# Algebraic Proof

## Notes

- The letter used is usually *n* but the choice of letter itself does not matter. However, avoid using the letters *i*, *s*, or *o* as these look too similar to 1, 5 and 0.
- If *n* represents an integer, then even numbers are represented by $2n$.
- If *n* represents an integer, then odd numbers are represented by $2n + 1$.
- Consecutive integers are represented by $n$, $n + 1$, $n + 2$ etc.
- Alternate numbers are represented by $n$, $n + 2$, $n + 4$ etc.
- Consecutive squares are represented by $n^2$, $(n + 1)^2$, $(n + 2)^2$ etc.

## Questions involving algebra…

1. Prove that $(2n + 1)^2 - (2n + 1)$ is an even number for all positive integer values of *n*.
2. Prove that $(4n + 2)^2 - (2n + 2)^2$ is a multiple of 4 for all positive integers.
3. Prove that $(2n + 3)^2 - (2n - 3)^2$ is a multiple of 8 for all positive integers of
4. Prove that $(3n + 1)(n + 3) - n(3n + 7) = 3(n + 1)$
5. Prove that $\frac{1}{8}(4n + 1)(n + 8) - \frac{1}{8}n(4n + 1) = 4n + 1$
6. Prove that $\frac{6n^2+30n}{3n^2+15}$ is an even number.
7. Tom Hanks says that $7n - (2n + 3)(n + 2)$ is always negative. Is he correct? Explain your answer.

**More wordy questions…**

8. Prove that the sum of two consecutive integers is always odd.

9. Prove that the result of multiplying an odd number by itself together is always odd.

   Prove that the result of multiplying two consecutive odd numbers is always odd.

   Prove that the result of multiplying **any** two odd numbers together is always odd.

10. Prove that the sum of four consecutive whole numbers is always even.

11. The product of two consecutive positive integers is added to the larger of the two integers. Prove that the result is always a square number.

12. Prove that the sum of two consecutive square numbers is always odd.

13. Prove that the sum of two consecutive square numbers is twice the product of two consecutive numbers +1.

14. Prove that the difference between the squares of any two consecutive integers is equal to the sum of these two integers.

15. Prove that the sum of 4 consecutive square numbers is divisible by 4 remainder 2.

16. Give an example to show that the sum of four consecutive integers is not always divisible by 4.

**Other questions…**

17. Show that the difference between $14^{20}$ and $21^{2}$ is a multiple of 7.

18. Show that $3^{60} - 25$ is not prime.

19. Is $3^{444} + 4^{333}$ a multiple of $5$? Prove this.

20. Prove that, for any prime number p greater than 3, $p^2 - 1$ is always a multiple of 24