Core 2 Differentiation Answers

	Solution	Marks	Total	Comments
1	$y'(x)=16-x^{-2}$	M1 A1		One term correct Both correct
	$y'(x)=16-\frac{1}{x^2}$	B1		$x^{-2}=\frac{1}{x^2}$ OE PI
	$y'(x)=0 \Rightarrow 16x^2=1$;	M1		c's $y'(x)$=0 and one relevant further step
	$\Rightarrow x=\pm\frac{1}{4}$	A1	5	Both answers required.
	Total		**5**	

8(a)	$\frac{dy}{dx}=\frac{3}{2}x^{\frac{1}{2}}-3$	M1 A1	2	One term correct Both correct
(b)(i)	When $x=0$, $\frac{dy}{dx}=-3$	B1F√		Ft provided answer < 0.
	Eqn of tangent at O is $y=-3x$	B1F√	2	OE Ft on $y'(0)$
(ii)	At (9,0) $\frac{dy}{dx}=\frac{3}{2}(9)^{\frac{1}{2}}-3$	M1		Attempt to find $y'(9)$
	Eqn tangent at A is $y-0=y'(9)[x-9]$	m1		OE
	$\Rightarrow y=\frac{3}{2}(x-9) \Rightarrow 2y=3x-27$	A1	3	CSO. AG
(iii)	Eliminating $y \Rightarrow -6x=3x-27$	M1		OE method to one variable (eg $2y=-y-27$)
	$9x=27 \Rightarrow x=3$	A1F		[A1F for each coordinate; only ft on $y=kx$ tangent in (b)(i) for $k<0$]
	When $x=3$, $y=-9$. $\{P(3,-9)\}$	A1F	3	
(c)	$\int\left(x^{\frac{3}{2}}-3x\right)dx=\frac{2}{5}x^{\frac{5}{2}}-\frac{3x^2}{2}$ $(+c)$	M1 A2,1,0	3	One power correct Condone absence of "+c" and unsimplified forms
(d)	$\int_0^9\left(x^{\frac{3}{2}}-3x\right)dx=$	B1		PI
	$=\frac{2}{5}\times 9^{\frac{5}{2}}-\frac{3}{2}\times 9^2-0$ $=-24.3$	M1		Correct use of limits following integration
	Area of triangle $OPA=\frac{1}{2}\times 9\times\lvert y_P\rvert$	M1		
	Sh.Area $=\frac{1}{2}\times 9\times\lvert y_P\rvert-\left\lvert\int_0^9\left(x^{\frac{3}{2}}-3x\right)dx\right\rvert$	M1		OE
	$=40.5-24.3=16.2$	A1	5	
	Total		**18**	

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7(a)(i)	When $x = 4$, $\frac{dy}{dx} = 3(2) + \frac{16}{16} - 7 = 0$	B1	1	AG Be convinced
(ii)	$\frac{16}{x^2} = 16x^{-2}$	B1	1	Accept $k = -2$
(iii)	$\frac{d^2y}{dx^2} = 3 \times \frac{1}{2}x^{-\frac{1}{2}} + 16 \times (-2)x^{-3} - 0$	M1		A power decreased by 1
	$\frac{d^2y}{dx^2} = \frac{3}{2}x^{-\frac{1}{2}}$; $-32x^{-3}$	A1; A1√	3	candidate's negative integer k [−1 for >2 term(s)]
(iv)	When $x = 4$, $\frac{d^2y}{dx^2} = \frac{3}{4} - \frac{32}{64} = \frac{1}{4}$	M1		Attempt to find $y''(4)$ reaching as far as two simplified terms
	Minimum since $y''(4) > 0$	E1√	2	candidate's sign of $y''(4)$
	[**Alternative:** Finds the sign of $y'(x)$ either side of the point where x=4, need evidence rather than just a statement: (M1) Correct ft conclusion with valid reason E1√] [In both, condone absent statement $y'(4)=0$]			
(b)(i)	At $P(1,8)$, $\frac{dy}{dx} = 3(1)^{\frac{1}{2}} + \frac{16}{1^2} - 7$ =12	B1	1	AG Be convinced
(ii)	Gradient of normal = $-\frac{1}{12}$	M1		Use of or stating $m \times m' = -1$
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	Equation of normal is $y - 8 = m[x - 1]$	M1		Can be awarded even if m=12
	$y - 8 = -\frac{1}{12}(x-1) \Rightarrow 12y - 96 = -x + 1$ $\Rightarrow 12y + x = 97$	A1	3	Any correct form of the equation
(c)(i)	$\int 3x^{\frac{1}{2}} + \frac{16}{x^2} - 7\,dx =$			
	…….. $= 3\frac{x^{\frac{3}{2}}}{\frac{3}{2}} + 16\frac{x^{-1}}{-1} - 7x + c$	M1 A2,1,0 √	3	One power correct. A1 if 2 of 3 terms correct candidate's negative integer k Condone absence of "+ c"
(ii)	$y = 2x^{\frac{3}{2}} - 16x^{-1} - 7x + c$ (*)	B1√		y = candidate's answer to (c)(i) with tidied coefficients and with '+c'. ('y =' PI by next line)
	When $x = 1, y = 8 \Rightarrow 8 = 2–16–7+c$	M1		Substitute. (1,8) in attempt to find constant of integration
	$y = 2x^{\frac{3}{2}} - 16x^{-1} - 7x + 29$	A1	3	Accept c = 29 after (*), including y =, stated
	Total		**17**	

Q	Solution	Marks	Total	Comments
6(a)(i)	$y = x + 1 + 4x^{-2} \Rightarrow \frac{dy}{dx} = 1 - 8x^{-3}$	M1 A2,1,0	3	Power $p \to p-1$ (A1 if $1 + ax^n$ with $a = -8$ or $n = -3$)
(ii)	$1 - 8x^{-3} = 0$	M1		Puts c's $\frac{dy}{dx} = 0$
	$x^3 = 8$	m1		Using $x^{-k} = \frac{1}{x^k}$ to reach $x^a = b$, $a>0$ or correct use of logs.
	$x = 2$ When $x = 2$, $y = 4$	A1 A1ft	4	
(iii)	At (1, 6), $\frac{dy}{dx} = 1 - 8 = -7$	M1		Attempt to find $y'(1)$
	Gradient of normal = $\frac{1}{7}$	M1		Use of or stating $m \times m' = -1$
	Equation of normal is $y - 6 = m[x-1]$	M 1		m numerical
	$y - 6 = \frac{1}{7}(x-1)$ $\{\frac{y-6}{x-1} = \frac{1}{7};\ 7y = x + 41\}$	A1ft	4	OE ft on c's answer for (a)(i) provided at least A1 given in (a)(i) and previous 3M marks awarded
(b)(i)	$\int x\left(+1 + \frac{4}{x^2}\right)dx =$ $\ldots\ldots = \frac{x^2}{2} + x - 4x^{-1}$ $\{+c\}$	M1 A2,1,0	3	One of three terms correct. For A2 need all <u>three</u> terms as printed or better (A1 if 2 of 3 terms correct)
(ii)	{Area=} $\int_1^4 x + 1 + \frac{4}{x^2} dx =$ $\left[\frac{x^2}{2} + x - \frac{4}{x}\right]_1^4 = (8 + 4 - 1) - \left(\frac{1}{2} + 1 - 4\right)$	M1		Dealing correctly with limits; F(4)–F(1) (must have integrated)
	$= 13.5$	A1	2	
	Total		**16**	

5(a)	$y_P = 4$	B1	1	
(b)	$y = 1 + \frac{2}{x} + \frac{2}{x} + \frac{4}{x^2}$ $y = 1 + 4x^{-1} + 4x^{-2}$	B2,1,0	2	(B1 if only one error in the expansion) For B2 the last line of the candidate's solution must be correct
(c)	$\frac{\mathrm{d}y}{\mathrm{d}x} = -4x^{-2} - 8x^{-3}$	M1 A1ft A1	 3	Index reduced by 1 after differentiating x to a negative power At least 1 term in x correct ft on expn CSO Full correct solution. ACF
(d)	When $x = 2$, $\frac{\mathrm{d}y}{\mathrm{d}x} = -4 \times 2^{-2} - 8 \times 2^{-3}$ Gradient $= -1 \ -1 = -2$	M1 A1	 2	Attempt to find $y'(2)$. AG (be convinced-no errors seen)
(e)	$-2 \times m' = -1$ $y - 4 = m(x - 2)$ $y - 4 = \frac{1}{2}(x - 2)$ $x - 2y + 6 = 0$	M1 M1 A1ft A1	 4	$m_1 \times m_2 = -1$ OE stated or used. PI C's y_P from part (a) if not recovered; m must be numerical. Ft on candidate's y_P from part (a) if not recovered. CAO Must be this or $0 = x - 2y + 6$
	Total		**12**	