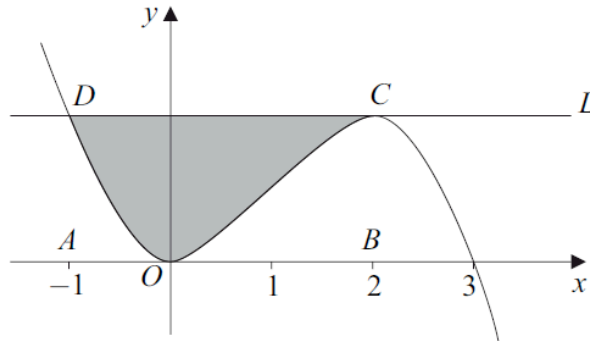# Core 1 Integration Questions

**8**   The diagram shows the curve with equation $y = 3x^2 - x^3$ and the line L.

The points A and B have coordinates $(-1, 0)$ and $(2, 0)$ respectively. The curve touches the x-axis at the origin O and crosses the x-axis at the point $(3, 0)$. The line L cuts the curve at the point D where $x = -1$ and touches the curve at C where $x = 2$.

   (a)   Find the area of the rectangle $ABCD$.   *(2 marks)*

(b)   (i)   Find $\displaystyle\int (3x^2 - x^3)\,dx$.   *(3 marks)*

        (ii)   Hence find the area of the shaded region bounded by the curve and the line L.
                                                                                                                                        (4 marks)

(c)   For the curve above with equation $y = 3x^2 - x^3$:

        (i)   find $\dfrac{dy}{dx}$;   *(2 marks)*

        (ii)   hence find an equation of the tangent at the point on the curve where $x = 1$;
                                                                                                                                        (3 marks)

        (iii)   show that y is decreasing when $x^2 - 2x > 0$.   *(2 marks)*

(d)   Solve the inequality $x^2 - 2x > 0$.   *(2 marks)*
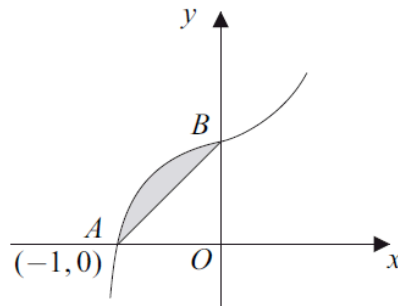
5 The curve with equation $y = x^3 - 10x^2 + 28x$ is sketched below.
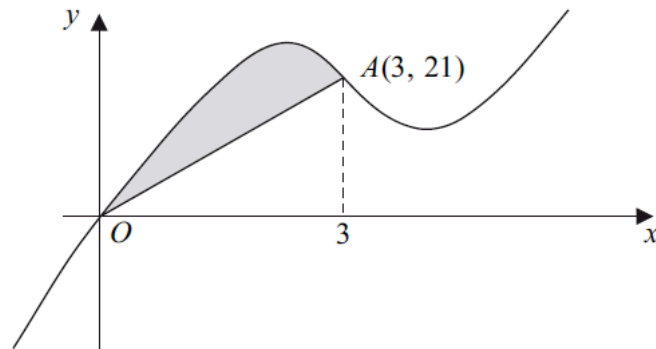
The curve crosses the x-axis at the origin O and the point $A(3, 21)$ lies on the curve.

(b) (i) Find $\int (x^3 - 10x^2 + 28x)\, dx$. (3 marks)

   (ii) Hence show that $\int_0^3 (x^3 - 10x^2 + 28x)\, dx = 56\frac{1}{4}$. (2 marks)

   (iii) Hence determine the area of the shaded region bounded by the curve and the line OA. (3 marks)

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6 The curve with equation $y = 3x^5 + 2x + 5$ is sketched below.

The curve cuts the x-axis at the point $A(-1, 0)$ and cuts the y-axis at the point B.

(a) (i) State the coordinates of the point B and hence find the area of the triangle AOB, where O is the origin. (3 marks)

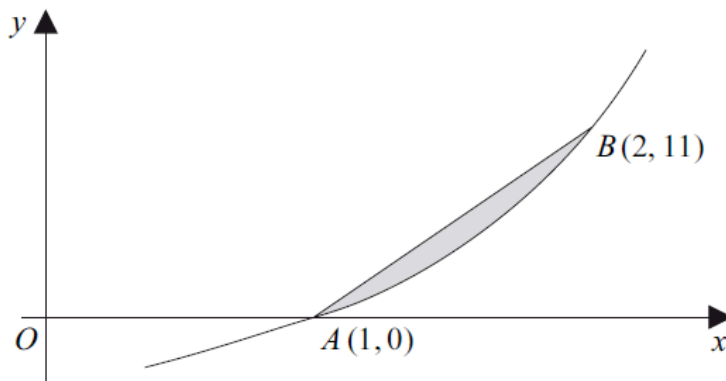   (ii) Find $\int (3x^5 + 2x + 5)\, dx$. (3 marks)

(iii) Hence find the area of the shaded region bounded by the curve and the line AB.

(4 marks)

(b) (i) Find the gradient of the curve with equation $y = 3x^5 + 2x + 5$ at the point $A(-1, 0)$.

(3 marks)

(ii) Hence find an equation of the tangent to the curve at the point A.

(1 mark)

(b) The curve with equation $y = x^3 + 4x - 5$ is sketched below.

The curve cuts the x-axis at the point $A(1, 0)$ and the point $B(2, 11)$ lies on the curve.

(i) Find $\int (x^3 + 4x - 5)\,dx$.

(3 marks)

(ii) Hence find the area of the shaded region bounded by the curve and the line AB.

(4 marks)