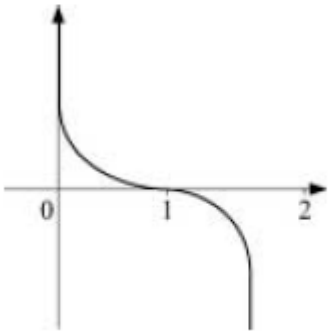
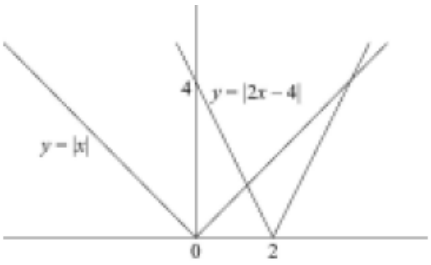
Core 3 Functions Answers

7(a)	$\left(1, \frac{\pi}{2}\right)$ OE in decimals	B1		Or for -1 and 1
	$\left(-1, -\frac{\pi}{2}\right)$	B1	2	
(b)		M1		Translation in + ve x direction
		M1		Correct shape
		A1	3	Correct Graph Through (1,0) touching $y-$axis

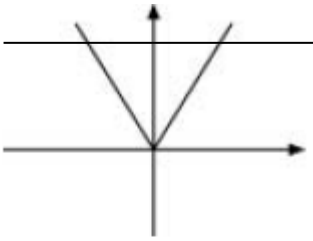
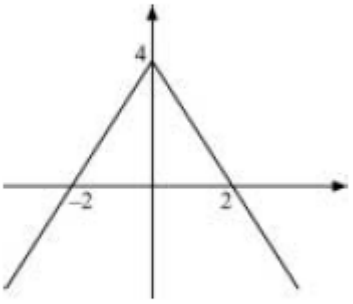
8(a)	(Range of f) $\geqslant 0$	B1	1	
(b)(i)	$\text{fg}(x) = \frac{1}{(x+2)^2}$	B1	1	OE Maybe in part (ii)
(ii)	$\frac{1}{(x+2)^2} = 4$			
	$(x+2)^2 = \frac{1}{4}$	M1		Or $4(x+2)^2 = 1$
	$x+2 = (\pm)\frac{1}{2}$	M1		$(2x+5)(2x+3) = 0$
	$x = -\frac{5}{2}, -\frac{3}{2}$	A1 A1	4	
(c)(i)	Not one to one	E1	1	OE
(ii)	$x = \frac{1}{y+2}$	M1		$x \Leftrightarrow y$
	$y+2 = \frac{1}{x}$	M1		Attempt to isolate
	$y = \frac{1}{x} - 2 \quad \left(\frac{1-2x}{x}\right)$	A1	3	
	Total		**10**	

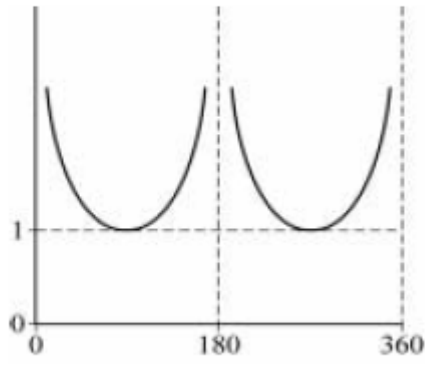
4(a)(i)		B1	1	$y=\|x\|$
(ii)		M1		2 branches mod graph $x>0$ for $y=0$
		A1	2	for 2, 4
(b)(i)	$x=2x-4,\ x=4$	B1		
	$-x=2x-4$	M1		
	$x=\frac{4}{3}$	A1	3	OE one value only
	Alternative:			
	$x^2=(2x-4)^2$	M1		
	$x=4,\frac{4}{3}$	A1A1		
(ii)	$\frac{4}{3}<x<4$	M1		$\frac{4}{3}$, 4 (ft) identified as extremes
		A1	2	CAO
	Total		**8**	

8(a)	$f(x)=2e^{3x}-1$			
	Range: $f(x)>-1$ (or $y>-1$ or $f>-1$)	M1		for –1 only
		A1	2	exactly correct
(b)	$y=2e^{3x}-1$			
	$x=2e^{3y}-1$	M1		$x\leftrightarrow y$
	$2e^{3y}=x+1$			
	$e^{3y}=\frac{x+1}{2}$	M1		attempt to isolate
	$y=\frac{1}{3}\ln\left(\frac{x+1}{2}\right)$	A1	3	all correct with no error AG (be convinced)
(c)	$f'^{-1}(x)=\frac{1}{3}\left(\frac{2}{x+1}\right)\times\frac{1}{2}$ OE	M1		for differentiation of ln; $\frac{k}{\text{their}(x\pm1)}$
		A1		for $\frac{1}{2}$
		A1		all correct
	$x=0$			
	$f'^{-1}(x)=\frac{1}{3}$	A1	4	CSO
	Alternative			
	$f^{-1}(x)=\frac{1}{3}\ln(x+1)-\frac{1}{3}\ln 2$	M1A1		
	$f'^{-1}(x)=\frac{1}{3(x+1)}$	A1		
	$f'^{-1}(0)=\frac{1}{3}$	A1		CSO
	Total		**9**	

9(a)	$x=\frac{1}{2}\quad y=\frac{\pi}{2}\quad \left(\text{or } 1.57, \sin^{-1}1\right)$	B1	1	ignore 90°

3(a)	$\text{f}(x)\leq 3$	M1A1	2	M1 for $\text{f} < 3$, $x \leq 3$ Condone y, f, range
(b)(i)	$y=\frac{2}{x+1}$			
	$x+1=\frac{2}{y}$	M1		Attempt to obtain x as a function of y or y as a function of x
	$x=\frac{2}{y}-1$	M1		$x \leftrightarrow y$ at any stage
	$y/\text{g}^{-1}(x)=\frac{2}{x}-1=\frac{2-x}{x}$	A1	3	Any correct form
(ii)	$\left(\text{g}^{-1}(x)\right)\neq -1$	B1	1	
(c)(i)	$\text{h}(x)=\frac{2}{3-x^2+1}$	M1		
	$=\frac{2}{4-x^2}=\frac{2}{(2-x)(2+x)}$	A1	2	
(ii)	$(x\in\mathbb{R}),\ x\neq+2,\ x\neq-2$	B1	1	Condone omit 'x is real' Allow $x^2 \neq 4$
	Total		**9**	

7(a)		B1	1	
(b)		M1 A1 A1	3	Shape inverted V in all four quadrants Symmetrical about y axis Coordinates
(c)	$4-\lvert 2x\rvert = x$			
	$4-2x=x \qquad x=\frac{4}{3}$	M1 A1		Attempt to solve
	$4+2x=x \qquad x=-4$	A1	3	And no others
(d)	$-4<x<\frac{4}{3}$	M1 A1	2	Either correct Other solution and no extras SC $-4\leq x\leq\frac{4}{3}$ B1

3(a)	$\text{cosec}\, x = 2$ $\Rightarrow \sin x = \frac{1}{2}$		M1		30° scores M1 implied
	$x = 30, 150$		A1	2	and no extras in range
(b)(i)	1		B1	1	
(ii)			M1		all positive, 2 U shapes
			A1	2	minima consistent > 0, **not** intersecting with each other or y-axis
(c)	$x = 30, 150, 210, 330$		B1F		3 correct values from their (a), which must be $\theta, 180 - \theta$
			B1	2	all correct and no extras in range

5(a)	$f(x) \geq 0$	allow $y \geq 0$	M1		> 0 or $f \geq 0$ or ≥ 0
			A1	2	
(b)(i)	$\sqrt{\frac{1}{x} - 2}$		B1	1	
(ii)	$\frac{1}{x} - 2 = 1$		M1		squaring their (b)(i) in an equation
	$\frac{1}{x} = 3$	OE	A1		
	$x = \frac{1}{3}$		A1	3	CSO
(c)	$y = \sqrt{x - 2}$				
	$y^2 = x - 2$		M1		attempt to isolate; condone 1 slip
	$x^2 = y - 2$		M1		reverse $x \Leftrightarrow y$
	$y = x^2 + 2$		A1	3	
		Total		**9**	