Core 1 Polynomials Questions

6 The polynomial p(x) is given by

$$p(x) = x^3 + x^2 - 10x + 8$$

(a) (i) Using the factor theorem, show that $x - 2$ is a factor of p(x). *(2 marks)*

(ii) Hence express p(x) as the product of three linear factors. *(3 marks)*

(b) Sketch the curve with equation $y = x^3 + x^2 - 10x + 8$, showing the coordinates of the points where the curve cuts the axes.

(You are not required to calculate the coordinates of the stationary points.) *(4 marks)*

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6 The polynomial p(x) is given by p(x) = $x^3 - 4x^2 + 3x$.

(a) Use the Factor Theorem to show that $x - 3$ is a factor of p(x). *(2 marks)*

(b) Express p(x) as the product of three linear factors. *(2 marks)*

(c) (i) Use the Remainder Theorem to find the remainder, r, when p(x) is divided by $x - 2$. *(2 marks)*

(ii) Using algebraic division, or otherwise, express p(x) in the form

$$(x - 2)(x^2 + ax + b) + r$$

where a, b and r are constants. *(4 marks)*

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1 The polynomial p(x) is given by

$$p(x) = x^3 - 4x^2 - 7x + k$$

where k is a constant.

(a) (i) Given that $x + 2$ is a factor of p(x), show that $k = 10$. *(2 marks)*

(ii) Express p(x) as the product of three linear factors. *(3 marks)*

(b) Use the Remainder Theorem to find the remainder when p(x) is divided by $x - 3$. *(2 marks)*

(c) Sketch the curve with equation $y = x^3 - 4x^2 - 7x + 10$, indicating the values where the curve crosses the x-axis and the y-axis. (You are **not** required to find the coordinates of the stationary points.) *(4 marks)*

6 (a) The polynomial $f(x)$ is given by $f(x) = x^3 + 4x - 5$.

    (i) Use the Factor Theorem to show that $x - 1$ is a factor of $f(x)$. *(2 marks)*

    (ii) Express $f(x)$ in the form $(x - 1)(x^2 + px + q)$, where p and q are integers. *(2 marks)*

    (iii) Hence show that the equation $f(x) = 0$ has exactly one real root and state its value. *(3 marks)*