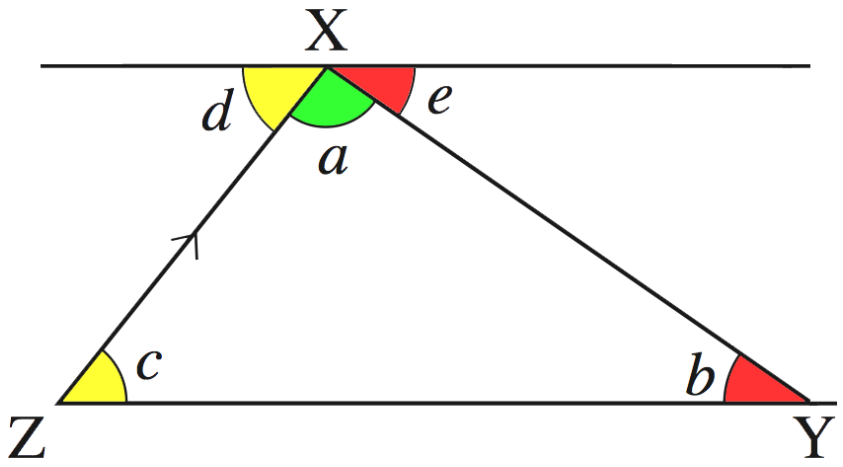
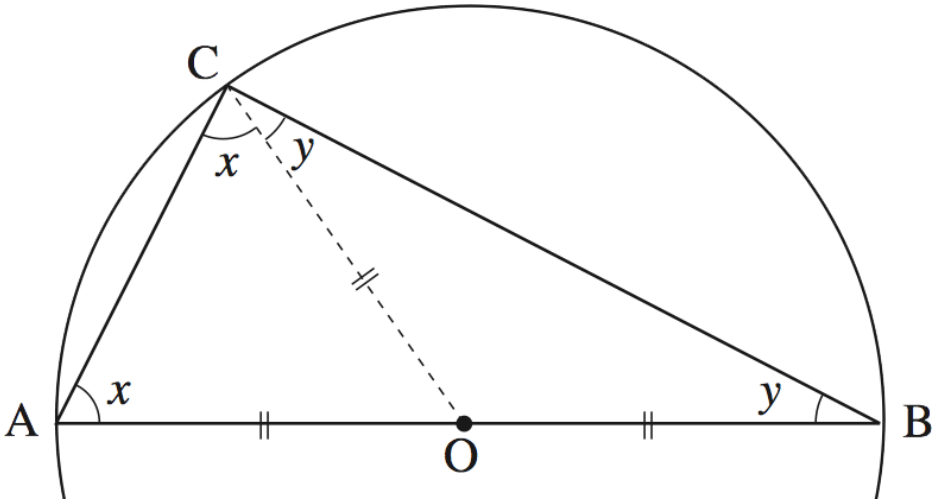
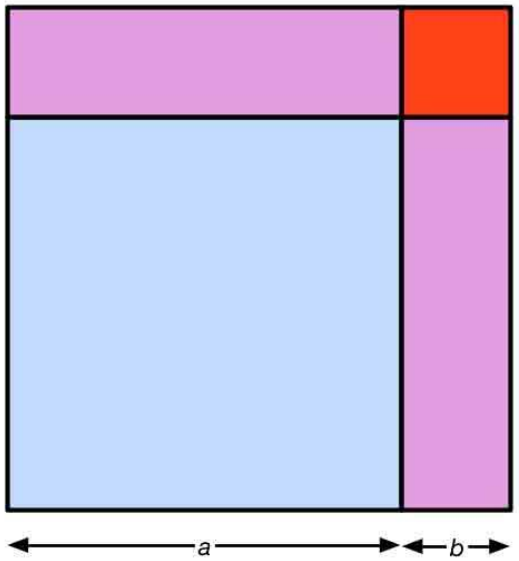
What does this prove, and why?



What does this prove, and why?



What does this prove, and why?



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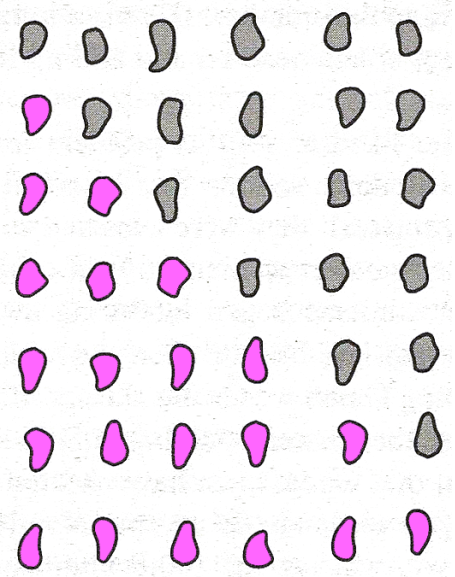

1. Prove that the result of multiplying two odd numbers together is always odd.

2. Prove that the sum of two consecutive square numbers is always odd.

3. Prove that $(x+y)^2+(x-y)^2 \equiv 2(x^2+y^2)$

4. Prove that $(x+y)^2-(x-y)^2 \equiv 4xy$

5. Prove that $x^3+y^3 \equiv (x+y)(x^2-xy+y^2)$

6. Prove that $x^3-y^3 \equiv (x-y)(x^2+xy+y^2)$

7. Prove that, for any prime number p greater than 3, p^2-1 is always a multiple of 24.