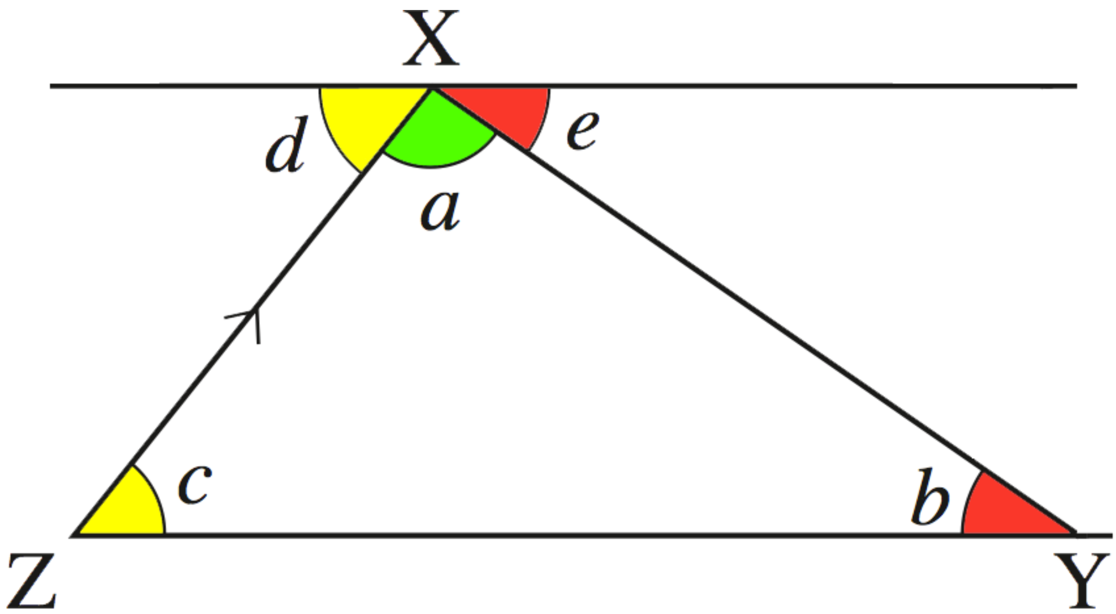
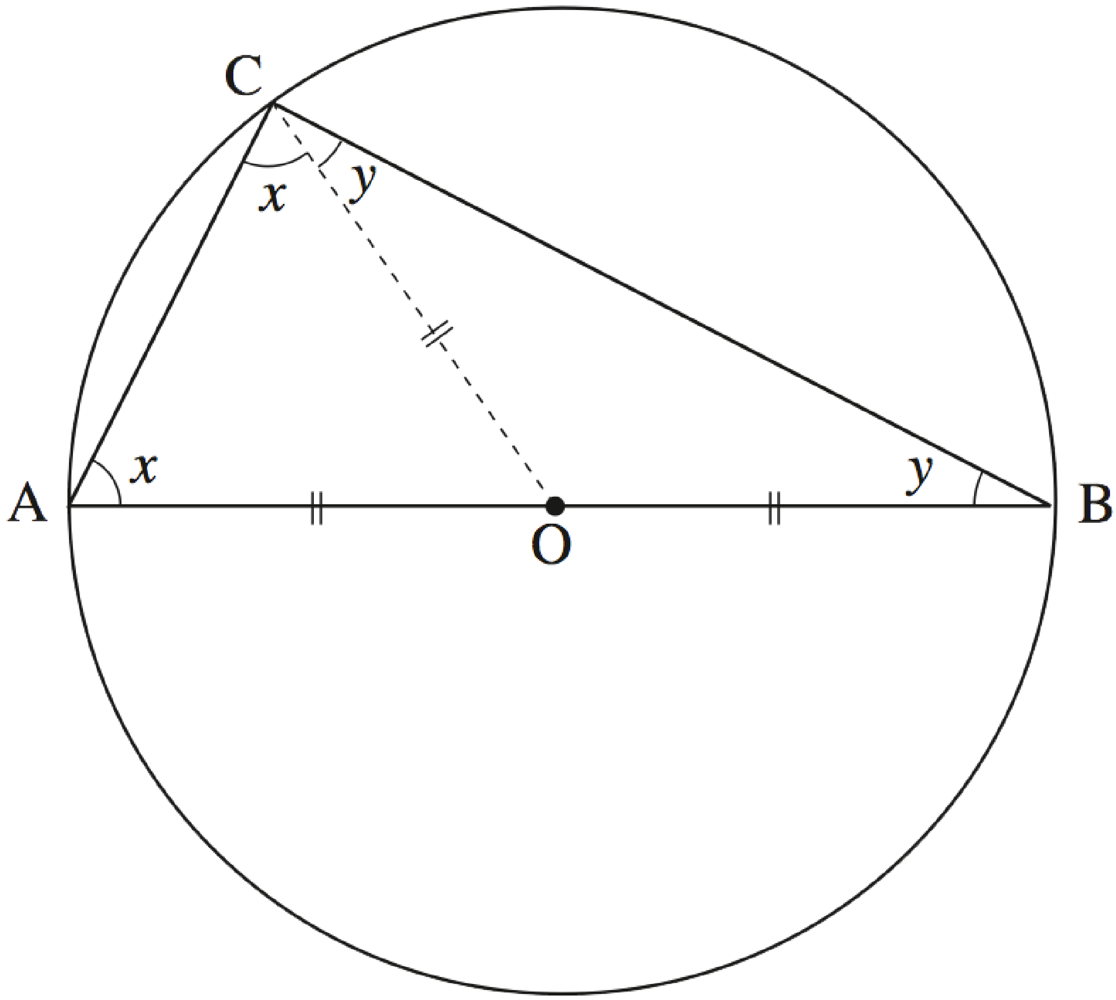
Geometric Proof Question 3



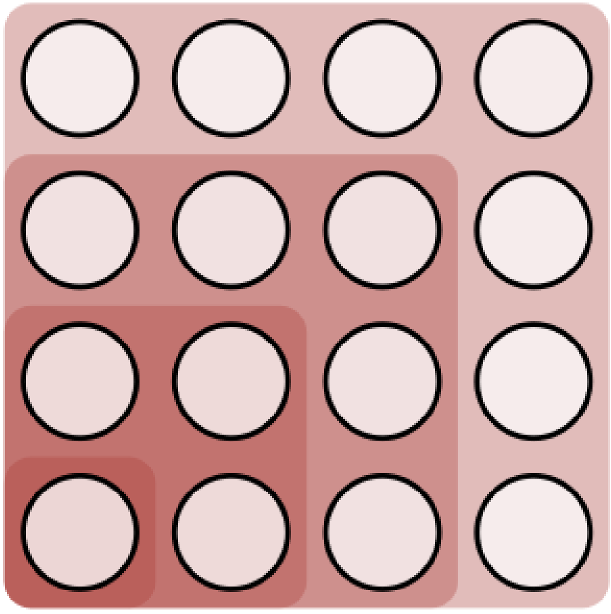
What does the above diagram prove?

Angles on a Semicircle



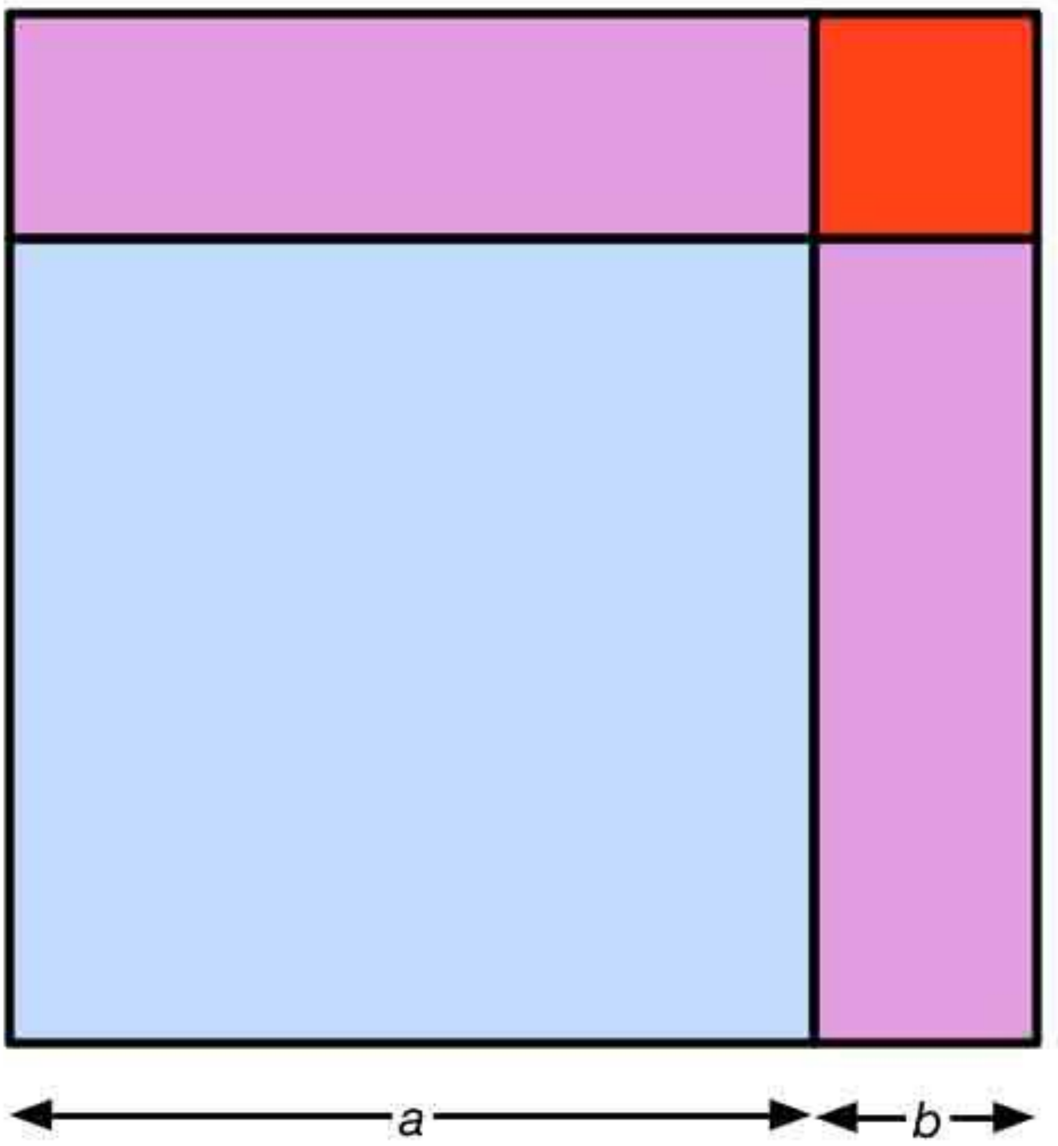
The angle subtended on the circumference from either end of a diameter is…

Geometric Proof Question 1



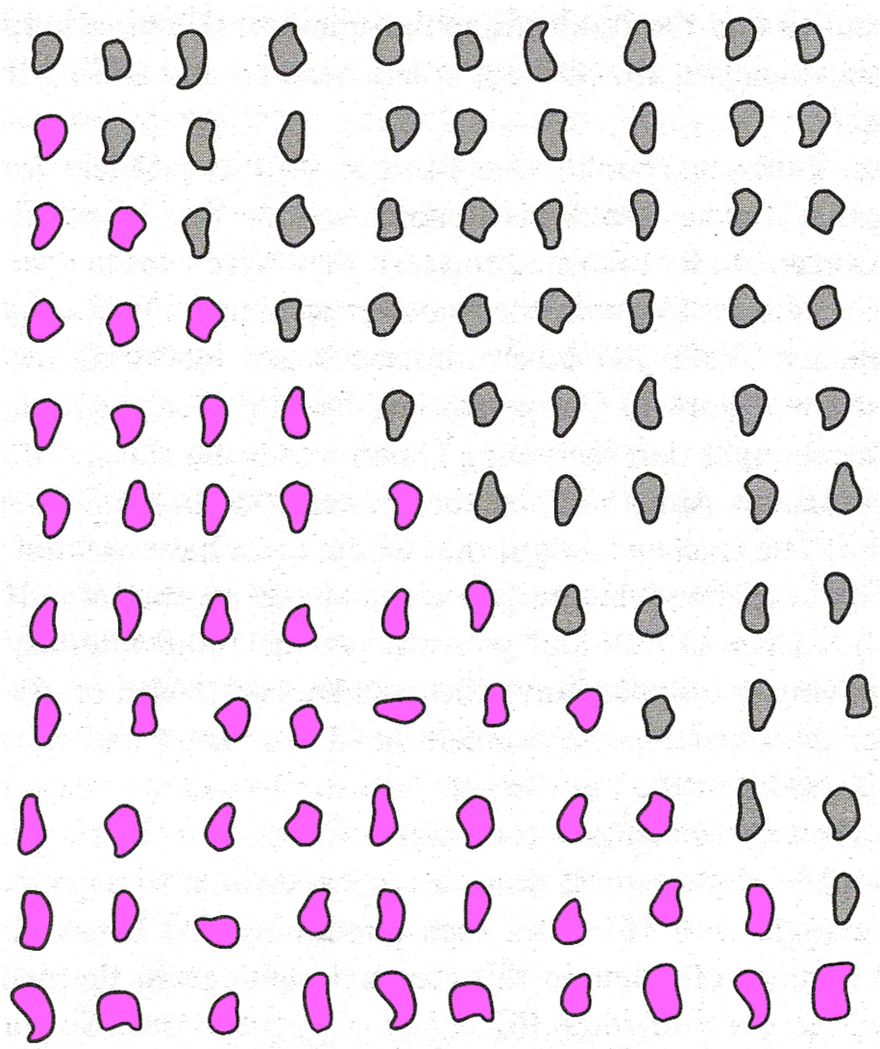
$$1 + 3 + 5 + \cdots + (2n - 1) = ?$$

Geometric Proof Question 2



What does the above diagram prove?

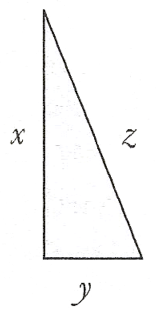
Geometric Proof Question 4

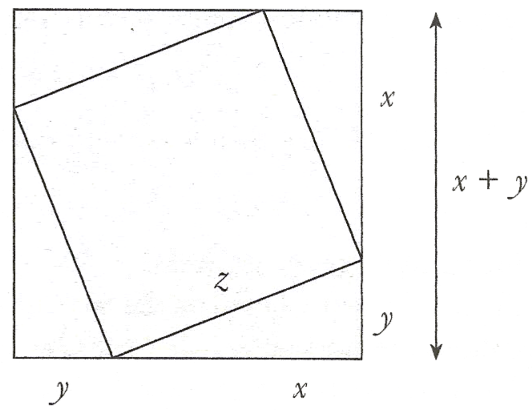


What does the above diagram prove?

Pythagoras

Appendix 1. The Proof of Pythagoras' Theorem





The aim of the proof is to show that Pythagoras' theorem is true for all right-angled triangles. The triangle shown above could be any right-angled triangle because its lengths are unspecified, and represented by the letters x, y and z.

Also above, four identical right-angled triangles are combined with one tilted square to build a large square. It is the area of this large square which is the key to the proof.

The area of the large square can be calculated in two ways.

Method 1: Measure the area of the large square as a whole. The length of each side is $x + y$. Therefore, the area of the large square $= (x + y)^2$.

Method 2: Measure the area of each element of the large square. The area of each triangle is $\frac{1}{2}xy$, i.e. $\frac{1}{2} \times$ base $\times$ height. The area of the tilted square is z^2. Therefore,

$$\begin{aligned}\text{area of large square} &= 4 \times (\text{area of each triangle}) + \text{area of tilted square}\\ &= 4\left(\tfrac{1}{2}xy\right) + z^2.\end{aligned}$$

Methods 1 and 2 give two different expressions. However, these two

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expressions must be equivalent because they represent the same [illegible]
Therefore,

$$\begin{aligned} \text{area from Method 1} &= \text{area from Method 2} \\ (x+y)^2 &= 4(\tfrac{1}{2}xy) + z^2. \end{aligned}$$

The brackets can be expanded and simplified. Therefore,

$$x^2 + y^2 + 2xy = 2xy + z^2.$$

The $2xy$ can be cancelled from both sides. So we have

$$x^2 + y^2 = z^2,$$

which is Pythagoras' theorem!

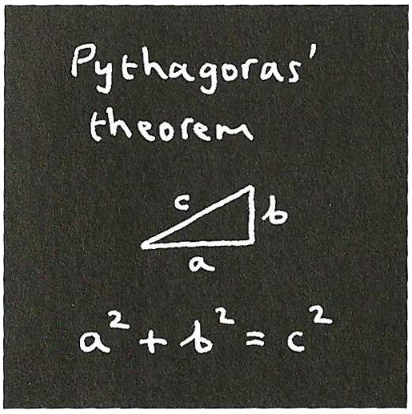
The argument is based on the fact that the area of the large square [illegible] be the same no matter what method is used to calculate it. We [illegible] logically derive two expressions for the same area, make them [illegible], and eventually the inevitable conclusion is that $x^2 + y^2 = z^2$, i.e. the [illegible] on the hypotenuse, z^2, is equal to the sum of the squares on the [illegible] sides, $x^2 + y^2$.

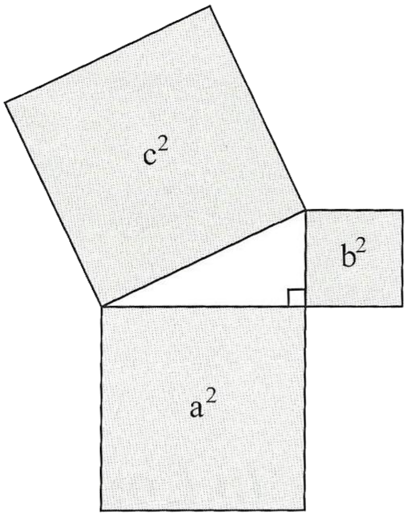
This argument holds true for all right-angled triangles. The [illegible] triangle in our argument are represented by x, y and z, and can [illegible] represent the sides of any right-angled triangle.

Pythagoras

Here, then, is an example of mathematics at its best, for Hobbes found the result so stunning that he couldn't quite believe it.

The result in question was, in fact, none other than *Pythagoras' theorem*: if a, b and c are the sides of a right-angled triangle, and c is the longest side, then $a^2 + b^2 = c^2$.





And Hobbes didn't just take somebody's word for this; he read a *proof*. It was this proof, as much as anything else, that made him

> . . . in love with Geometry.

So we too will now prove Pythagoras' theorem.

I can see, of course, that this might prompt the question: *why bother*? After all, the theorem has been around

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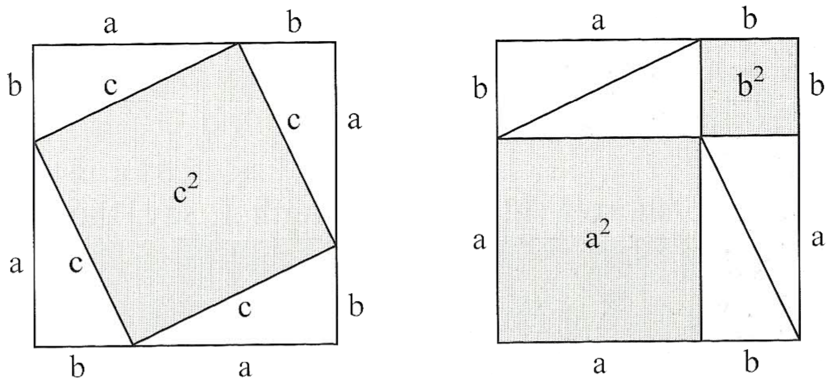
for over 2000 years. Everybody knows Pythagoras' theorem. Surely, if it were not true, if there were anything wrong with it, *somebody would have noticed by now*.

In mathematics, however, this kind of argument is virtually worthless.

And in any case, the following delightfully simple proof of Pythagoras' theorem is almost fun.



Take a square of side $a + b$ and place 4 copies of the original right-angled triangle within it, as shown. This leaves a square area c^2. Now think of the triangles as white tiles on a dark floor, and move three of them so that they occupy the new positions indicated. The floor area *not* occupied by triangles is now $a^2 + b^2$, yet must be the same as before.

So $a^2 + b^2 = c^2$.