Parametric Equations

Be able to find a Cartesian equation / eliminate the third variable (or find an implicit equation).

The smaller part

Be able to differentiate, find $\frac{dy}{dx}$ (usually in terms of t).

The bigger part

Simpler examples such as…

$$x = t + 1 \qquad y = t^2$$

Harder examples such as…

$$x = 2sint \qquad y = 5 - 4cost$$

$$x = 3t + \frac{1}{t} \qquad y = 3t - \frac{1}{t}$$

Note that…

$x^2 - y^2 = -8$ is an implicit function

$y = \sqrt{x^2 + 8}$ is an explicit function

$$\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx} \qquad \frac{dy}{dx} = \frac{dy}{dt} \div \frac{dx}{dt}$$

Rearrangements of Examples

The Simpler Example

$$x = t + 1 \qquad\qquad y = t^2$$

$$t = (x-1) \Rightarrow y = (x-1)^2$$

Harder Example 1:

$$x = 2sint \qquad\qquad y = 5 - 4cost$$

$$\Rightarrow \frac{x}{2} = sint \qquad\qquad \Rightarrow y - 5 = -4cost$$

$$\Rightarrow \frac{x^2}{4} = sin^2t \qquad\qquad \Rightarrow \frac{5-y}{4} = cost$$

$$\Rightarrow \frac{(5-y)^2}{16} = cos^2t$$

$$\frac{x^2}{4} + \frac{(5-y)^2}{16} = 1$$

$$4x^2 + (5-y)^2 = 16$$

Harder Example 2:

$$x = 3t + \frac{1}{t} \qquad\qquad y = 3t - \frac{1}{t}$$

$$x + y = 6t$$

$$x - y = \frac{2}{t}$$

$$(x+y)(x-y) = 6t \times \frac{2}{t} = 12$$

$$(x+y)(x-y) = 12$$

Differentiation of Examples

E.g. 1

$$x = 2t + 1 \qquad\qquad y = t^2$$

$$\frac{dx}{dt} = 2 \qquad\qquad \frac{dy}{dt} = 2t$$

And therefore…

$$\frac{dy}{dx} = \frac{dy}{dt} \div \frac{dx}{dt} = 2t \div 2 = t$$

E.g.2

$$x = 2sint \qquad y = 5 - 4cost$$

$$\frac{dx}{dt} = 2cost \qquad \frac{dy}{dt} = 4sint$$

And therefore…

$$\frac{dy}{dx} = \frac{dy}{dt} \div \frac{dx}{dt} = \frac{4sint}{2cost} = 2tant$$

E.g.3

$$x = 3t + \frac{1}{t} \qquad y = 3t - \frac{1}{t}$$

$$\frac{dx}{dt} = 3 - \frac{1}{t^2} \qquad \frac{dy}{dt} = 3 + \frac{1}{t^2}$$

$$\frac{dt}{dx} = \frac{1}{3 - \frac{1}{t^2}}$$

And therefore…

$$\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx} = \left(3 + \frac{1}{t^2}\right) \times \left(\frac{1}{3 - \frac{1}{t^2}}\right) = \frac{3 + \frac{1}{t^2}}{3 - \frac{1}{t^2}} = \frac{3t^2 + 1}{3t^2 - 1}$$

Which rearranges to…

$$\frac{3 + \frac{1}{t^2}}{3 - \frac{1}{t^2}} \times \frac{t^2}{t^2} = \frac{3t^2 + 1}{3t^2 - 1}$$