1

$11^3 = 1331$

1 3 3 1

$11^4 = 14641$

1 4 6 4 1

1 5 10 10 5 1

$11^5 = ?$

1 6 15 20 15 6 1

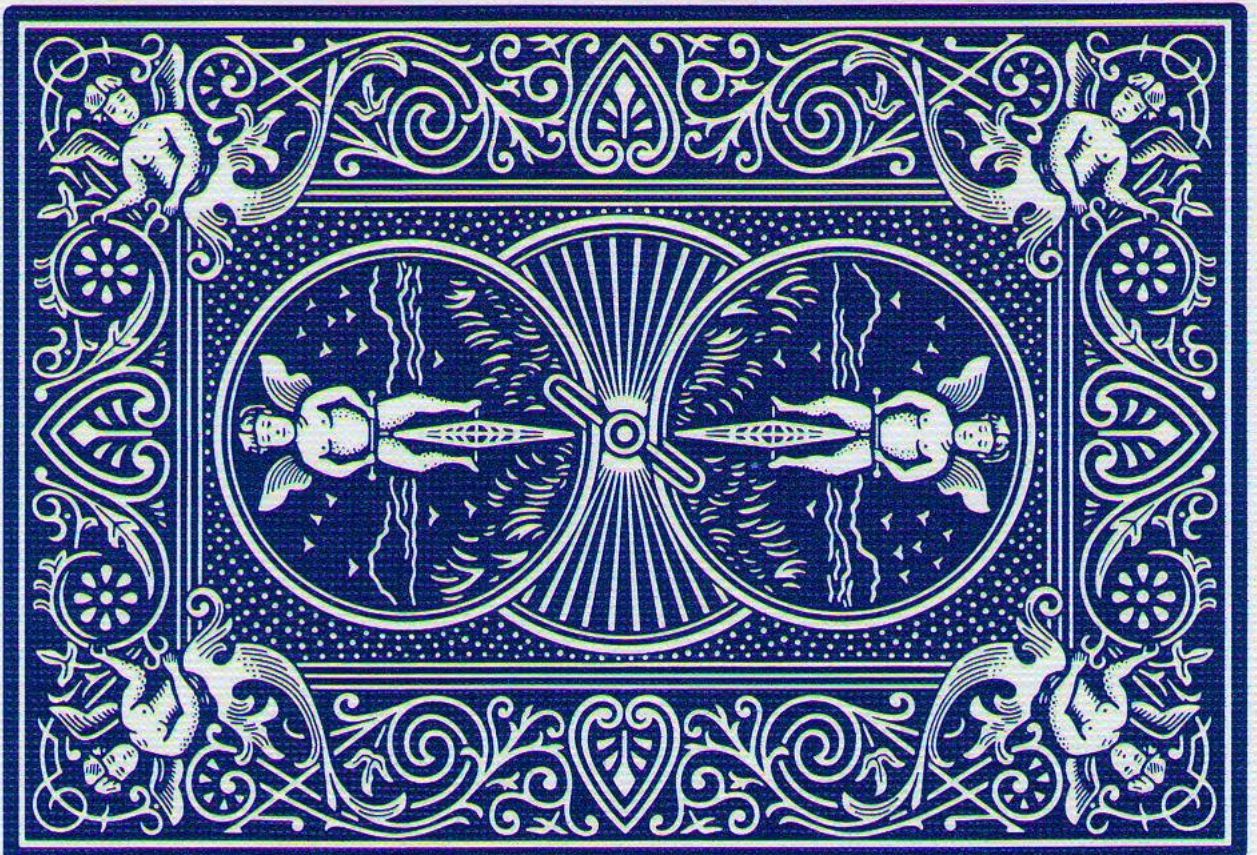
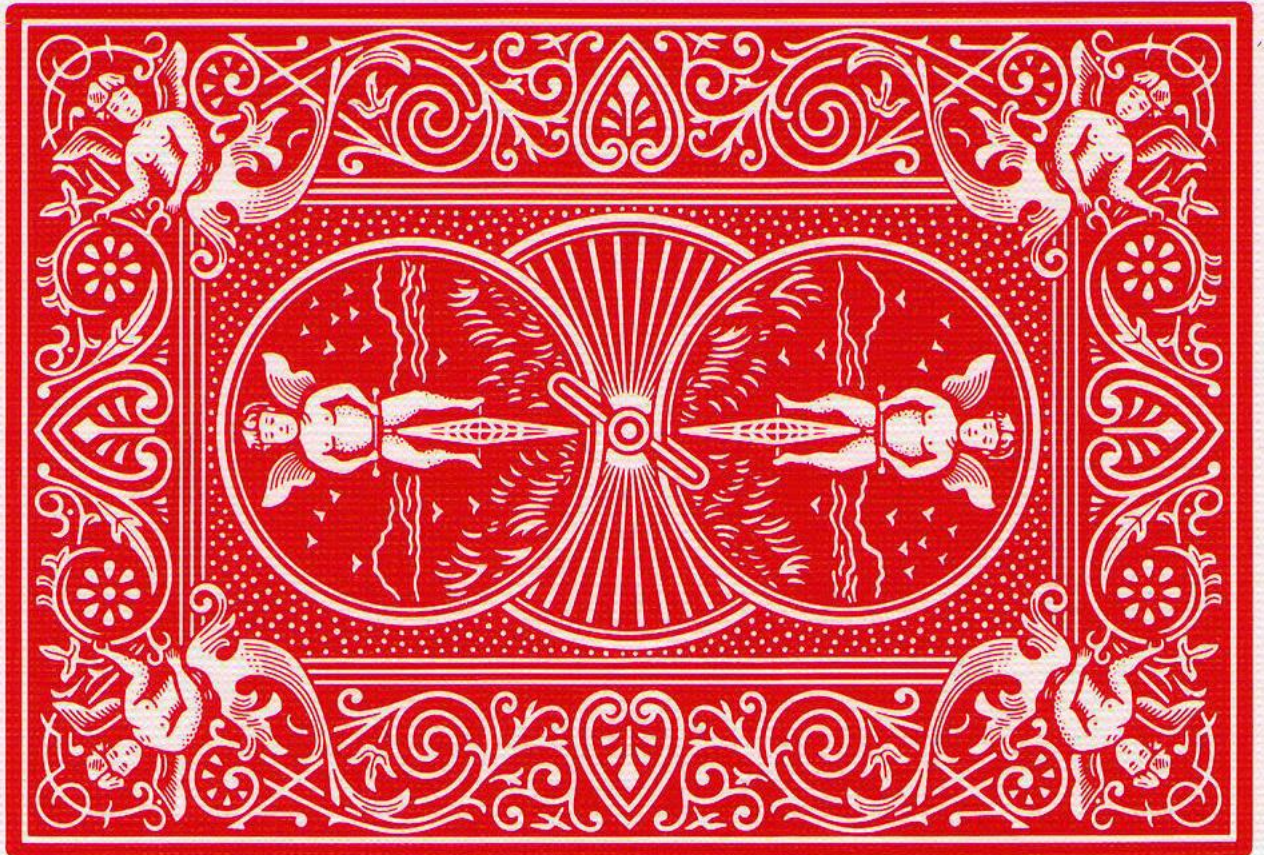
$11^6 = ?$

1 7 21 35 35 21 7 1

$11^7 = ?$

1 8 28 56 70 56 28 8 1

1 9 36 84 126 126 84 36 9 1



5

$$2 = 1$$

Let $a = 1$ and $b = 1$

$$a = b$$

multiply both sides by a

$$a^2 = ab$$

subtract b^2 from both sides

$$a^2 - b^2 = ab - b^2$$

factorise both sides

$$(a + b)(a - b) = b(a - b)$$

divide both sides by $(a - b)$

$$a + b = b$$

But $a = 1$ and $b = 1$ so

$$2 = 1$$

What's gone wrong?

6

1 2 5 10

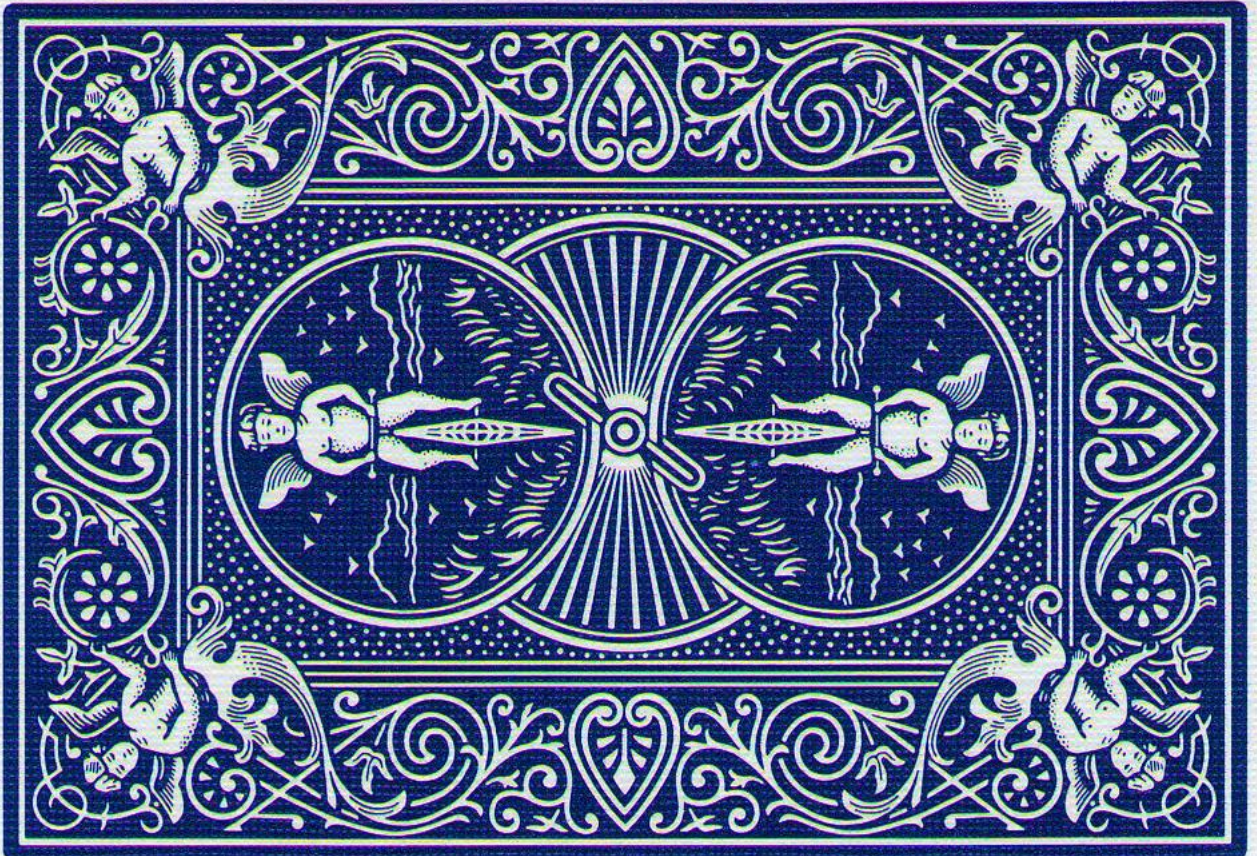
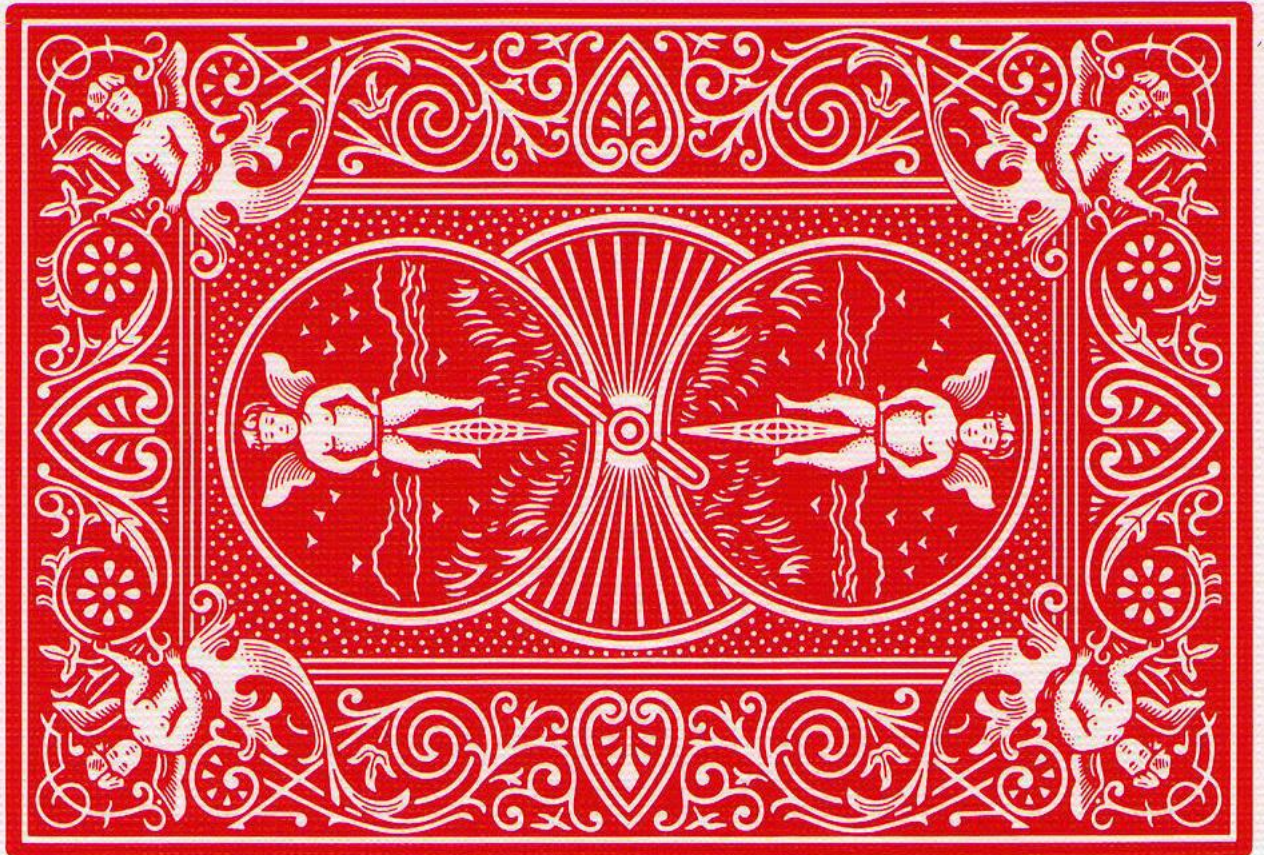
Four people, in various states of fitness, have been trekking all day and come upon a rope bridge that must be crossed to reach home.

The bridge only holds two people. It is dark and they have just one torch so their only option is to lead each other across, back and forth, until they are all across.

The fittest person claims they can cross the bridge in 1 minute, the next person in 2 minutes, the next in 5 minutes and the last, who is really unfit, will take 10 minutes.

As each pair crosses they go at the slower persons speed.

How long does it take them to cross the bridge?
(19 minutes is not good enough)



7

$$1^3 = 1$$

$$6^3 = 216$$

$$2^3 = 8$$

$$7^3 = 343$$

$$3^3 = 27$$

$$8^3 = 512$$

$$4^3 = 64$$

$$9^3 = 729$$

$$5^3 = 125$$

$$10^3 = 1000$$

What to do: Pass a calculator to a student. Ask them to cube any two-digit number and to tell you the answer but not the original number. You will be able to tell them their two-digit number almost instantly.

How its done: The digit unit of their answer gives the digit unit of their original number (note that 3 & 7 and 2 & 8 are interchanged) and by comparing the number of thousands in their answer against the cubes you'll know their tens digit.

8

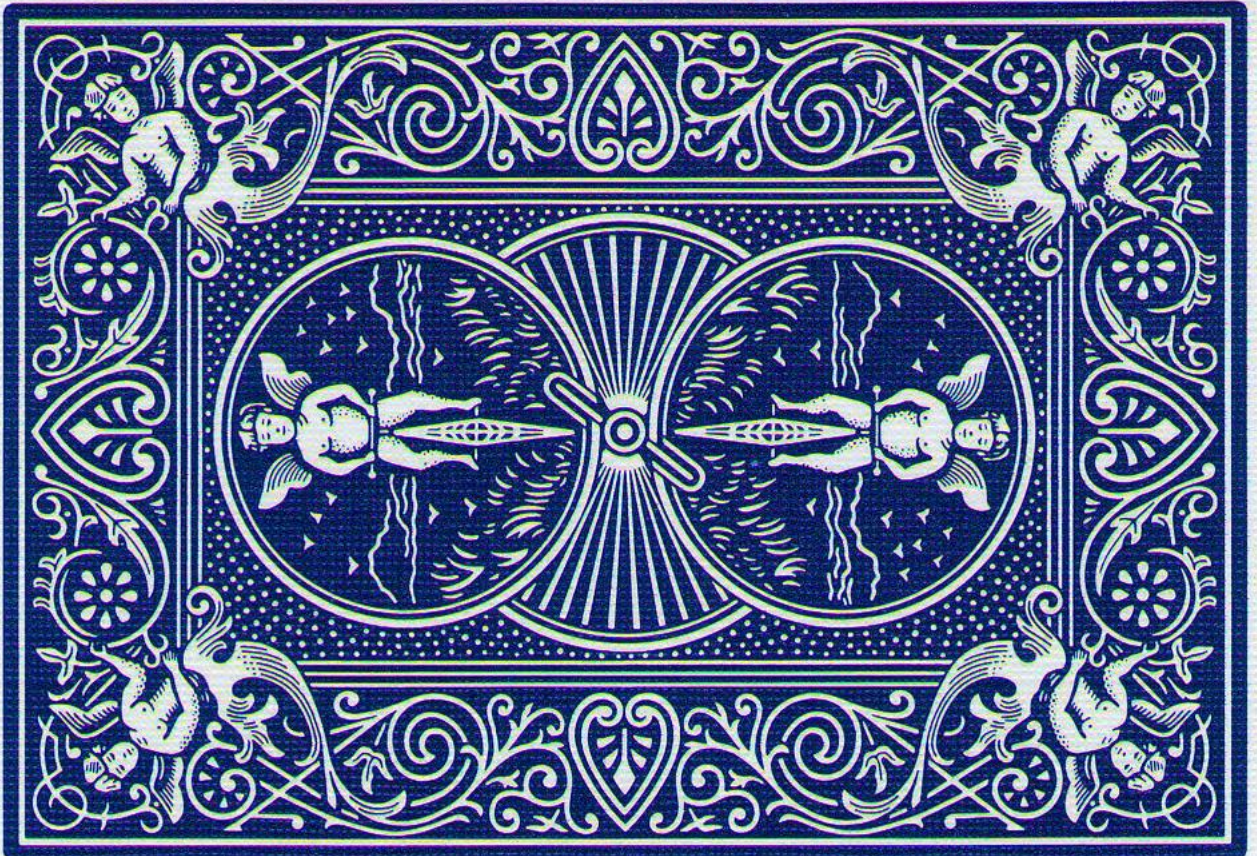
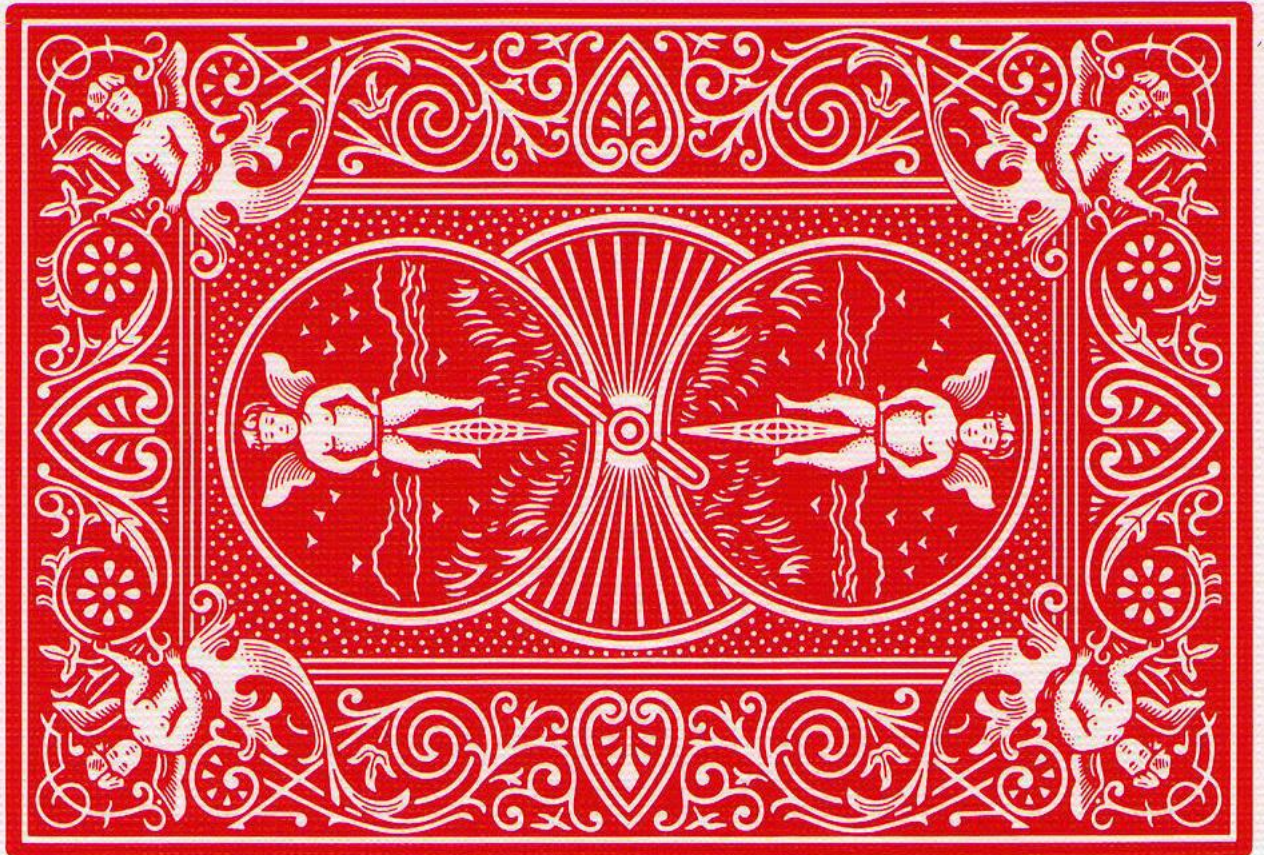


What to do: Establish that since barcodes are used to identify a product, every product has its own unique barcode which is a seemingly random and particular number.

Ask a student to find something with a barcode on it and check that this begins with a single digit separate from the first block of digits (as above). Instruct them to read it out but to leave out any one digit, replacing it with the word 'blank' so you know where this digit occurs. You will tell them what the missing digit is.

How its done: As the student reads out the barcode, write the digits on two alternating lines as $\begin{matrix} b & b & b & b & b & b \\ a & a & X & a & a & a \end{matrix}$. Total each row, multiply the top row by three then add these totals together.

The total of these numbers should end in a zero but because one digit is missing they won't. The missing digit is whatever must be added to make the next multiple of ten (i.e. end in a zero). If the missing digit is in the top row then this figure will need to be divided by three.



9

$$\begin{array}{r}
 2454 \\
 \hline
 456 \\
 123 \\
 + 876 \\
 212 \\
 787 \\
 \hline
 \end{array}$$

What to do: Ask a student to give you a three digit number. Write this down then quickly draw a line above it and write above this their number plus 1998 (by adding 2000 and subtracting 2).

Now explain that you will both take it in turns to give more three digit numbers. You'll do this twice each, they'll go first.

Each time its your turn, choose a number that complements their previous number to 999 (each pair of digits should add to 9).

When you're finished, draw a line beneath the list of numbers and ask students to add up the numbers between the lines.

Challenge them to do it quicker than you and then subtly put your pen down.

How did you do it?

32	33	34	35
36	37	38	39
40	41	42	43
44	45	46	47
48	49	50	51
52	53	54	55
56	57	58	59
60	61	62	63

16	17	18	19
20	21	22	23
24	25	26	27
28	29	30	31
48	49	50	51
52	53	54	55
56	57	58	59
60	61	62	63

8	9	10	11
12	13	14	15
24	25	26	27
28	29	30	31
40	41	42	43
44	45	46	47
56	57	58	59
60	61	62	63

What to do: Ask a student to think of a number between 1 – 63 but not to tell you what it is.

Show them these cards and ask which ones show their number.

By totaling the top left numbers of each card on which their number is shown you are now able to tell them their number.

Try with other students.

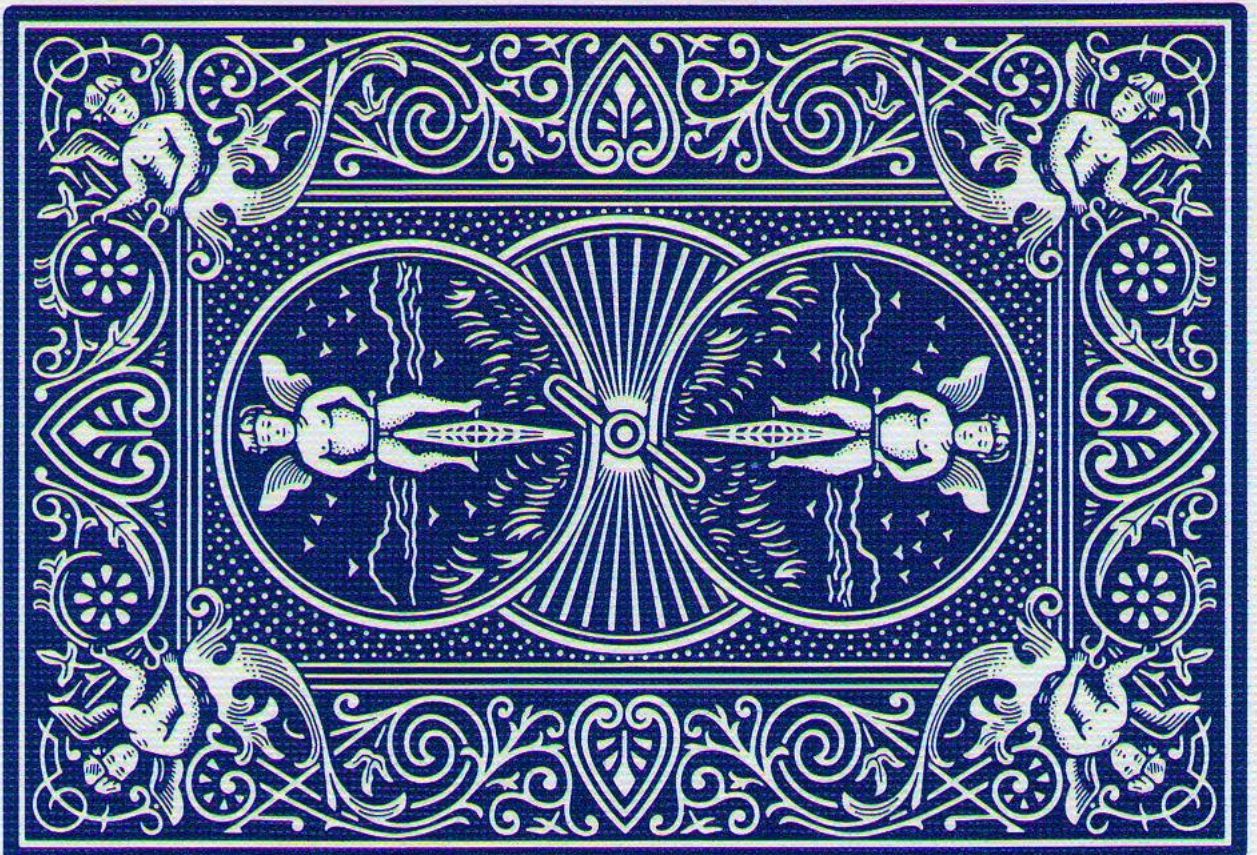
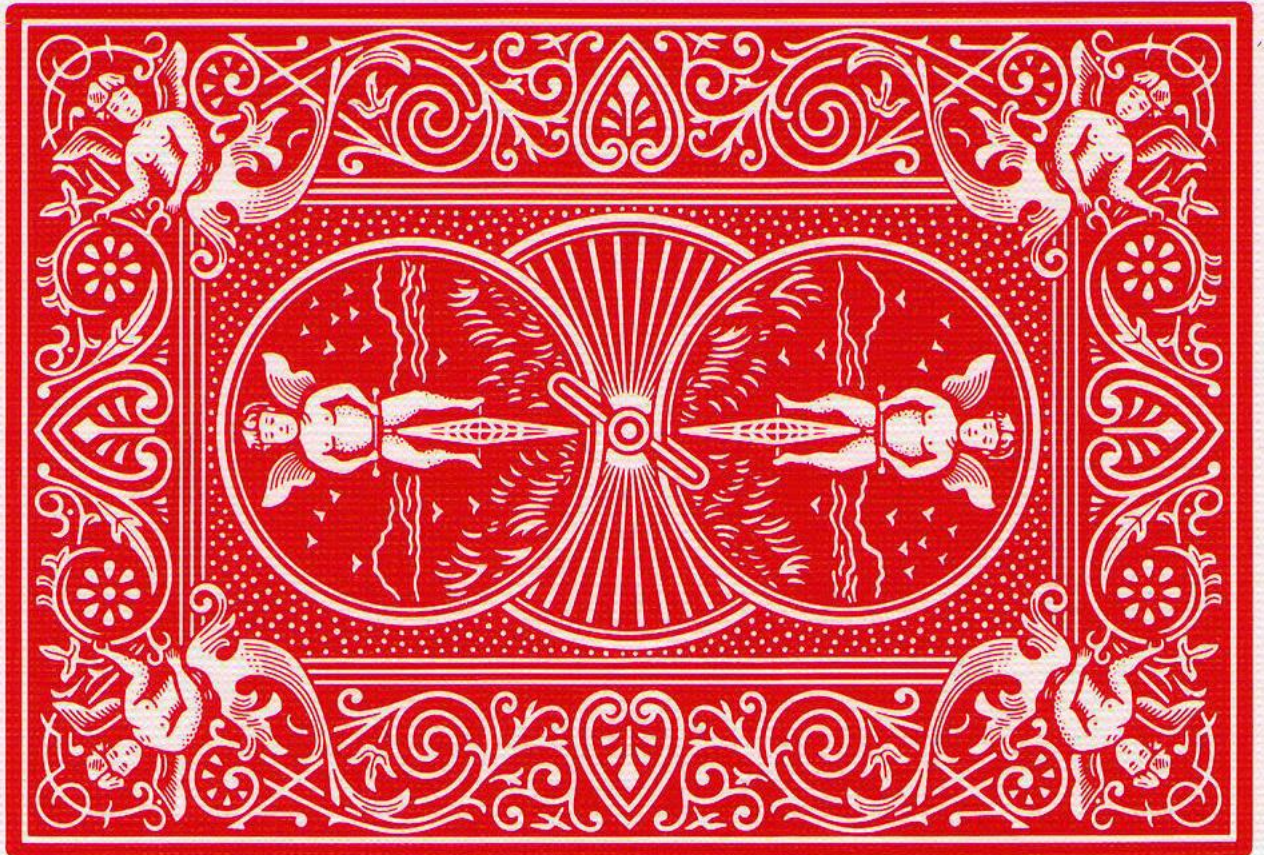
How does it work and why?

4	5	6	7
12	13	14	15
20	21	22	23
28	29	30	31
36	37	38	39
44	45	46	47
52	53	54	55
60	61	62	63

2	3	6	7
10	11	14	15
18	19	22	23
26	27	30	31
34	35	38	39
42	43	46	47
50	51	54	55
58	59	62	63

1	3	5	7
9	11	13	15
17	19	21	23
25	27	29	31
33	35	37	39
41	43	45	47
49	51	53	55
57	59	61	63

10



11

Use all of

1 3 4 6

and any of

+ - × ÷

to make

24

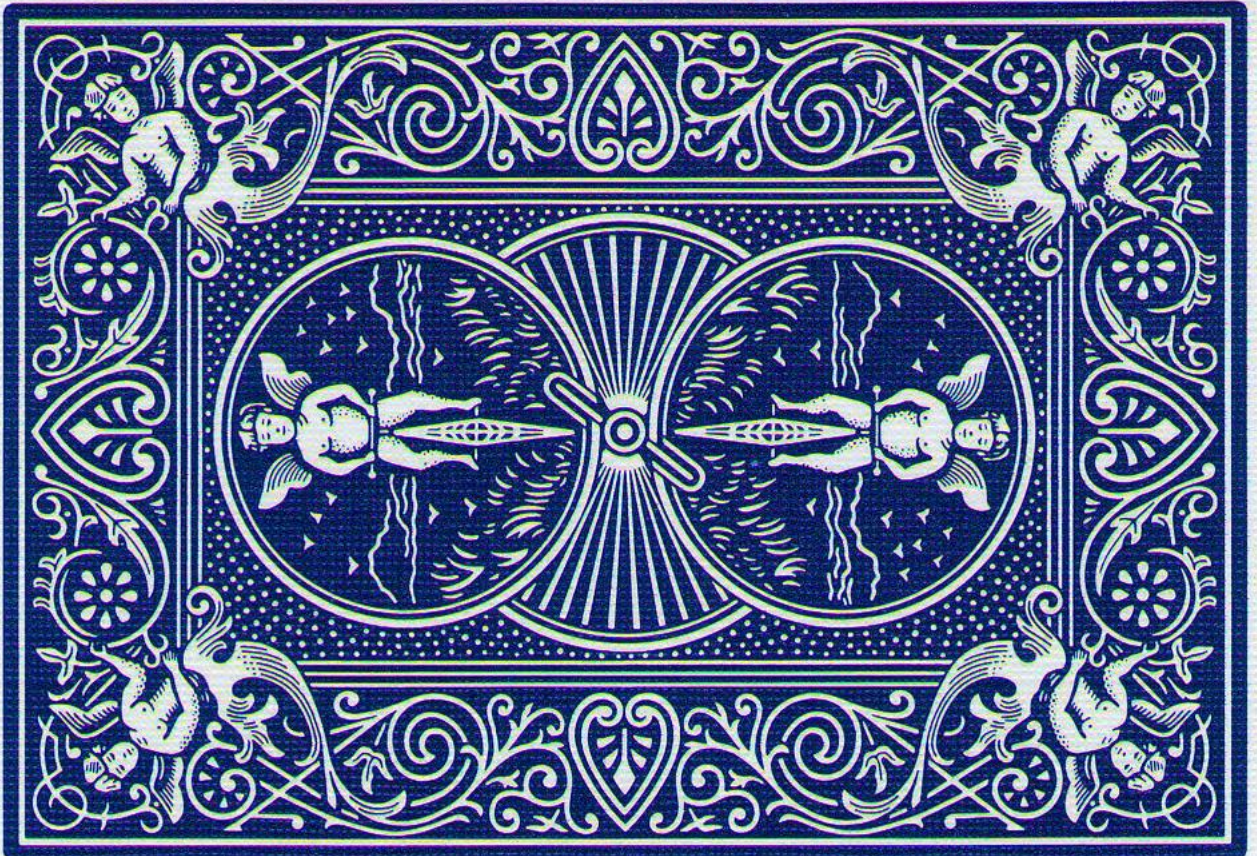
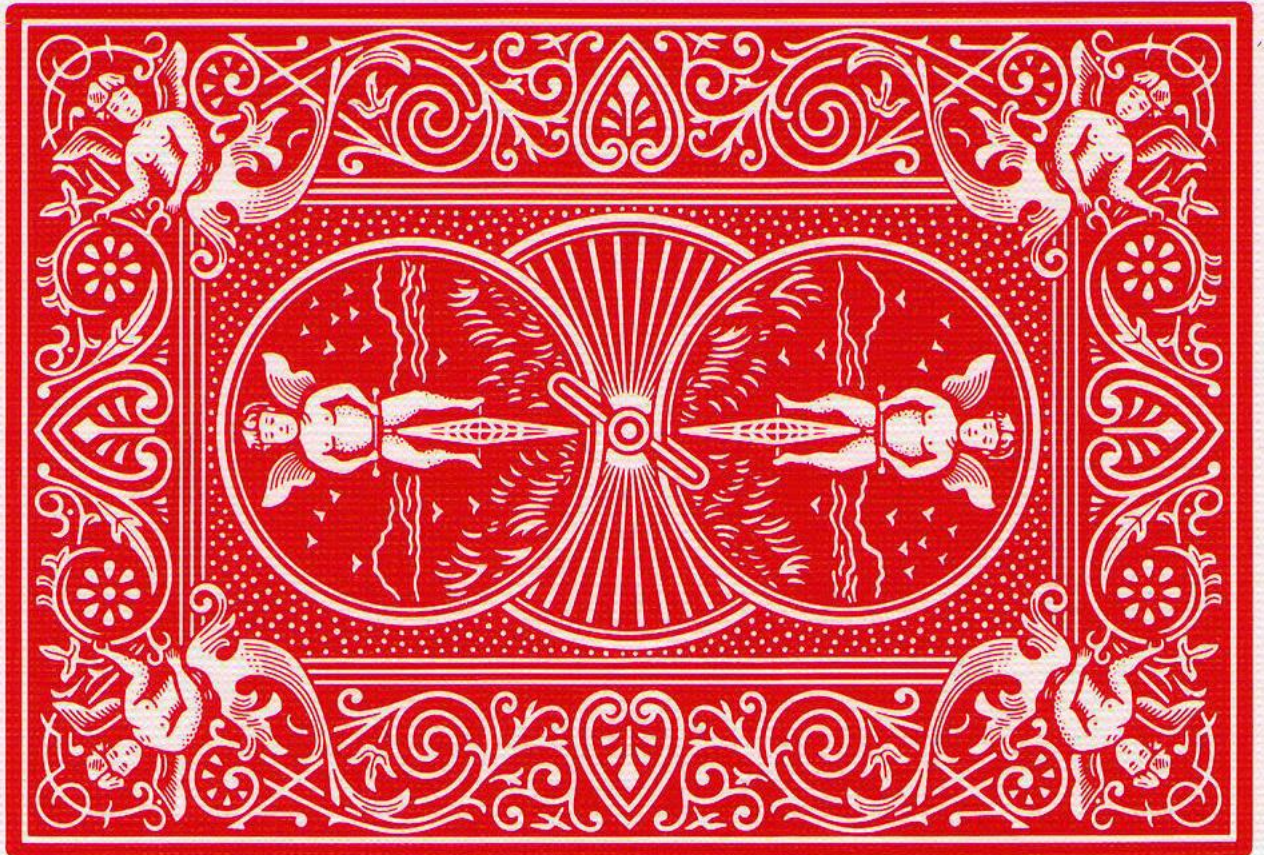
$\frac{1}{9}$

12

Three people share the cost of a £30 meal by paying £10 each to the waiter. As he returns to the kitchen the waiter realises that the bill should've been £25 so fetches some change to give back to the customers. As the waiter gives the customers their change they give him £2 of it as a tip, keeping £1 each.

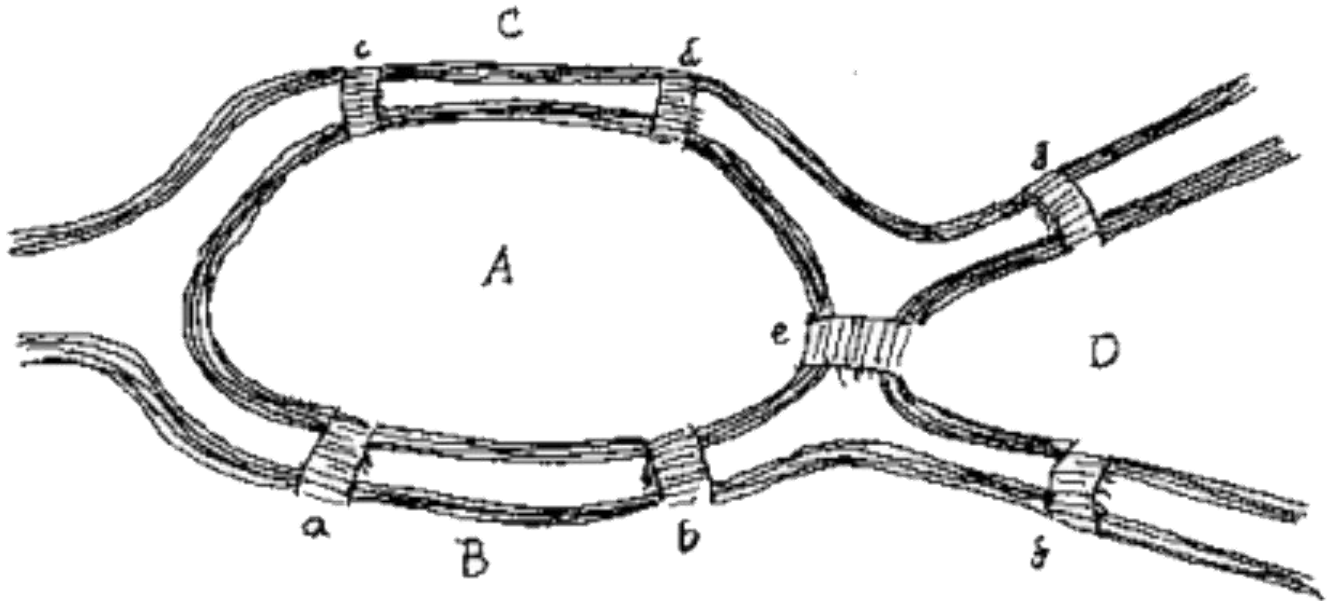
Having each paid £10 and got £1 change, the customers then realise that they've paid £27 between them, the waiter has £2 and this totals £29 instead of £30. They accuse the waiter of stealing from them and vow never to return to the restaurant.

Were they correct to do this?



13

The Bridges of Königsberg

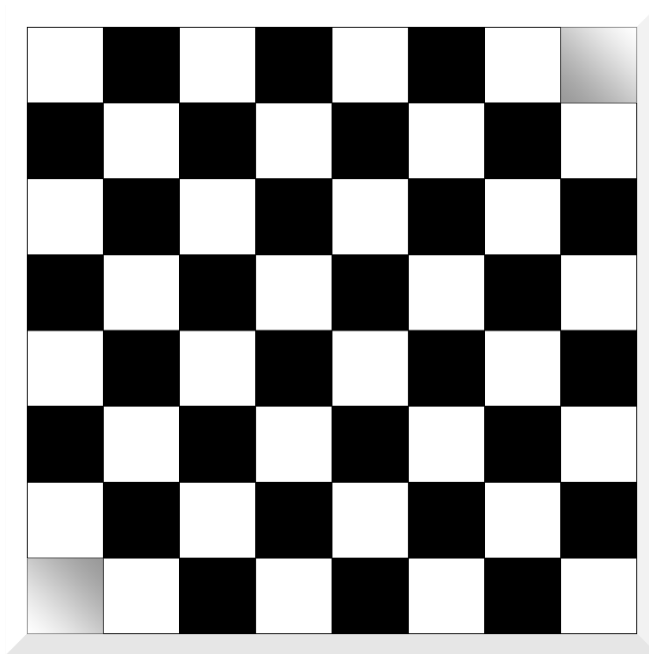


Can you travel across every bridge and visit every area without crossing any bridges more than once?

If so, how? If not, why not?



14

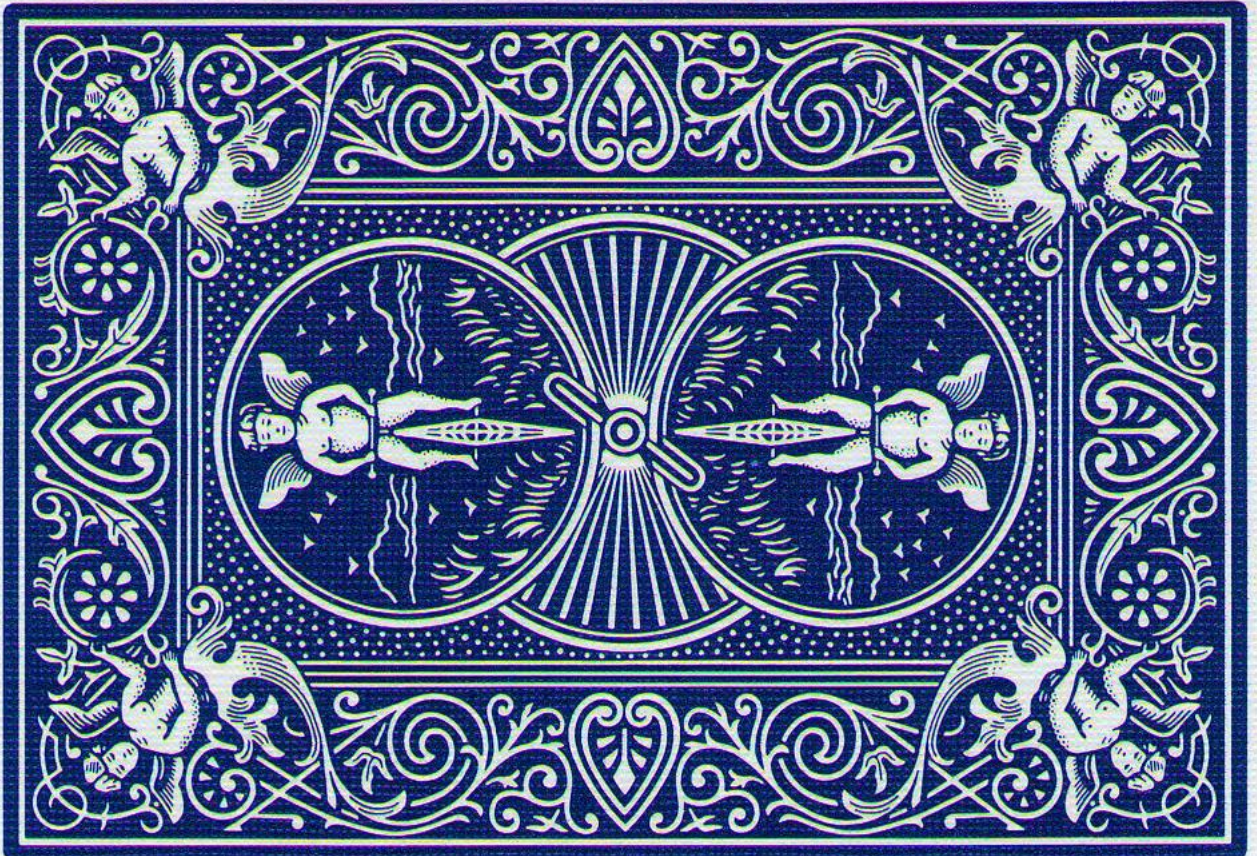
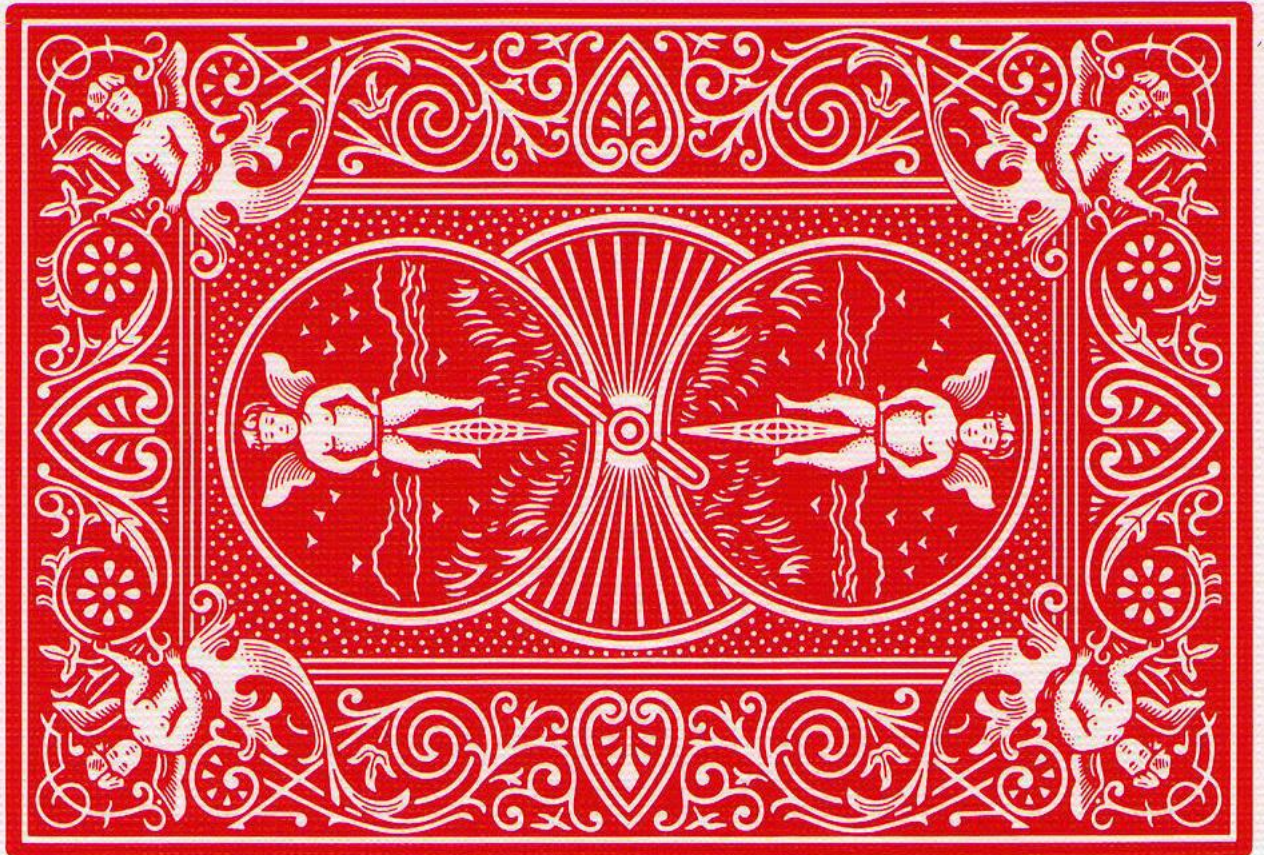


The chessboard above has had the two black corner squares removed.

Is it possible to place dominoes on the board, one domino per two squares, to cover the board exactly?



If so how? If not, why not?

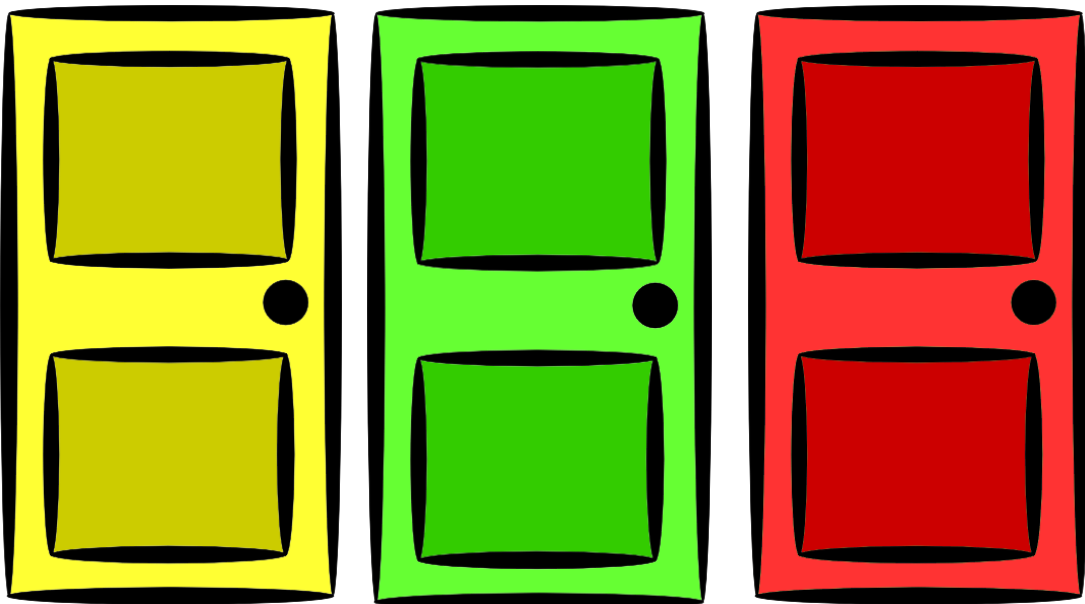


15

123, 123

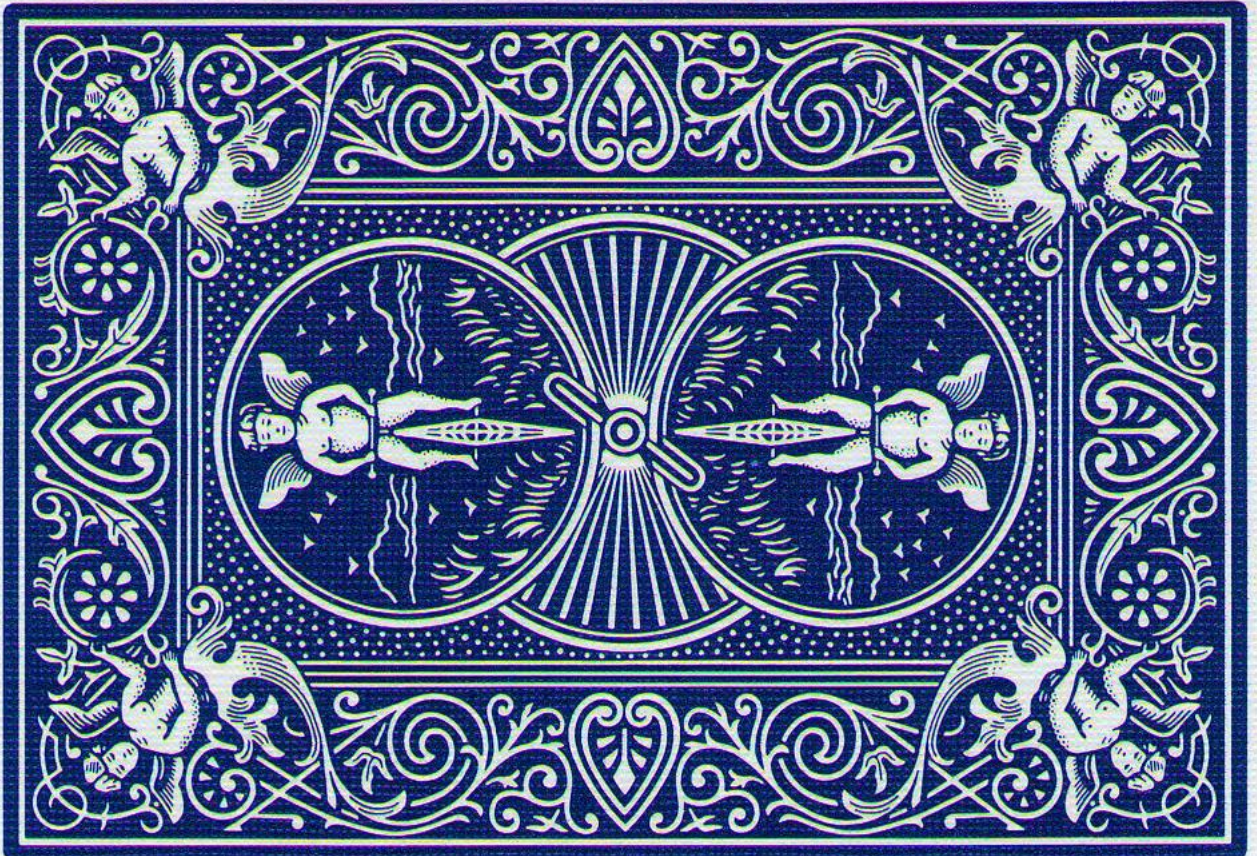
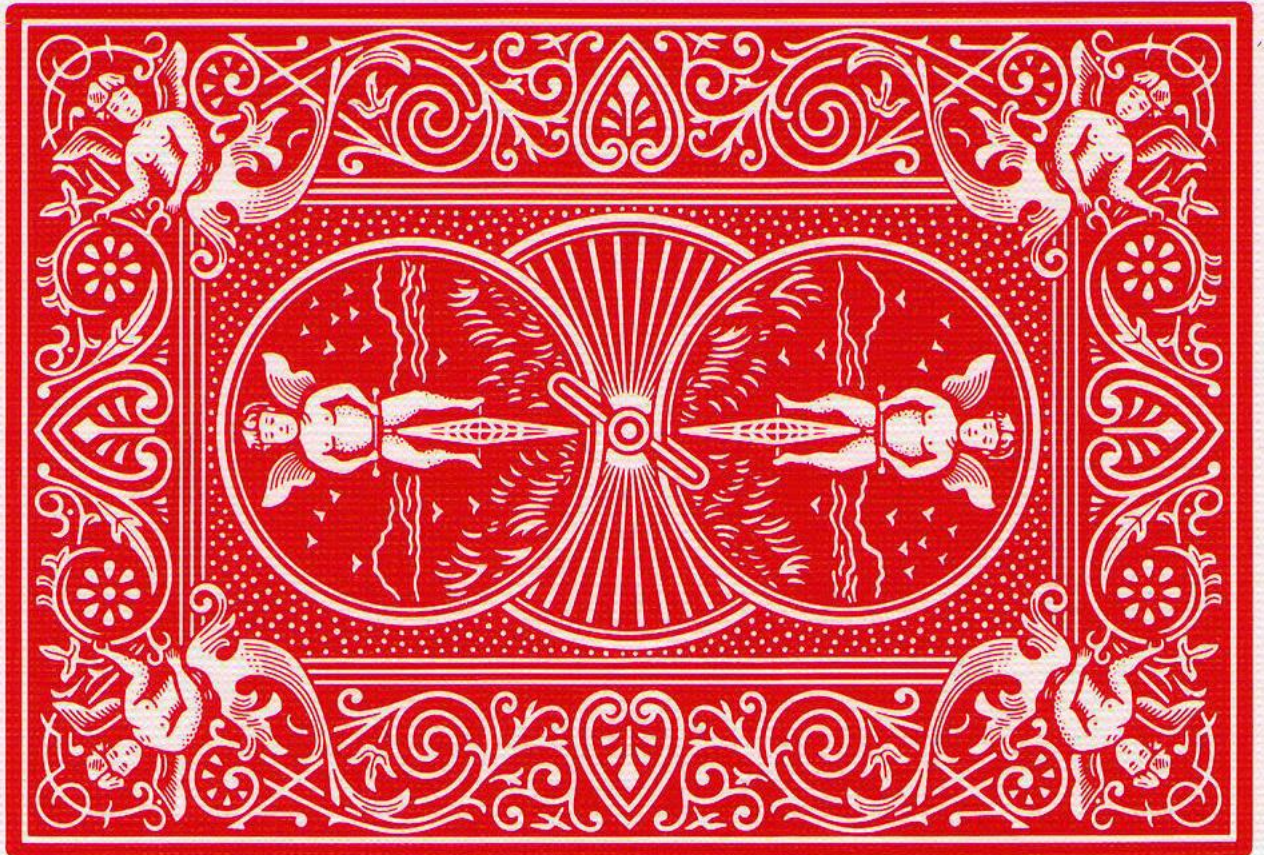
What to do: Ask a student to think of any three digit number and to make a six digit number by repeating this twice. Tell them that you predict it can be divided by 7 exactly and ask them to check, perhaps using a calculator. Further to this, tell them that you predict this number can now be divided by 11 exactly and ask them to check. Even further, you predict this number is divisible by 13 and that this will give a surprising answer! Ask them to check and to explain how you did it.

16



You're on a game show, in the final round. The host offers you the choice of three doors behind two of which are goats and the other is the star prize. Having chosen one door but not opened it, the host, who knows what's behind each of the doors, opens another to reveal one of the goats. You're then given the option to *stick with your current choice* or to *switch to the other remaining door that is closed*.

Should you stick, switch or does it not matter?



Two digit multiples of 11

$$27 \times 11 =$$

$$\begin{array}{r} 2 \quad 2+7 \quad 7 \\ \hline \end{array}$$

$$\begin{array}{r} 2 \quad 9 \quad 7 \\ \hline \end{array}$$

$$297$$

$$48 \times 11 =$$

$$\begin{array}{r} 4 \quad 4+8 \quad 8 \\ \hline \end{array}$$

$$\begin{array}{r} 4 \quad 2 \quad 8 \\ \hline 1 \quad \quad \end{array}$$

$$528$$

Does this always work? How come? Can you prove it?

(17)

Squaring a Two Digit Number Ending in 1.

$$31^2 =$$

$$\begin{array}{r} 900 \quad 30 \times 30 \\ + \quad 60 \quad 2 \times 30 \\ \quad 1 \quad 1 \times 1 \\ \hline \end{array}$$

$$961$$

$$71^2 =$$

$$\begin{array}{r} 4900 \quad 70 \times 70 \\ + \quad 140 \quad 2 \times 70 \\ \quad 1 \quad 1 \times 1 \\ \hline \end{array}$$

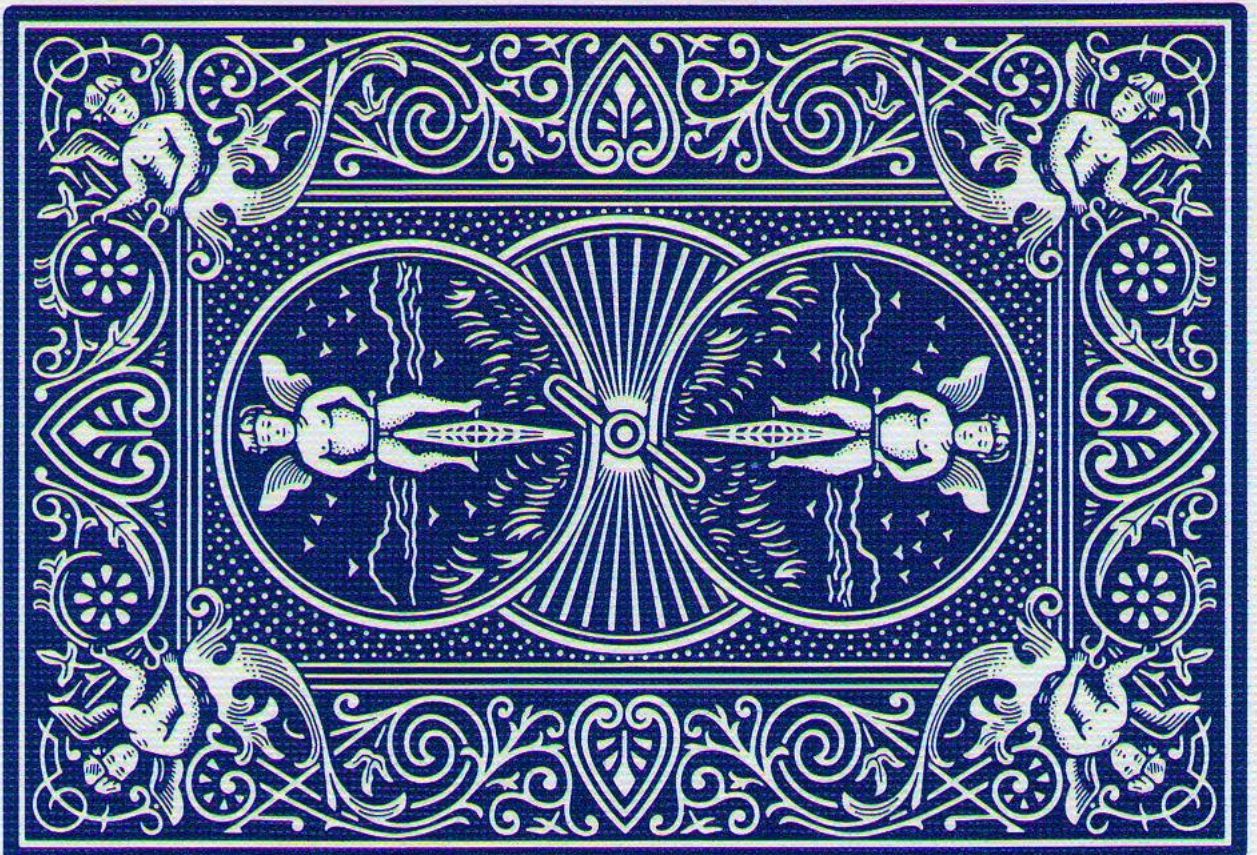
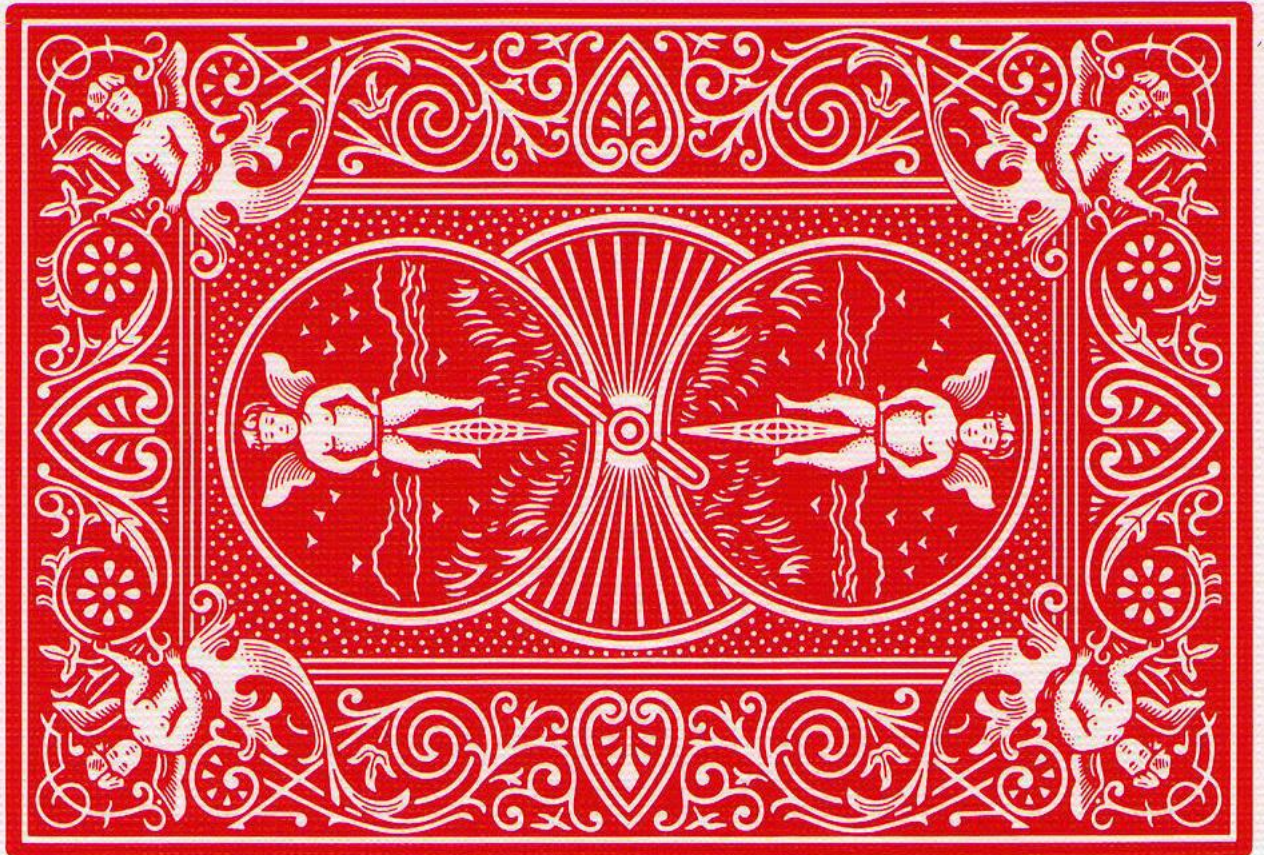
$$5041$$

Why does this work?

Find a method for squaring two digit numbers ending in 5.

Extend to find a method for squaring any two digit number.

(18)



19

Make that Number

What to do: Ask a student to choose a target number (or use the number 50, as below). The game is to take turns adding any number between one and five (inclusive) until one of you reach the target number.

The winner is the person who says the target number.

How its done: To ensure that you are the winner, begin by working out the difference between the *target number* and the *nearest multiple of six below the target number*. As soon as possible, make the total equal to any multiple of six plus this difference. From here just keep adding the number that makes theirs up to six and you'll be sure to win.

An example: Target number = 50. Difference between 50 and 48 (nearest multiple of six below 50) is 2. Aim to make any of the following values as soon as possible:

2, 8, 14, 20, 26, 32, 38, 44, 50

How could you generalise the game?

20

Electricity

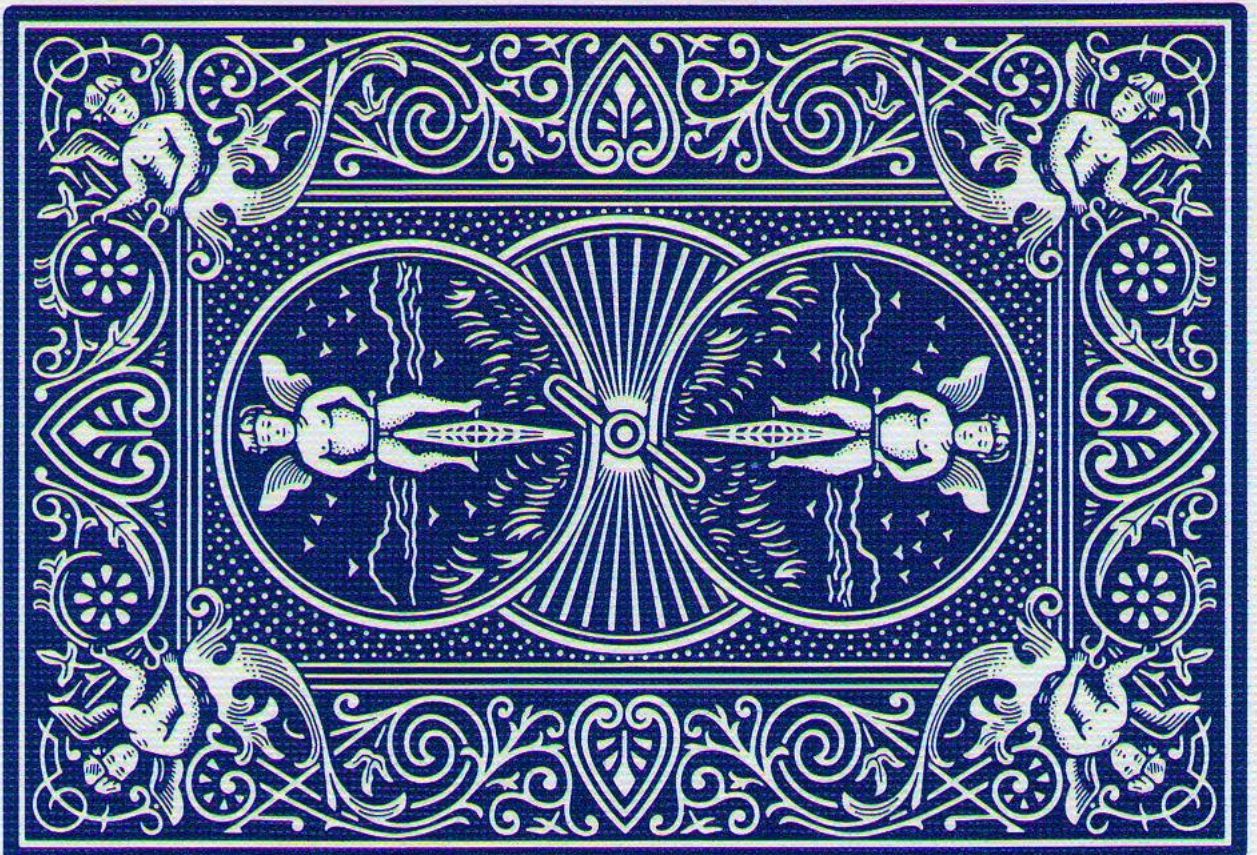
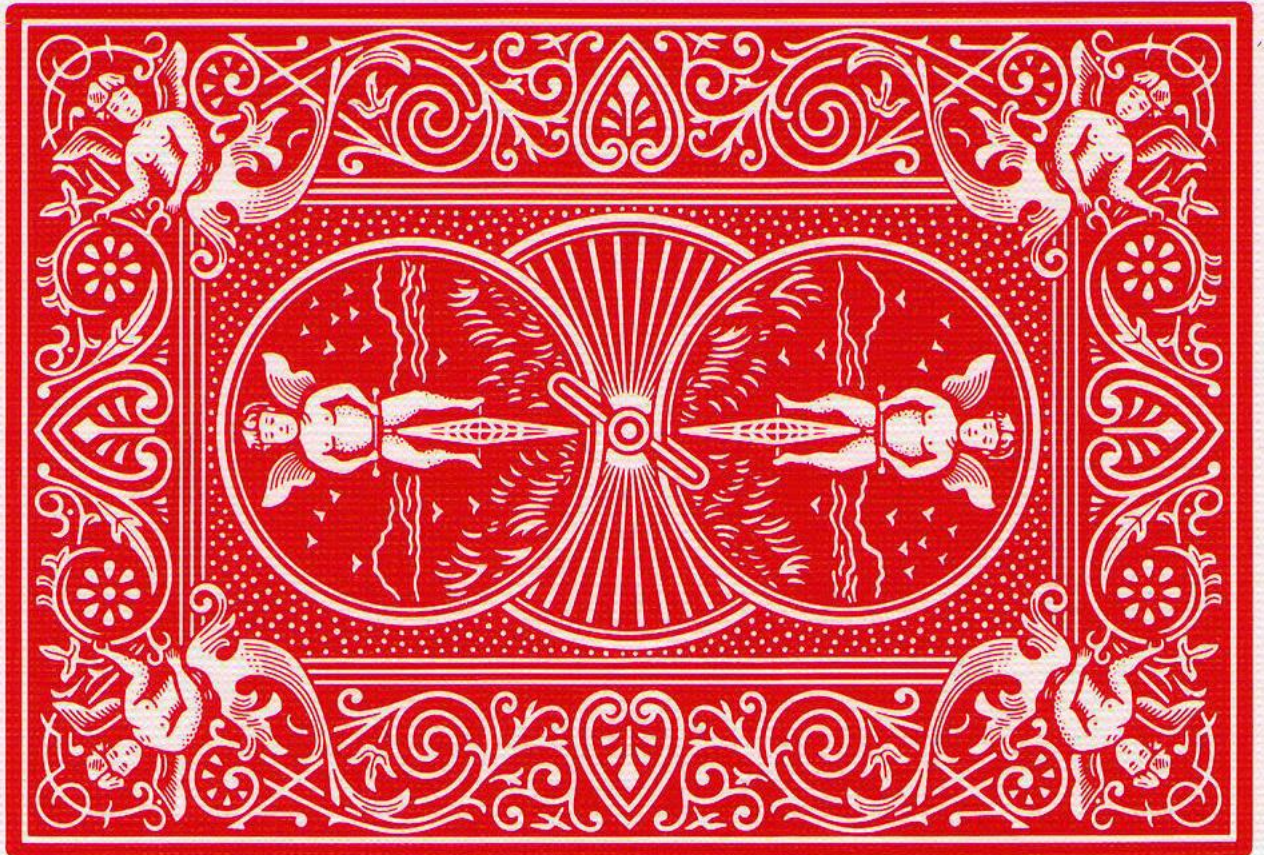
Water

Gas



Can you connect up all three utilities to all three houses without crossing any lines?

If not, why not?



21

$$p^2 - 1 = 24m$$

Take any prime number greater than 3, square it and subtract 1.

Is the answer a multiple of 24?

Try again, and again, and again.

Why is that?

$p^2 - 1 = (p+1)(p-1) \Rightarrow p-1, p+1$ are three consecutive integers.

Since p is a prime > 3 , then either $p-1$ or $p+1$ is a multiple of three.

Furthermore, both $p-1$ and $p+1$ are also multiples of two
and either $p-1$ or $p+1$ is a multiple of four.

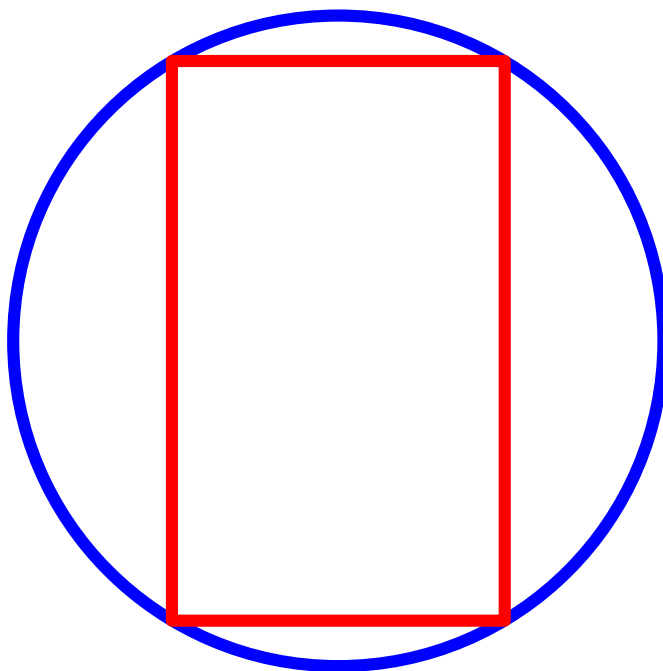
$\therefore p^2 - 1$ is a multiple of $2 \times 3 \times 4 = 24$.

A nice proof of this is also possible in base 12.

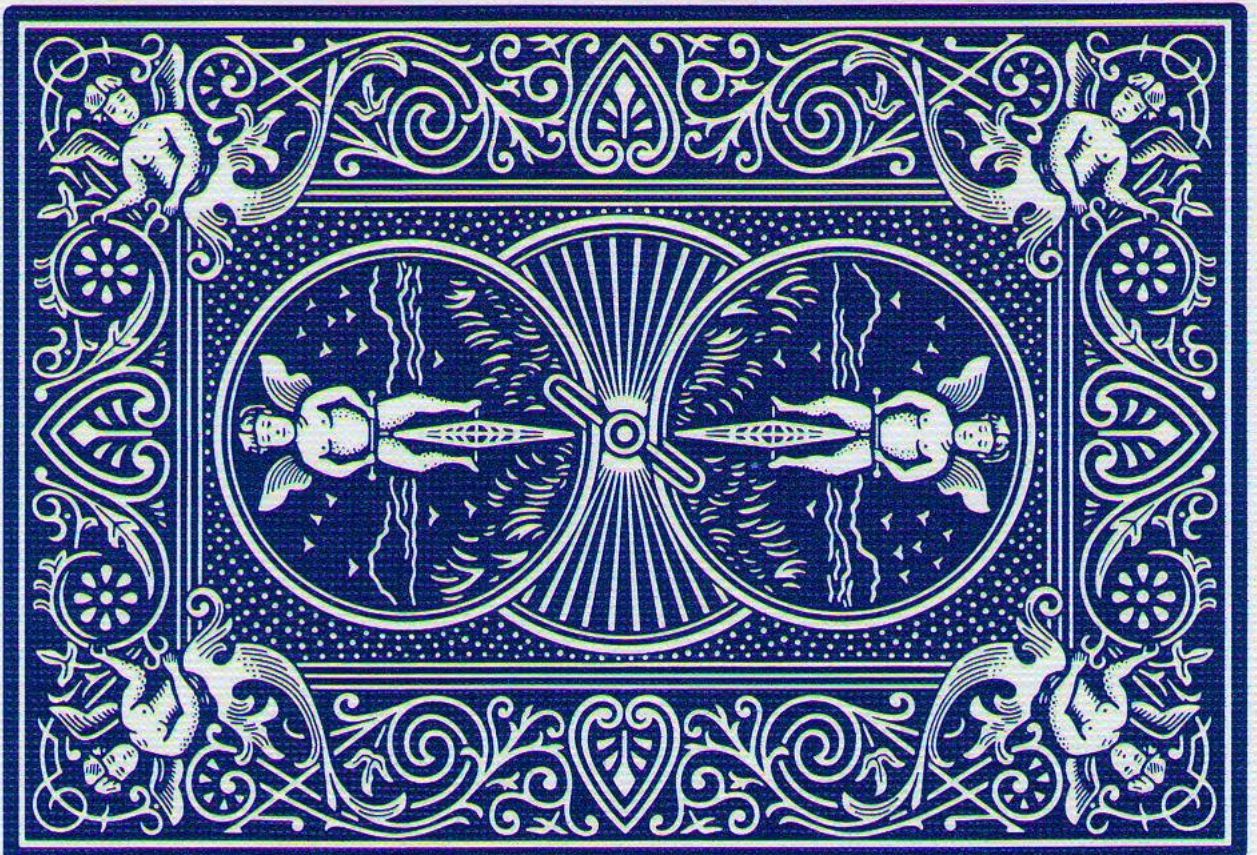
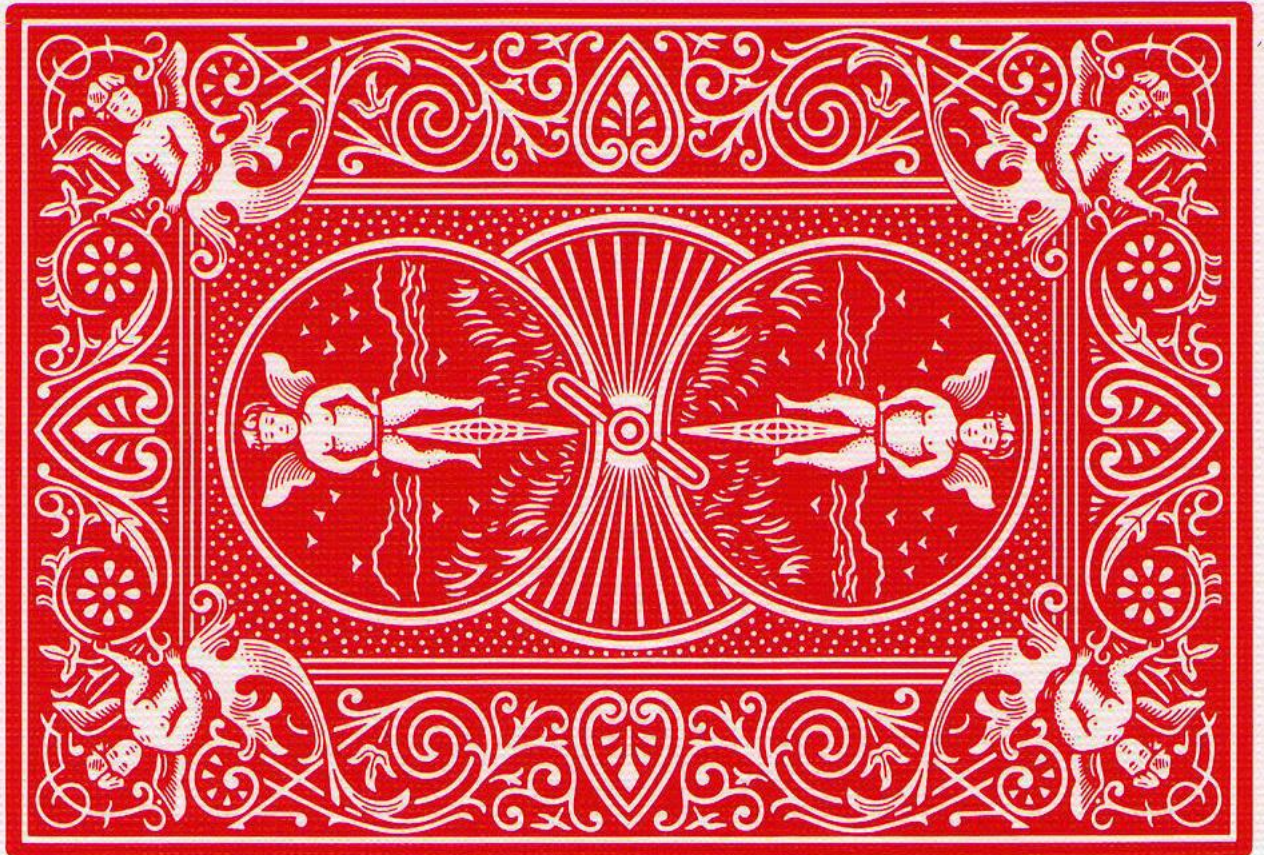
22

The circle in the diagram has radius 6 cm.

The rectangle has a perimeter of 28 cm.



Find the area of the rectangle.



23

A school summer fayre has a stall offering two games.

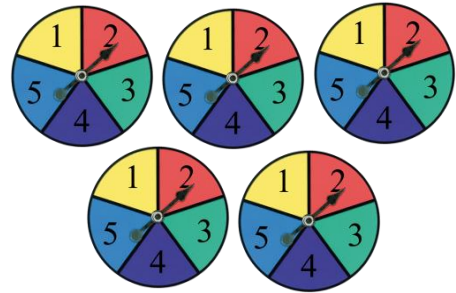
£1 per go

Flip all 10 coins,
if you get 10 heads,
win an ipad (value £400).



£1 per go

Spin all 5 spinners,
if you get 5 fives,
win a Macbook (value £1000).



Which game are you most likely to win?
Which game should the school encourage you to play?

3

24

5

$$3 + 5 = 8$$

$$5 + 8 = 13$$

$$8 + 13 = 21$$

$$13 + 21 = 34$$

$$21 + 34 = 55$$

$$34 + 55 = 89$$

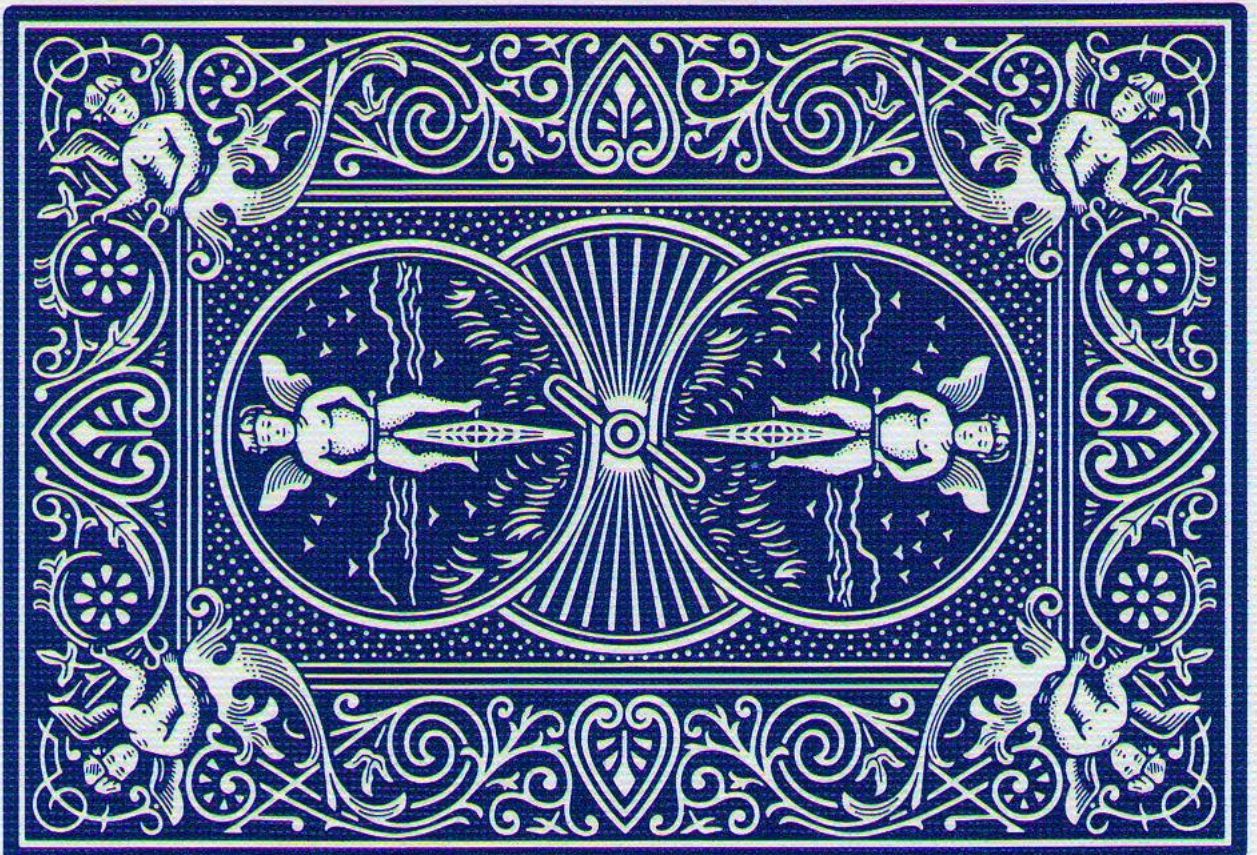
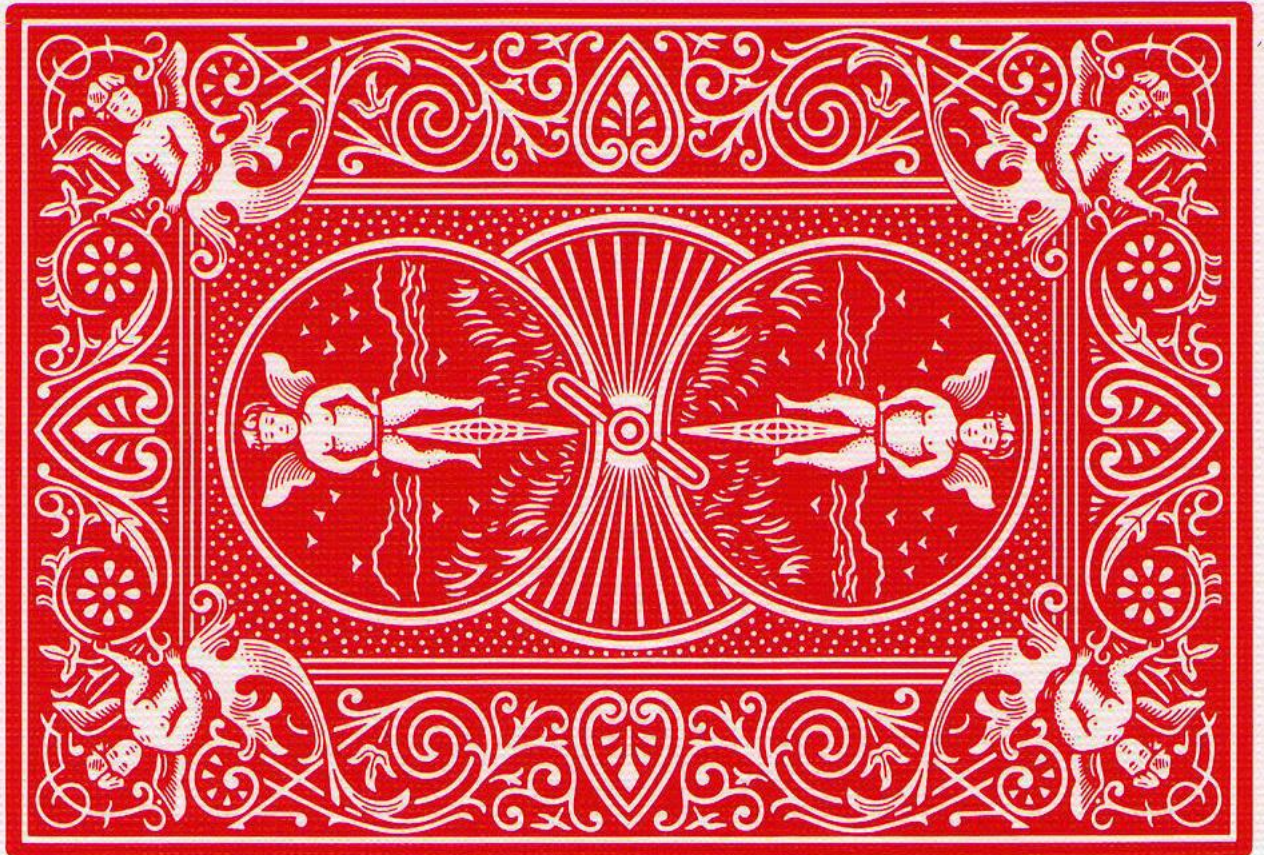
$$55 + 89 = 144$$

$$89 + 144 = 233$$

What to do: Ask a student for any two single digit numbers, write these down and then continue by adding the previous two numbers until you have a list of ten numbers. You will be able to find the total of these ten numbers almost instantly.

How its done: Multiply the seventh row by eleven (see card no. 17) ... but why?

NB: the seventh row is the fourth row from the bottom.



25



At the end of the football season, the ball boys wanted their team to walk under some bunting that spanned the length of the pitch. Sadly, their bunting was only 101m long and didn't stretch.

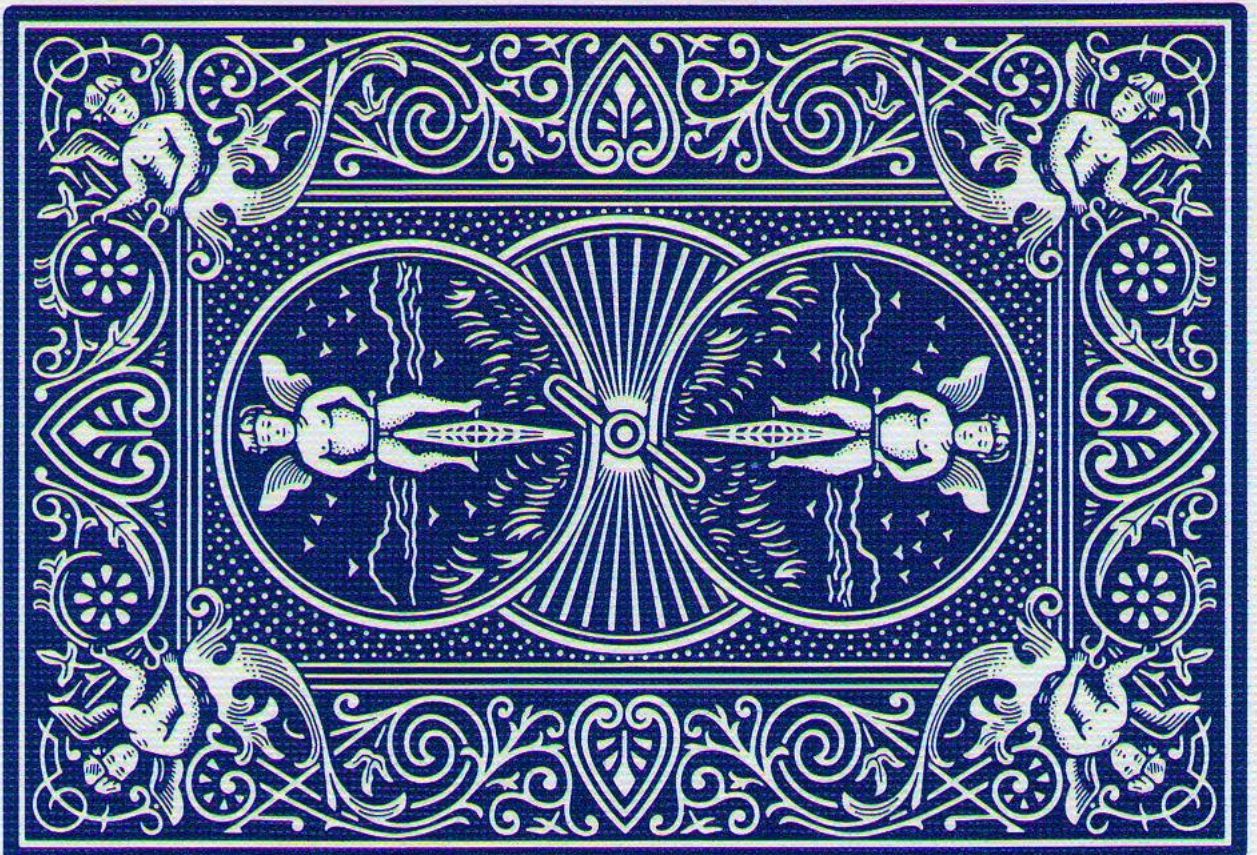
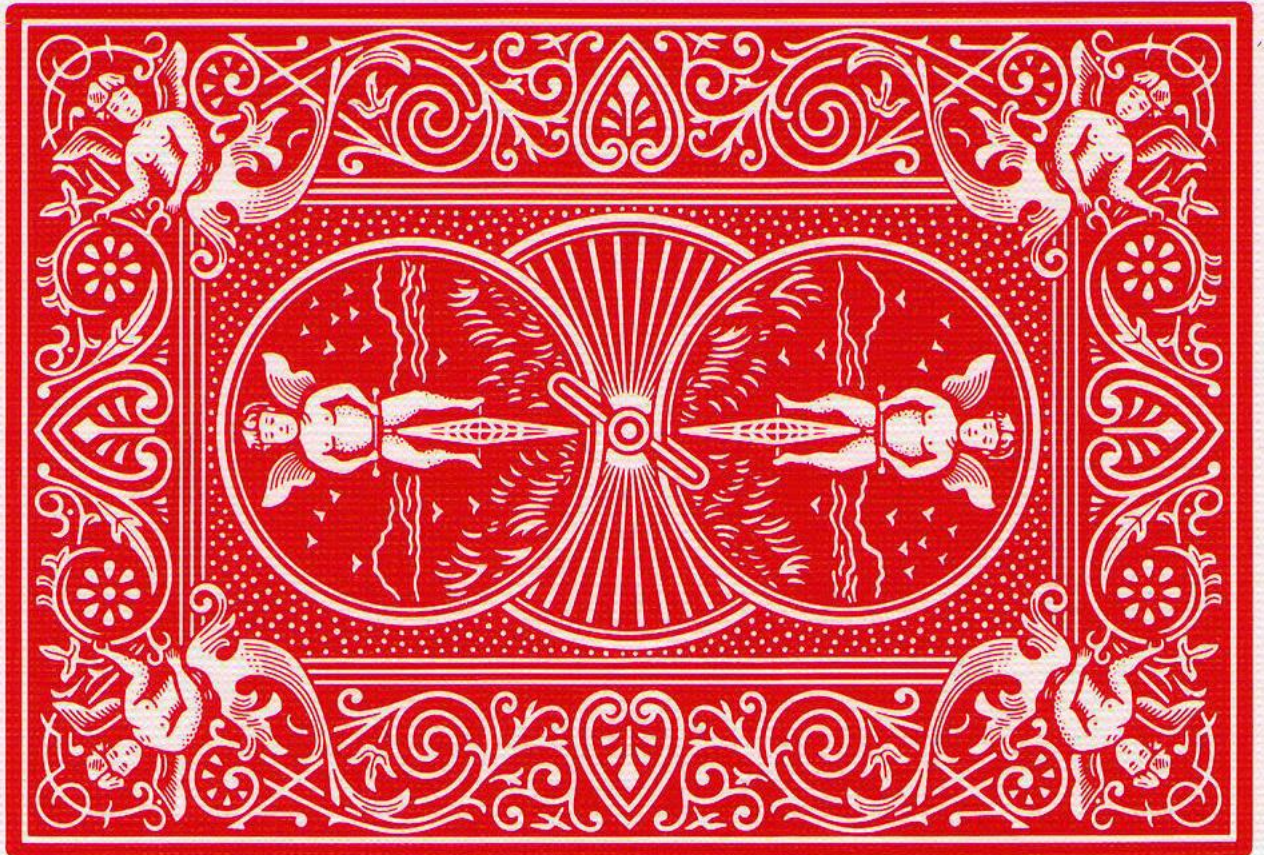
Was this long enough for the players to fit beneath?

26

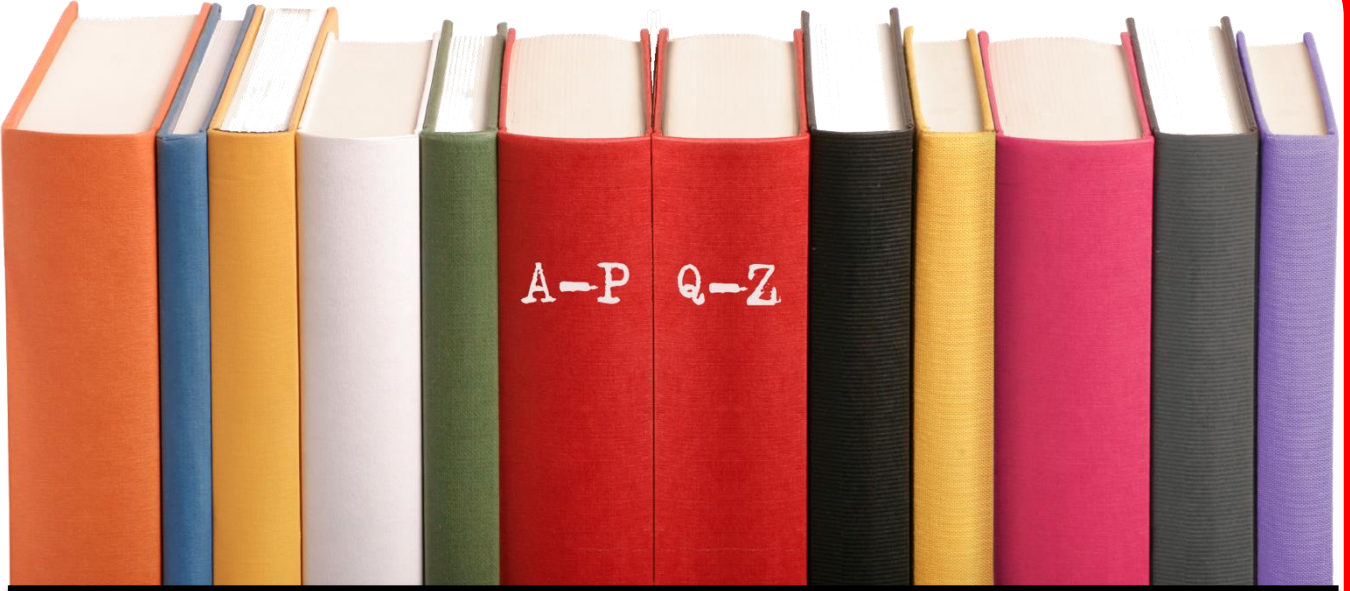
Choose any single digit number.
Write it down three times to form a three digit number.
Divide this by the sum of those three digits.

$$\begin{array}{r} XXX \\ \hline X + X + X \end{array} = ?$$

Try again and again.
Why is that?



4mm 4mm 4mm 4mm



10cm 10cm

Amongst my books is a two volume encyclopedia. Each volume is 10cm wide including the covers which are each 4mm thick.

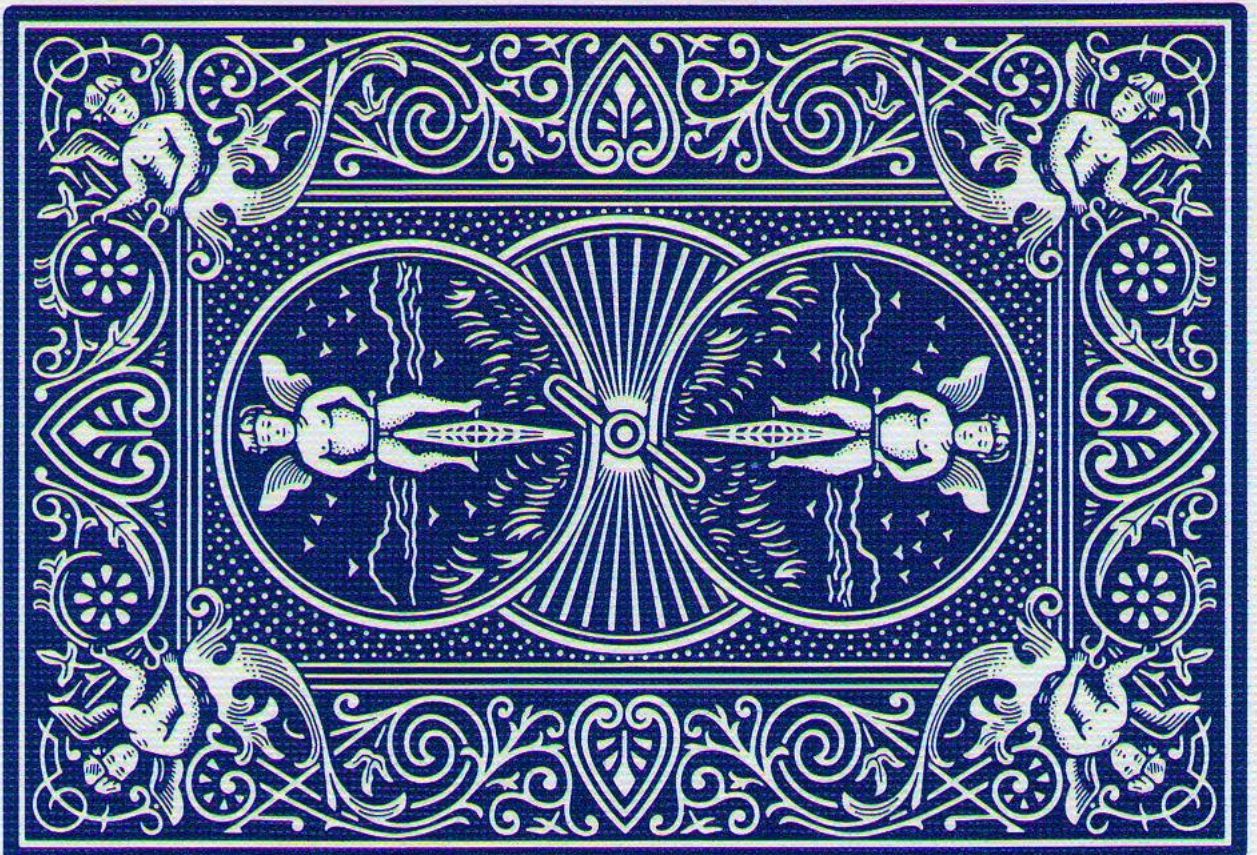
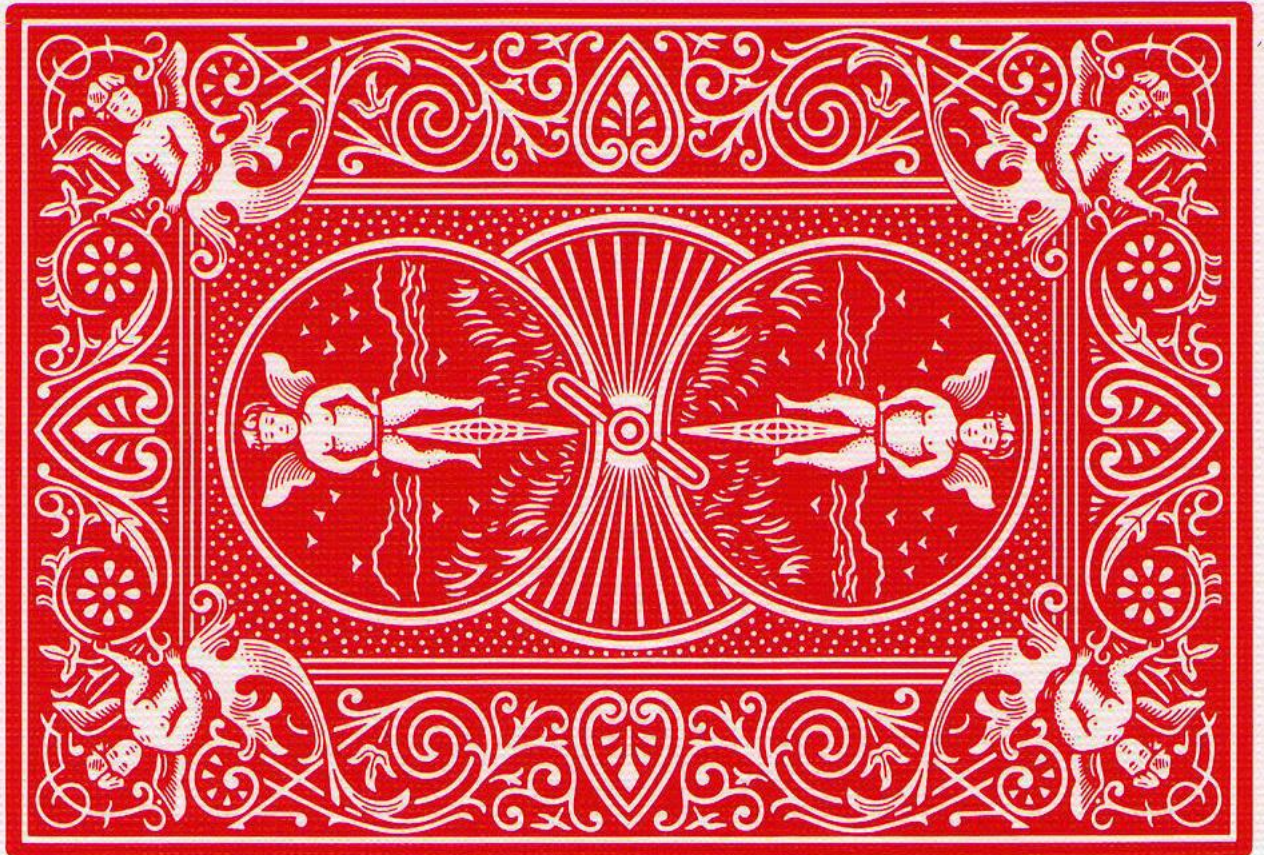
(27)

How far apart are the entries for Aardvark and Zebra?

(28)

$$3^{444} + 4^{333}$$

Multiple of 5?



29

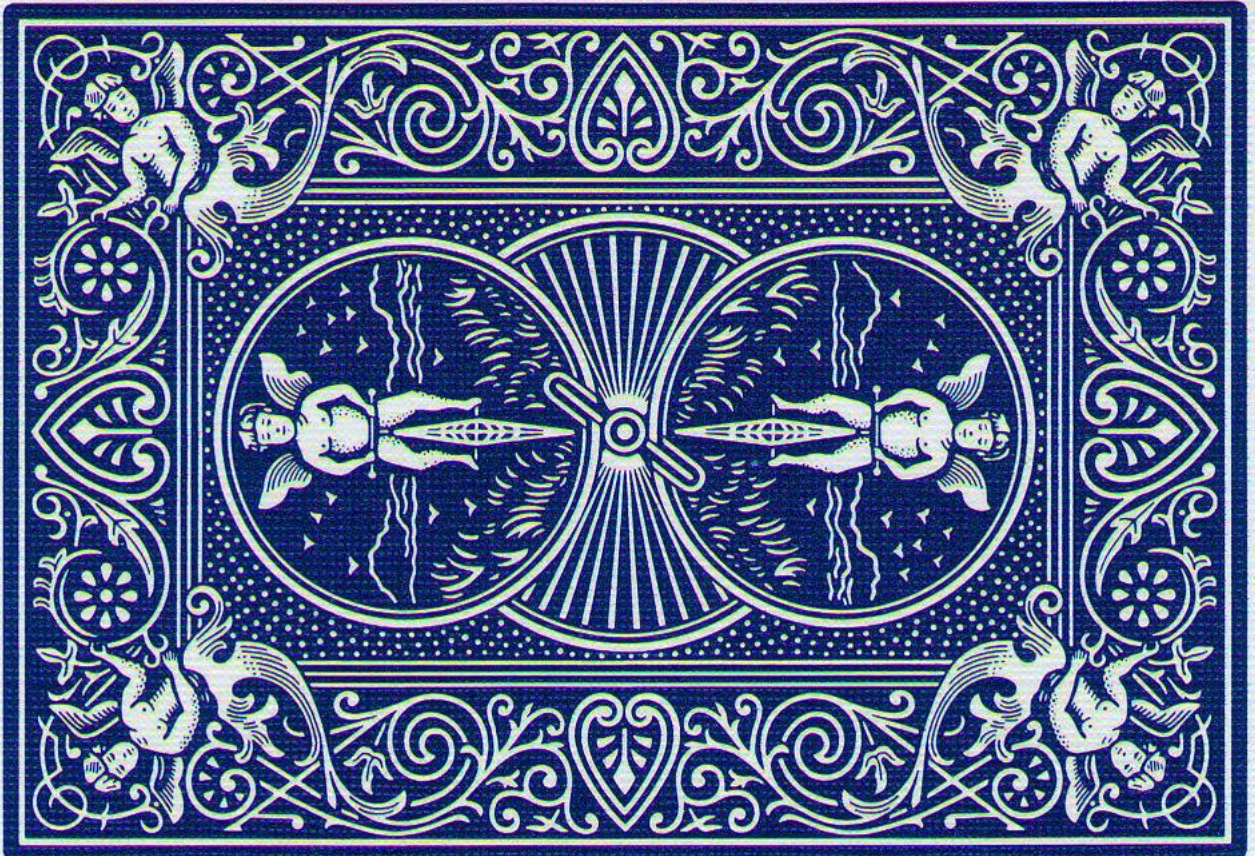
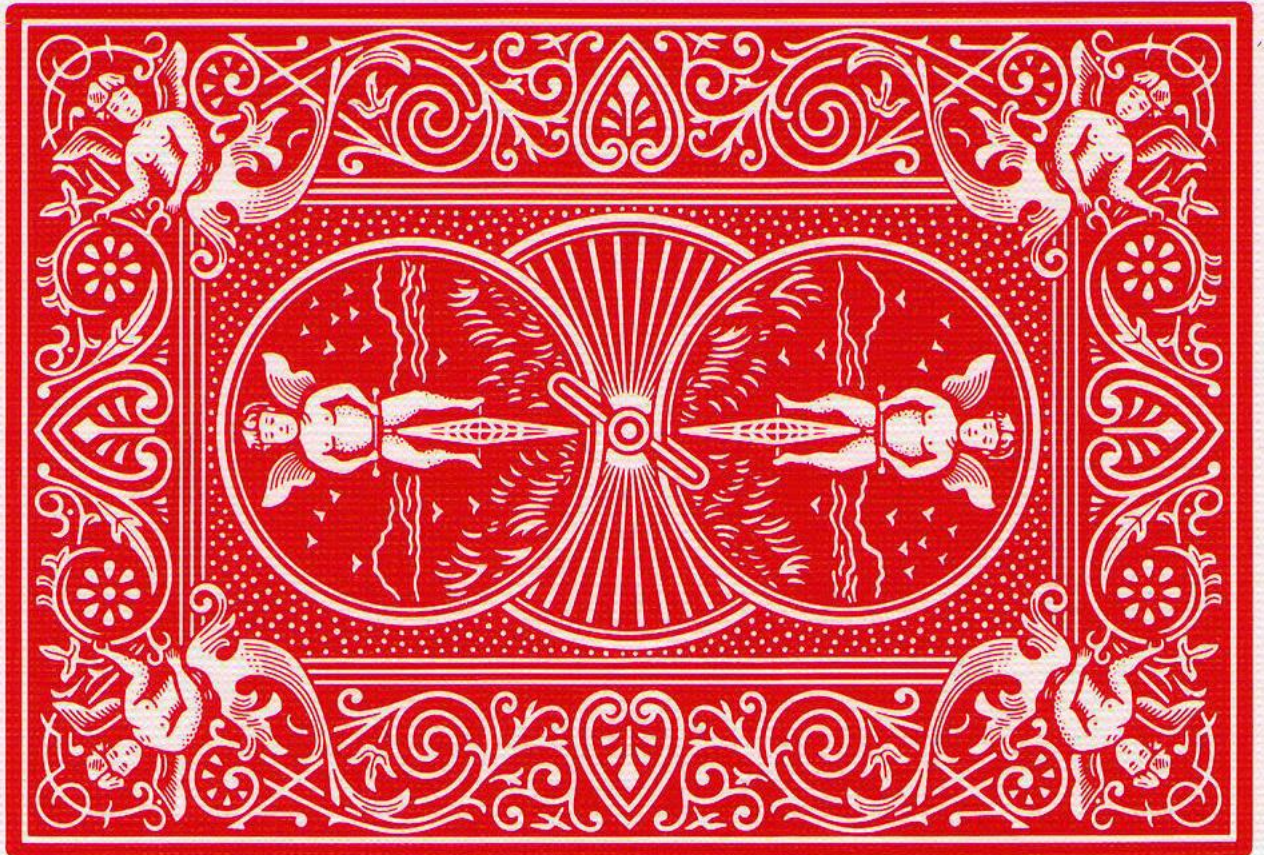


A camel crosses a desert 1000 km wide. It can carry a maximum of 1000 bananas and eats 1 banana for every kilometre that it walks. There are 3000 bananas in total. What is the maximum that can be carried to the end?

30



What way round would the queens head be if we rolled the left hand coin around the other?
At what angle would the queens head be if we rolled the left hand coin **behind** the other?

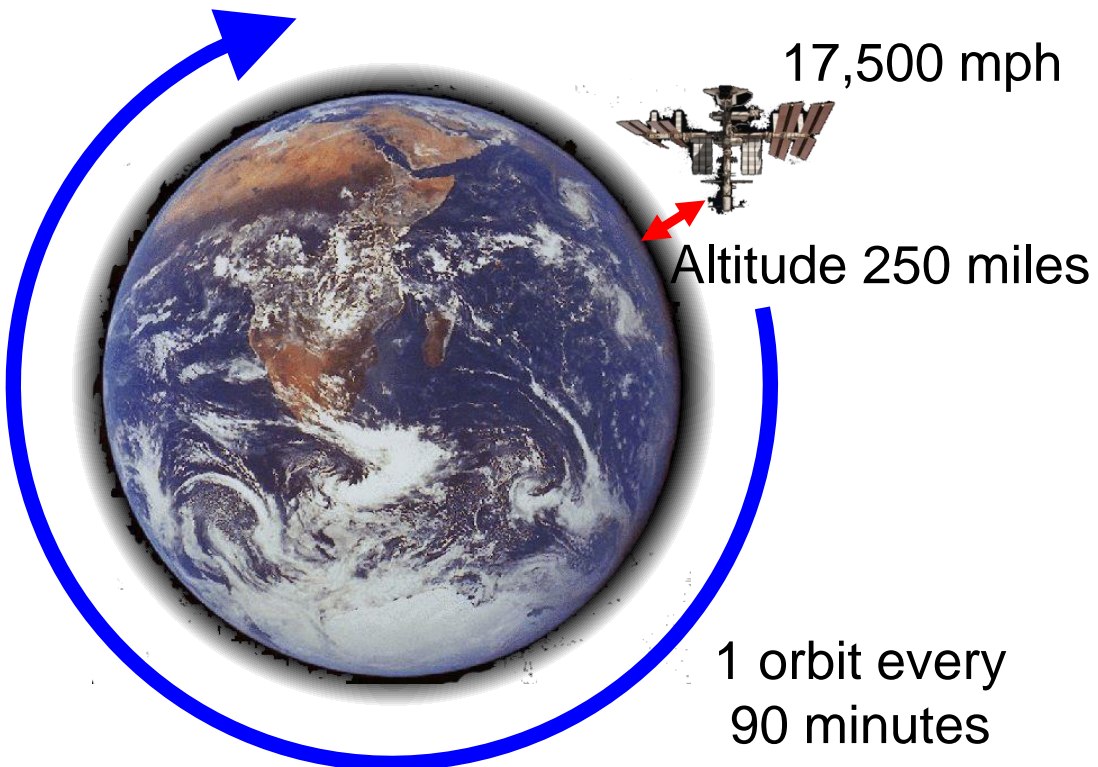


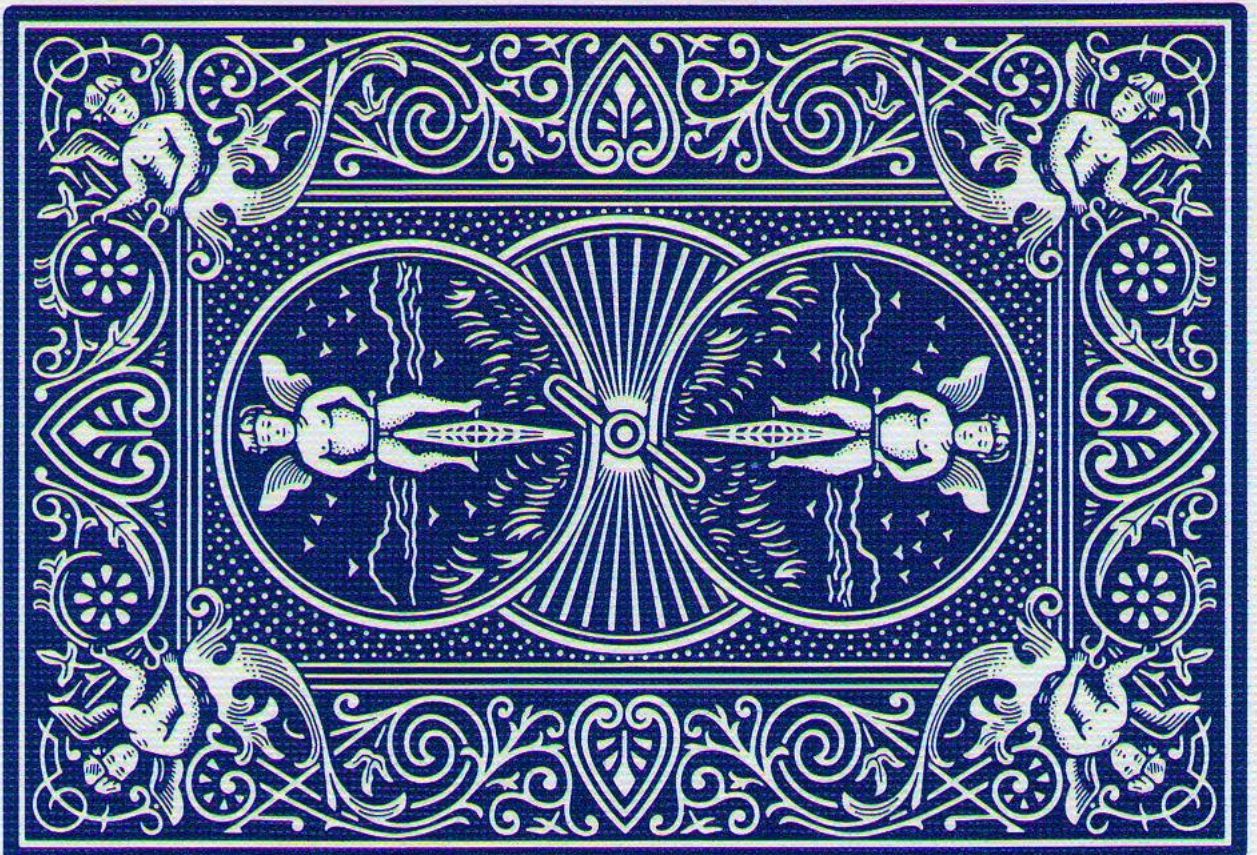
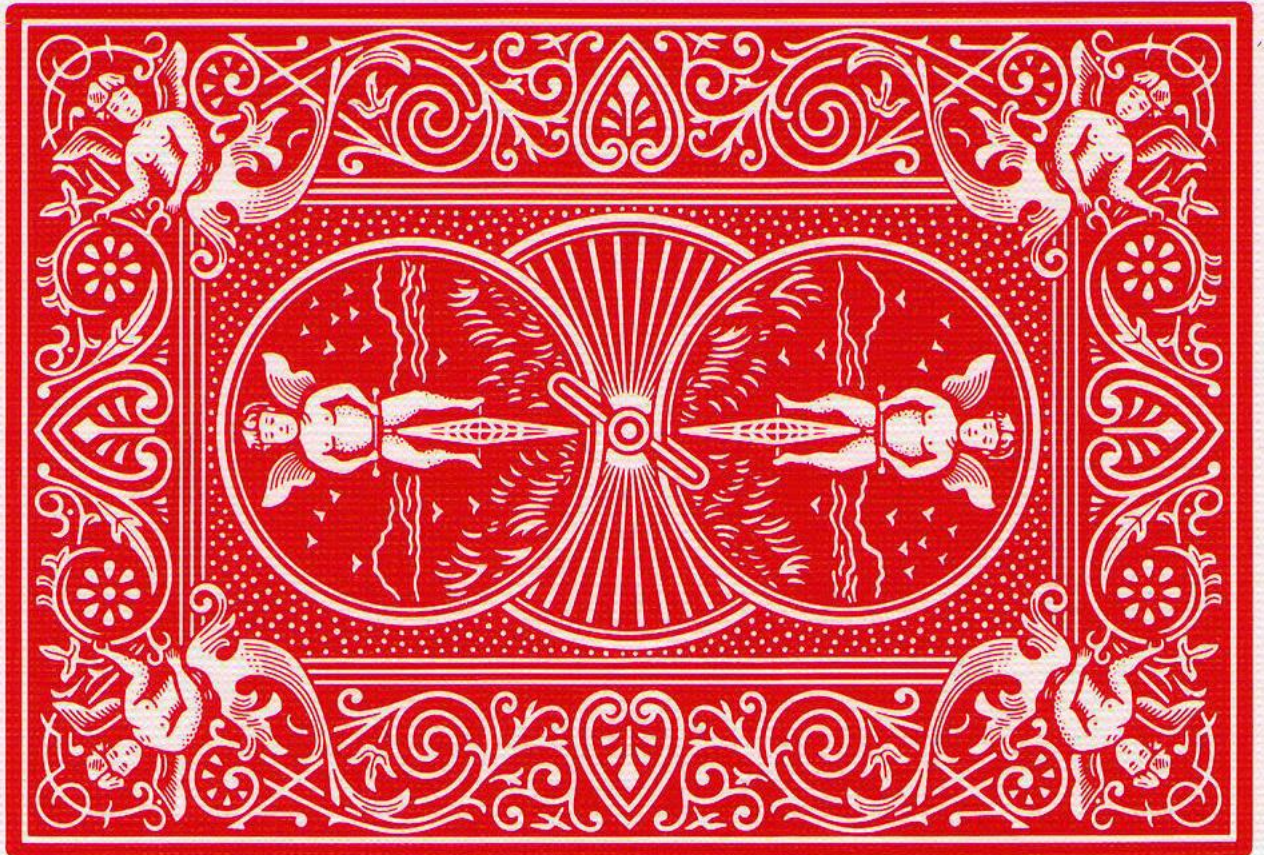
31

What is the biggest product that you can make using numbers which sum to 10?

32

What is the Circumference of the Earth?





33

1

2

3

4

5

6

7

8

9

10

What is the lowest common multiple of the numbers 1-10?

1

2

3

4

5

6

7

8

9

10

11

12

13

14

15

16

17

18

19

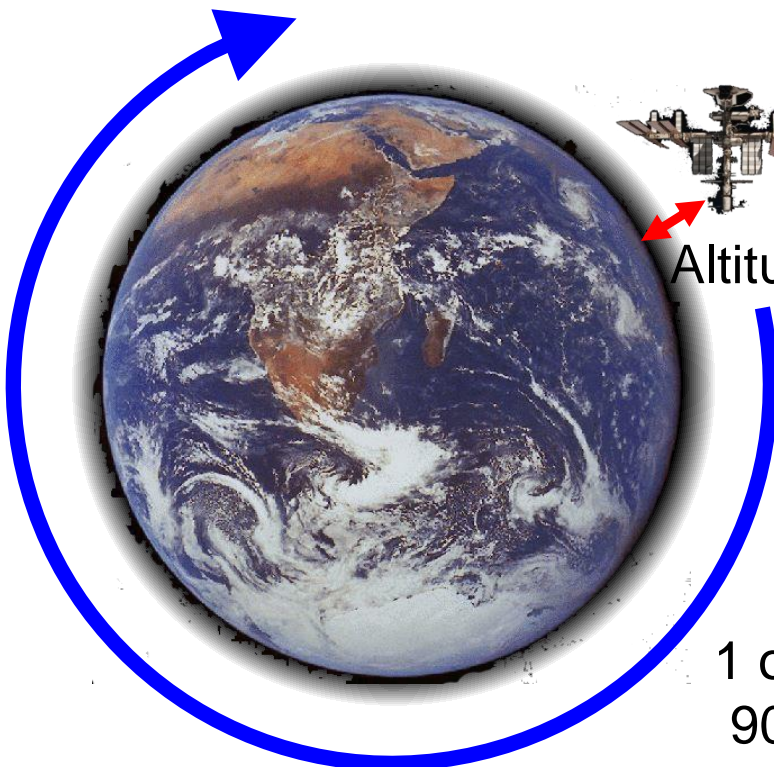
20

What is the lowest common multiple of the numbers 1-20?

Lowest common multiple of 1-100?

34

What is the Circumference of the Earth?



17,500 mph

Altitude 250 miles

1 orbit every
90 minutes