

The Summer Fayre

On each go, the probability of winning with the coins is $\frac{1}{1024}$. The expected pay-out per go is therefore $\frac{1}{1024} \times 400 = 39\text{p}$.

On each go, the probability of winning with the spinners is $\frac{1}{3125}$. The expected pay-out per go is therefore $\frac{1}{3125} \times 1000 = 32\text{p}$.

Therefore players should choose the coins. (The trivial case is not to play at all since both games cost £1 to play and represent an overall loss in terms of pay-outs. But then this is neither fun nor in the spirit of supporting the school and its summer fayre.)

With the coins, the school would expect to pay-out every 1,024 goes and could expect that this payout occurs on the median 512th attempt. By this time they would have accumulated enough money to pay for the prize. The breakeven point is the 400th player. The probability of the school reaching this point follows a Poisson distribution

$$X \sim P\left(\frac{1023}{1024}\right)$$
$$P(X > 400) = \left(\frac{1023}{1024}\right)^{400} = 0.676$$

With the spinners, the school would expect to pay-out every 3,125 goes and could expect that this payout occurs on the median 1,563rd attempt. By this time they would have accumulated enough money to pay for the prize. The breakeven point is the 1000th player. The probability of the school reaching this point follows a Poisson distribution

$$X \sim P\left(\frac{3124}{3125}\right)$$
$$P(X > 1000) = \left(\frac{3124}{3125}\right)^{1000} = 0.726$$

How much money can the school expect to make through each game in the long term? How much money can the school expect to make through the stall (both games) in the long term?

Possible extensions include:

- What if both games were changed to 'get all showing the same result'? Implications for players, implications for the school?
- What about another game where contestants can flip a coin three times and, if they get three heads they win £100 but if they get three tails they pay £10. Would students choose to play this game or not? Is this game a good idea for the school to offer?

See also <http://www.youtube.com/watch?v=rwvIGNXY21Y>