Core 3 Integration Answers

3(a)(i)	$f' = \frac{dy}{dx} = 4x^3 + 2$	B1	1	
(ii)	$\int \frac{2x^3+1}{x^4+2x}\,dx$			
	$= \frac{1}{2}\ln\left(x^4+2x\right)\ (+c)$	M1 A1	2	For $k\ln\left(x^4+2x\right)$ By substitution $k\ln u$ M1 correct A1
(b)(i)	$u = 2x+1$			
	$du = 2\,dx$	B1		
	$\int x\sqrt{2x+1}\,dx =$			
	$\int \left(\frac{u-1}{2}\right)\sqrt{u}\,\frac{du}{2}$	M1		Must be in terms of u only incl. du
	$= \frac{1}{4}\int \left(u^{\frac{3}{2}} - u^{\frac{1}{2}}\right)du$	A1	3	AG
(ii)	$\int_0^4 dx = \int_1^9 du$	B1		Or changing u's to x's at end
	$\frac{1}{4}\int u^{\frac{3}{2}} - u^{\frac{1}{2}} = \frac{1}{4}\left[\frac{u^{\frac{5}{2}}}{\frac{5}{2}} - \frac{u^{\frac{3}{2}}}{\frac{3}{2}}\right]$	M1 A1		
	$= \frac{1}{4}\left[\left(\frac{2}{5}(9)^{\frac{5}{2}} - \frac{2}{3}(9)^{\frac{3}{2}}\right) - \left(\frac{2}{5} - \frac{2}{3}\right)\right]$			Sight of any of these 3 lines
	$= \frac{1}{4}\left[79.2 + 0.2\dot{6}\right]$			
	$= 19.86$			
	$= 19.9$	A1	4	AG
	Total		**10**	

(b)	$\int x^{-2}\ln x\,dx \quad u=\ln x \quad dv=x^{-2}$	M1		Attempt at integration by parts
	$du=\frac{1}{x} \quad v=-x^{-1}$	A1		
	$\int = -\frac{1}{x}\ln x+\int x^{-2}\,dx$	A1		
	$=-\frac{1}{x}\ln x-\frac{1}{x}(+c)$	A1	4	
(ii)	$R=\left[-\frac{1}{x}(\ln x+1)\right]_1^5$	M1		$R=\left[\text{ Their}(b)\right]_1^5$
	$=-\frac{1}{5}(\ln 5+1)+(\ln 1+1)$	A1		OE
	$=\frac{1}{5}(4-\ln 5)$	A1	3	convincing argument; AG

(b)	$\int x(2x+1)^8\,dx$ $u=2x+1$ $du=2\,dx$	B1		OE
	$\int=\int\left(\frac{u-1}{2}\right)u^8\left(\frac{du}{2}\right)$	M1		all in terms of u. Condone omission of du
	$=\frac{1}{4}\int u^9-u^8\,du$			
	$=\frac{1}{4}\left[\frac{u^{10}}{10}-\frac{u^9}{9}\right]$	B1		$p\frac{u^{10}}{10}+q\frac{u^9}{9}$
	$=\frac{(2x+1)^{10}}{40}-\frac{(2x+1)^9}{36}(+c)$	A1	4	OE; CAO SC: correct answer, no working/parts in x (B1)

4(a)	$\int x\sin x\,dx \quad u=x$			
	$\frac{dv}{dx}=\sin x$	M1		For differentiating one term and integrating other
	$\frac{du}{dx}=1 \quad v=-\cos x$			
	$\int = -x\cos x-\int -\cos x(dx)$	m1		For correctly substituting their terms into parts formula
		A1		
	$=-x\cos x+\sin x\ (+c)$	A1	4	CSO
(b)	$u=x^2+5$			
	$du=2x\,dx$			
	$\int = \int \frac{1}{2}u^{\frac{1}{2}}(du)$	M1		$\int ku^{\frac{1}{2}}(du)$ condone omission of du but M0 if dx
		A1		$k=\frac{1}{2}$ OE
	$=\frac{u^{\frac{3}{2}}}{3}$	A1√		Ft $\int ku^{\frac{1}{2}}du$
	$=\frac{1}{3}\sqrt{(x^2+5)^3}\ (+c)$	A1	4	CSO SC $\frac{2}{6}\sqrt{(x^2+5)^3}$ with no working B3
(c)	$y=x^2-9$			
	$x^2=y+9$			
	$V=\pi\int x^2dy$	B1		Must have π and x^2, condone omission of dy, but B0 if dx
	$=\pi\int(y+9)dy$			
	$=(\pi)\left[\frac{y^2}{2}+9y\right]_1^2$ or $(\pi)\left[\frac{(y+9)^2}{2}\right]_1^2$	M1		$\int$"their x^2"dy integrated; Limits 2 and 1 substituted in correct order including – sign } π not necessary
	$=(\pi)[20-9\frac{1}{2}]$	m1		
	$=10\frac{1}{2}\pi$	A1	4	CSO
	Total		**12**	

6(a)	$\int x e^{5x} dx$			
	$u = x \quad dv = e^{5x}$	M1		integrate one term, differentiate one term
	$du = 1 \quad v = \frac{1}{5}e^{5x}$	A1		
	$\int = \frac{1}{5}xe^{5x} - \int \frac{1}{5}e^{5x}dx$	A1		
	$= \frac{1}{5}xe^{5x} - \frac{1}{25}e^{5x} \ (+c)$	A1	**4**	
(b)(i)	$u = x^{\frac{1}{2}}$			
	$du = \frac{1}{2}x^{-\frac{1}{2}} \ dx$	M1		
	$\int = \int \frac{1}{1+u} \times 2du$	A1	2	correct with no errors; AG
(ii)	$\int_1^9 dx = \int_1^3 \frac{2}{1+u}du$	m1		correct limits used in correct expression, ignoring k
	$= \left[2\ln(1+u)\right]_1^3$	M1		for $k\ln(1+u)$
	$= 2\ln 4 - 2\ln 2$ $(= \ln 4)$	A1	3	ISW OE
	Total		**9**	