Scalar Products of Vectors

Magnitude of a vector:

$$\underline{\mathbf{a}} = \begin{pmatrix} x \\ y \end{pmatrix} \quad \Rightarrow \quad |\underline{\mathbf{a}}| = \sqrt{x^2 + y^2}$$

The size of $\underline{\mathbf{a}}+\underline{\mathbf{b}}$ will be the size of $|(\underline{\mathbf{a}}+\underline{\mathbf{b}})|$, not $|\underline{\mathbf{a}}|+|\underline{\mathbf{b}}|$. I.e.:

$$\underline{\mathbf{a}} = \begin{pmatrix} 3 \\ 2 \end{pmatrix}, \underline{\mathbf{b}} = \begin{pmatrix} 1 \\ -4 \end{pmatrix}$$

$$|(\underline{\mathbf{a}}+\underline{\mathbf{b}})| = \left|\begin{pmatrix} 3 \\ 2 \end{pmatrix} + \begin{pmatrix} 1 \\ -4 \end{pmatrix}\right| = \left|\begin{pmatrix} 4 \\ -2 \end{pmatrix}\right| = \sqrt{4^2 + -2^2} = \sqrt{20} = \sqrt{4}\sqrt{5} = 2\sqrt{5}$$

Scalar Product:

$$\underline{\mathbf{p}}\bullet\underline{\mathbf{q}} = |\underline{\mathbf{p}}|\,|\underline{\mathbf{q}}|\cos\theta$$

$$\underline{\mathbf{p}}\bullet\underline{\mathbf{p}} = |\underline{\mathbf{p}}|^2 \qquad \text{(obvious)}$$

$$\underline{\mathbf{p}}\bullet\underline{\mathbf{q}} = 0 \quad \Rightarrow \quad \text{p \& q perpendicular}$$

$$(\underline{\mathbf{p}}+\underline{\mathbf{q}})\bullet\underline{\mathbf{r}} = \underline{\mathbf{p}}\bullet\underline{\mathbf{r}} + \underline{\mathbf{q}}\bullet\underline{\mathbf{r}} \qquad \text{(obvious)}$$

Scalar products in Vector Form:

$$\underline{\mathbf{j}}\bullet\underline{\mathbf{k}} = \underline{\mathbf{k}}\bullet\underline{\mathbf{i}} = \underline{\mathbf{i}}\bullet\underline{\mathbf{j}} = 0 \quad \text{(perpendicular, obvious)}$$

$$\begin{pmatrix} l \\ m \\ n \end{pmatrix} \bullet \begin{pmatrix} u \\ v \\ w \end{pmatrix} = lu + mv + nw$$

$$lu + mv + nw = |\underline{\mathbf{p}}|\,|\underline{\mathbf{q}}|\cos\theta$$