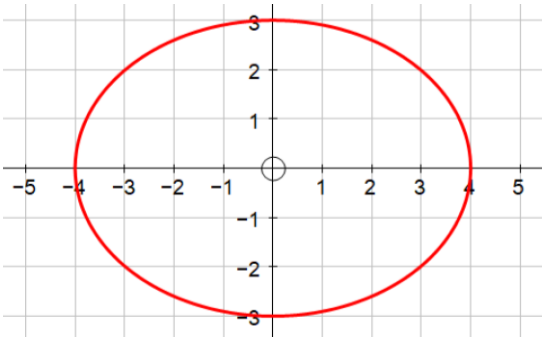
Parametric Integration

Find $\int y\,dx$ between $t = 0$ and $t = 2\pi$, for the parametric equations below…

$x = 4sint \qquad y = 3cost$



$$\int y\,dx = \int 3cost\,dx$$

$$x = 4sint$$
$$\Rightarrow \frac{dx}{dt} = 4cost$$
$$\Rightarrow dx = 4cost\,dt$$

$$\int y\,dx = \int 3cost\,dx$$
$$= \int 3cost \times 4cost\,dt$$
$$= 12\int cos^2t\;dt$$

$$cos2t = 2cos^2t - 1 \Rightarrow cos^2t = \frac{cos2t + 1}{2}$$

Integration becomes…

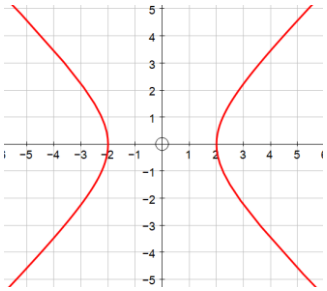
$$12\int cos^2t\;dt = 6\int cos2t + 1\;dt$$
$$= 6\left[\frac{sin2t}{2} + t\right]_0^{2\pi}$$
$$= 6\left[\left(\frac{sin4\pi}{2} + 2\pi\right) - \left(\frac{sin0}{2} + 0\right)\right]$$
$$= 12\pi$$

Now try these…

1. Find $\int y\,dx$ between $t = 0$ and $t = 2\pi$, for $x = 3sint$ and $y = 5cost$.

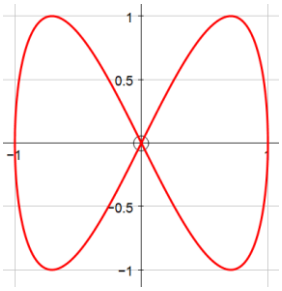
2. Find $\int y\,dx$ between $t = 2$ and $t = 3$, for the parametric equations below…

$$x = t + \frac{1}{t} \qquad y = t - \frac{1}{t}.$$



3. Find the area bounded by the x-axis and the curve with parametric equations below between $0 \leq t \leq \frac{\pi}{2}$.

$$x = sint \qquad y = sin2t$$



4. Find the positive integral bounded by the x-axis and the curve with parametric equations below…

$$x = t - 2sint \qquad y = 1 - 2cost$$

5. The trajectory of a ball thrown from the top of a tower is modelled by the parametric equations $x = 20t$ and $y = 50 + 15t - 2t^2$.

   Find the area of the region underneath the flight of the ball.

Answers =

1. 15π
2. $ln\left(\frac{4}{9}\right) + \frac{185}{72}$
3. $\frac{2}{3}$
4. $4\pi + 3\sqrt{3}$
5.