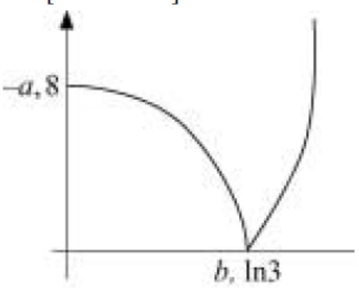
Core 3 Exponentials & Logarithms Answers

5(a)	$a=-8$	B1		
	$e^{2x}-9=0$	M1		
	$e^{2x}=9$			
	$2x=\ln 9$			
	$x=\ln 3$	A1	3	AG Condone verification
(b)	$\left(e^{2x}-9\right)^2=e^{4x}-18e^{2x}+81$	B1	1	AG
(c)	$V=\pi\int y^2\,(\mathrm{d}x)$	B1		
	$=(\pi)\int e^{4x}-18e^{2x}+81\ \mathrm{d}x$	M1		
	$=(\pi)\left[\frac{e^{4x}}{4}-9e^{2x}+81x\right]_0^{\ln 3}$	M1 A1		1^{ST} or 2^{nd} term correct All correct
	$=(\pi)\left[\frac{e^{4x}}{4}-9e^{2x}+81x\right]_0^{\ln 3}$	M1 A1		1^{ST} or 2^{nd} term correct All correct
	$=(\pi)\left[\left(\frac{e^{\ln 81}}{4}-9e^{\ln 9}+81\ln 3\right)-\left(\frac{1}{4}-9\right)\right]$	m1		Attempt at limits with ln3
	$=\pi[81\ln 3-52]$	A1	6	
(d)		M1		Modulus graph
		A1F	2	All correct
	−a			
	Total		**12**	

9(a)	$y=x^{-2}\ln x$			
	$\frac{\mathrm{d}y}{\mathrm{d}x}=x^{-2}\frac{1}{x}-2x^{-3}\ln x$	M1 A1 A1		Use of product or quotient each term
	$=\frac{1-2\ln x}{x^3}$	A1	4	Convincing argument $x^{-2}\times\frac{1}{x}=x^{-3}$ AG

(c)(i)	At A, $\frac{dy}{dx}=0$			
	$1-2\ln x=0$			
	$\ln x=\frac{1}{2}$	M1		Attempt at $\ln x = k$
	$x=e^{\frac{1}{2}}$	A1	2	

5(a)	$y = e^{2x} - 10e^{x} + 12x$			
(i)	$\frac{dy}{dx} = 2e^{2x} - 10e^{x} + 12$	B1		$2e^{2x}$
		B1	2	remaining terms correct, no extras
(ii)	$\frac{d^2y}{dx^2} = 4e^{2x} - 10e^{x}$	B1F	1	ft 1 slip
(b)(i)	$2e^{2x} - 10e^{x} + 12 = 0$ $e^{2x} - 5e^{x} + 6 = 0$	B1	1	AG (be convinced)
(ii)	$z^2 - 5z + 6 = 0$	M1		use of $z = e^{x}$ oe
	$z = 2, 3$ $z = 2,\ e^{x} = 2$ $x = \ln 2$	M1		finding $e^{x} =$ their 2,3
	$z = 3,\ e^{x} = 3$ $x = \ln 3$	A1	3	all correct AG SC: verification $\ln 2$ (B1) $\ln 3$ (B1)
(iii)	$x = \ln 2$: $y = e^{2\ln 2} - 10e^{\ln 2} + 12\ln 2$ or $2^2 - 10\times 2 + 12\ln 2$ $= 4 - 20 + 12\ln 2$	M1		either substitution of their $x = \ln 2$ $(e^{x} = 2)$ or their $x = \ln 3$ $(e^{x} = 3)$
	$= -16 + 12\ln 2$	A1		
	$x = \ln 3$: $y = e^{2\ln 3} - 10e^{\ln 3} + 12\ln 3$ $= 9 - 30 + 12\ln 3$ $= -21 + 12\ln 3$	A1	3	
(iv)	$x = \ln 2$: $\frac{d^2y}{dx^2} = 4e^{2\ln 2} - 10e^{\ln 2}$	M1		use of; in either of their $e^{x} = 2,3$ into their $\frac{d^2y}{dx^2}$

	$= 16 - 20 = -4$			
	$\therefore$ maximum	A1		CSO
	$x = \ln 3$:			
	$\frac{d^2y}{dx^2} = 4e^{2\ln 3} - 10e^{\ln 3}$			
	$= 36 - 30 = 6$			
	$\therefore$ minimum	A1	3	CSO
	Total		**13**	

(b)(i)	$y = x\ln x$			
	$\frac{dy}{dx} = x \times \frac{1}{x} + \ln x$	M1		use of product rule (only differentiating, 2 terms with + sign)
	$= \ln x + 1$	A1	2	
(ii)	$\int(\ln x + 1)dx = x\ln x$	M1		OE; attempt at parts with $u = \ln x$
	$\int \ln x \, dx = x\ln x - x(+c)$	A1	2	
(iii)	$\int_1^5 \ln x \, dx = [x\ln x - x]_1^5$			
	$= (5\ln 5 - 5) - (1\ln 1 - 1)$	M1		correct substitution of limits into their (ii) provided $\ln x$ is involved
	$5\ln 5 - 4$	A1	2	ISW
	Total		**9**	

(b)(i)	$x = 2y^3 + \ln y$			
	$\frac{dx}{dy} = 6y^2 + \frac{1}{y}$	B1	1	
(ii)	At $(2,1)$			
	$\frac{dx}{dy} = 6 + 1 = 7$	M1		
	$\frac{dy}{dx} = \frac{1}{7}$	A1√		May be implied
	$(y-1) = \frac{1}{7}(x-2)$	A1	3	OE

9(a)(i)	$\int\left(4-e^{2x}\right)dx$			
	$=4x-\frac{1}{2}e^{2x}\ (+c)$	B1		$4x$
		B1	2	$-\frac{1}{2}e^{2x}$
(ii)	$\int_0^{\ln 2}=\left[4x-\frac{1}{2}e^{2x}\right]_0^{\ln 2}$			
	$=\left[4\ln 2-\frac{1}{2}e^{2\ln 2}\right]-\left[(0)-\frac{1}{2}\left(e^0\right)\right]$	M1		Substitute both ln 2 and 0 correctly into an integrated expression
	$=4\ln 2-2+\frac{1}{2}$			Convincing
	$=4\ln 2-\frac{3}{2}$	A1	2	**AG**
(b)(i)	$x=0$			
	$y=4-1=\underline{\underline{3}}$	B1	1	
(ii)	At B, $y=0$			
	$4-e^{2x}=0$	M1		Or reverse argument
	$e^{2x}=4$			
	$x=\ln 2$	A1	2	**AG**
(c)	$\frac{dy}{dx}=-2e^{2x}$	B1		
	$x=\ln 2$, Gradient $=-2e^{2\ln 2}$	M1		$x=\ln 2$ into ke^{2x}
	$=-8$			
	Gradient normal $=\frac{1}{8}=\frac{1}{2e^{2\ln 2}}$	A1		OE
	Equation $y=\frac{1}{8}x-\frac{1}{8}\ln 2$	A1	4	OE
(d)	When $x=0$			
	$y=-\frac{1}{8}\ln 2$	M1		Attempt to integrate their line and substitute $x=0, \ln 2$
	Area Δ $=\frac{1}{16}(\ln 2)^2$ condone – ve sign	A1√		$\frac{1}{2}(\text{their } y)\times\ln 2$
	$=0.03$			
	Total area $=4\ln 2-\frac{3}{2}+\frac{1}{16}(\ln 2)^2=1.30$ AWRT	A1	3	CSO
	Total		**14**	

1(a)	$y=\ln x$ $\frac{dy}{dx}=\frac{1}{x}$	B1	1	penalise + c once on 1(a) or 2(a)
(b)	$y=(x+1)\ln x$ $\frac{dy}{dx}=(x+1)\times\frac{1}{x}+\ln x$	M1 A1	2	product rule
(c)	$y=(x+1)\ln x$ $\frac{dy}{dx}=\frac{1}{x}+1+\ln x$			
	$x=1$: $\frac{dy}{dx}=1+1=2$	M1		substitute $x = 1$ into their $\frac{dy}{dx}$
	Grad normal $=-\frac{1}{2}$	M1 A1		use of $m_1m_2=-1$ CSO
	$y=-\frac{1}{2}(x-1)$	A1	4	OE
	Total		**7**	

7(a)(i)	$y=(x^2-3)e^x$ $\frac{dy}{dx}=(x^2-3)e^x+2xe^x$	M1 A1	2	product rule
(ii)	$\frac{d^2y}{dx^2}=(x^2-3)e^x+2xe^x+2xe^x+2e^x$	M1 A1	2	product rule from their $\frac{dy}{dx}$
(b)(i)	$\frac{dy}{dx}=0$ $\Rightarrow e^x(x^2+2x-3)=0$	M1		$e^x f(x)=0$ from $\frac{dy}{dx}=0$
	$e^x(x+3)(x-1)=0$	m1		attempt at factorising or use of formula
	$\therefore x=-3, 1$	A1 A1	4	first correct solution second correct solution, and no others SC No working shown: $x=-3$ B2, $x=1$ B2
(ii)	$x=-3$ $y''=-4e^x$ max (-0.2)	M1		Condone slip
	$x=1$ $y''=4e^x$ min (10.9)	A1	2	
	Total		**10**	