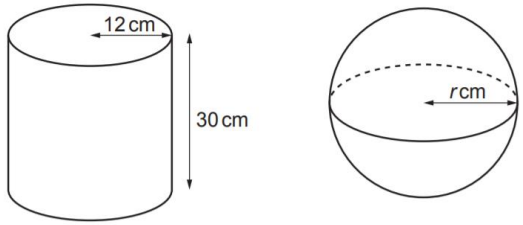
# Four Maths Questions at Different Levels – Question Set 8

## Easy higher tier GCSE




The cylinder has radius 12 cm and height 30 cm.
The cylinder and the sphere have the same volume.

Work out the radius $r$ cm of the sphere.

[The volume $V$ of a sphere with radius $r$ is $V = \frac{4}{3}\pi r^3$.]

OCR GCSE, Nov 2018, Paper 1

## Harder higher tier GCSE

**(a)** Prove that

$$(2x+1)(3x+2)+x(3x+5)+2$$

is a perfect square.

**(b)** Gemma says

The equation $(2x + 1)(3x + 2) + x(3x + 5) + 2 = -12$ has no solutions.

Explain Gemma's reasoning.

OCR GCSE, Nov 2018, Paper 2

## Something interesting

What is the biggest product that you can make using numbers which sum to 10?

(@colmanweb!)

## A Level

Given that $k \in \mathbb{Z}^+$

(a) show that $\int_k^{3k} \frac{2}{(3x-k)}\,dx$ is independent of $k$,

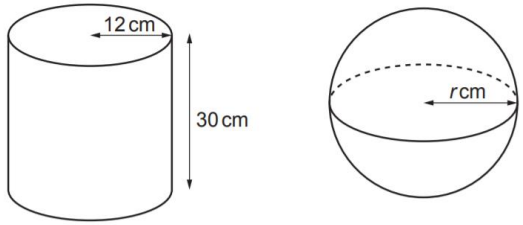
(b) show that $\int_k^{2k} \frac{2}{(2x-k)^2}\,dx$ is inversely proportional to $k$.

Edexcel, Paper 1, June 2018

Answers at www.colmanweb/easter2020

# Four Maths Questions at Different Levels – Answers Set 8

## Easy higher tier GCSE




The cylinder has radius 12 cm and height 30 cm.
The cylinder and the sphere have the same volume.

Work out the radius $r$ cm of the sphere.

$r \approx 14.8$

[The volume $V$ of a sphere with radius $r$ is $V = \frac{4}{3}\pi r^3$.]

OCR GCSE, Nov 2018, Paper 1

## Harder higher tier GCSE

**(a)** Prove that

$$(2x+1)(3x+2)+x(3x+5)+2$$

is a perfect square.

The perfect square is $(3x+2)^2$

**(b)** Gemma says

The equation $(2x+1)(3x+2)+x(3x+5)+2=-12$ has no solutions.

Explain Gemma's reasoning.

Reasoning involves $b^2 - 4ac < 0$

OCR GCSE, Nov 2018, Paper 2

## Something interesting

What is the biggest product that you can make using numbers which sum to 10?

If restricted to two numbers (and hence calculating the maximum area) then the answer is $5 \times 5 = 25$.

If restricted to three numbers (max volume) then we have $\left(\frac{10}{3}\right)^3 = \frac{1000}{27} \approx 37$.

Beyond this, $3^2 \times 2^2 = 36$ or $2^5 = 32$ are both interesting solutions which ultimately lead to $e^{\left(\frac{10}{e}\right)}$ and $\left(\frac{10}{e}\right)^e \approx 39.4$.

Graphs of these, such as $y = e^{\left(\frac{10}{e}\right)}$ and $y = \left(\frac{10}{e}\right)^e$ are both interesting, as is the three dimensional $z = x \times y \times (10 - x - y)$.

(@colmanweb!)

## A Level

Given that $k \in \mathbb{Z}^+$

(a) show that $\int_k^{3k} \frac{2}{(3x-k)}\,dx$ is independent of $k$,

(b) show that $\int_k^{2k} \frac{2}{(2x-k)^2}\,dx$ is inversely proportional to $k$.

Edexcel, Paper 1, June 2018