Core 1 Differentiation Questions

7 The volume, $V\ \mathrm{m}^3$, of water in a tank at time t seconds is given by

$$V = \tfrac{1}{3}t^6 - 2t^4 + 3t^2, \quad \text{for } t \geqslant 0$$

(a) Find:

(i) $\dfrac{\mathrm{d}V}{\mathrm{d}t}$; *(3 marks)*

(ii) $\dfrac{\mathrm{d}^2V}{\mathrm{d}t^2}$. *(2 marks)*

(b) Find the rate of change of the volume of water in the tank, in $\mathrm{m}^3\,\mathrm{s}^{-1}$, when $t = 2$. *(2 marks)*

(c) (i) Verify that V has a stationary value when $t = 1$. *(2 marks)*

(ii) Determine whether this is a maximum or minimum value. *(2 marks)*

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3 A curve has equation $y = 7 - 2x^5$.

(a) Find $\dfrac{\mathrm{d}y}{\mathrm{d}x}$. *(2 marks)*

(b) Find an equation for the tangent to the curve at the point where $x = 1$. *(3 marks)*

(c) Determine whether y is increasing or decreasing when $x = -2$. *(2 marks)*

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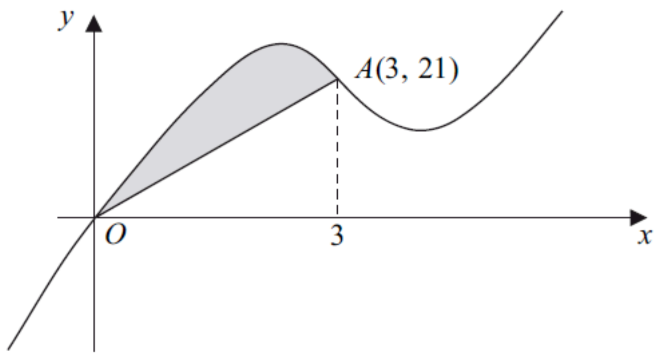
5 The curve with equation $y = x^3 - 10x^2 + 28x$ is sketched below.



The curve crosses the x-axis at the origin O and the point $A(3, 21)$ lies on the curve.

(a) (i) Find $\dfrac{dy}{dx}$. *(3 marks)*

(ii) Hence show that the curve has a stationary point when $x = 2$ and find the x-coordinate of the other stationary point. *(4 marks)*

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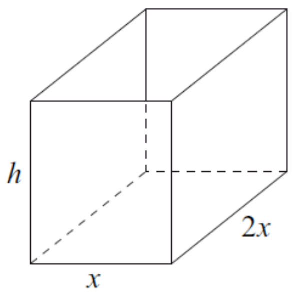
5 The diagram shows an **open-topped** water tank with a horizontal rectangular base and four vertical faces. The base has width x metres and length $2x$ metres, and the height of the tank is h metres.



The combined internal surface area of the base and four vertical faces is $54\,\text{m}^2$.

(a) (i) Show that $x^2 + 3xh = 27$. *(2 marks)*

(ii) Hence express h in terms of x. *(1 mark)*

(iii) Hence show that the volume of water, V m^3, that the tank can hold when full is given by

$$V = 18x - \frac{2x^3}{3}$$ *(1 mark)*

(b) (i) Find $\frac{dV}{dx}$. *(2 marks)*

(ii) Verify that V has a stationary value when $x = 3$. *(2 marks)*

(c) Find $\frac{d^2V}{dx^2}$ and hence determine whether V has a maximum value or a minimum value when $x = 3$. *(2 marks)*

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4 A model helicopter takes off from a point O at time $t = 0$ and moves vertically so that its height, y cm, above O after time t seconds is given by

$$y = \frac{1}{4}t^4 - 26t^2 + 96t, \qquad 0 \leqslant t \leqslant 4$$

(a) Find:

(i) $\frac{dy}{dt}$; *(3 marks)*

(ii) $\frac{d^2y}{dt^2}$. *(2 marks)*

(b) Verify that y has a stationary value when $t = 2$ and determine whether this stationary value is a maximum value or a minimum value. *(4 marks)*

(c) Find the rate of change of y with respect to t when $t = 1$. *(2 marks)*

(d) Determine whether the height of the helicopter above O is increasing or decreasing at the instant when $t = 3$. *(2 marks)*
