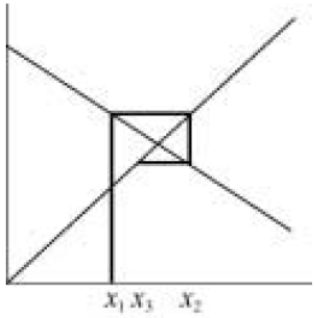
Core 3 Numerical Methods Answers

2	$\int_1^3 \frac{1}{\sqrt{1+x^3}}\,dx$			
	$\begin{array}{cc} x & y \\ 1 & 0.707\,(1) \\ 1.5 & 0.478\,(1) \\ 2 & 0.333\,(3) \\ 2.5 & 0.245\,(3) \\ 3 & 0.189\,(0) \end{array}$	B1 B1		3 correct } **SC** B1 for all correct expressions but all correct } wrongly evaluated
	$A = \frac{1}{3}\times 0.5\left[\begin{array}{l} y(1)+y(3)+ \\ 4\left(y(1.5)+y(2.5)\right)+2(y(2)) \end{array}\right]$	M1		use of Simpson's rule
	$=0.743$	A1	4	
	Total		**4**	

6(a)	$f(0.5) = -0.875$ $f(1) = 2$ Change of sign $\therefore$ root	M1 A1	 2	
(b)	$x^3 + 4x - 3 = 0$ $4x = 3 - x^3$ $x = \frac{3-x^3}{4}$	 B1	 1	 AG
(c)(i)	$x_1 = 0.5$ $x_2 = 0.71875$ 0.72 AWRT $x_3 = 0.66$	M1 A1 A1	 3	
(ii)	 	M1 A1 A1	 3	For cobweb, x_1 to curve For x_2 All correct
	Total		**9**	

1(a)	$f(2) = -1$ $f(2.1) = +0.161$ }	M1		both attempted
	change of sign $\therefore 2 < \alpha < 2.1$	A1	2	
(b)	$x^3 - x - 7 = 0$			
	$x^3 = x + 7$			
	$x = \sqrt[3]{x+7}$	B1	1	AG
(c)	$x_1 = 2$	M1		
	$x_2 = 2.0801...$	A1		AWRT 2.08
	$x_3 = 2.0862...$			AWRT 2.09
	$x_4 = 2.09$	A1	3	
	Total		**6**	

6(a)	$\therefore \int \ln x = 1(\ln 1.5 + \ln 2.5 + \ln 3.5 + \ln 4.5)$	M1 A1		use of 1.5, 2.5,… ; 3 or 4 correct x values AWFW 4 to 4.2
	$= 4.08$	A1	3	CAO

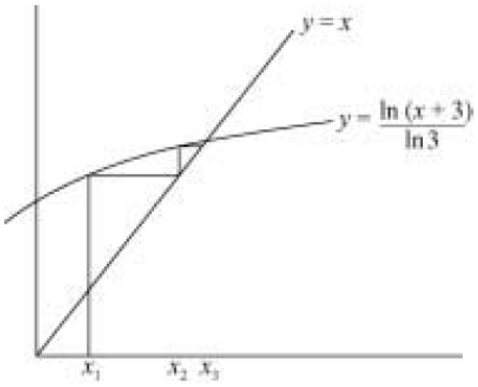
(c)	$V = (k)\int \sec^2 x \, dx$	M1		
	$= (k)[\tan x]_0^1$	A1		
	$= 4.89$	A1	3	CAO

1	$x =$ 1.5, 2.5, 3.5, 4.5	M1 A1		Method x values
	$y_1 = 0.7115$ 0.712 $y_2 = 0.5218$ 0.522 $y_3 = 0.4439$ 0.444 $y_4 = 0.3993$ 0.399 } AWRT	A1		3 correct y's
	$A = 1 \times (y_1 + y_2 + y_3 + y_4)$ $= 2.08$	A1	4	
	Total		**4**	

8(a)	$A(-1,\pi)$	B1		
	$B\left(0,\frac{\pi}{2}\right)$	B1	2	
(b)	$\cos^{-1}x-3x-1=0$			
	$f(0.1)=0.17$ allow 0.2, 0.1	M1		Or comparing 'sides'
	$f(0.2)=-0.23$ allow -0.2			
	Change of sign $\therefore$ root	A1	2	
(c)	$x_1=0.1$	M1		
	$x_2=0.1569=0.157$	A1		
	$x_3=0.1378=0.138$			
	$x_4=0.144$	A1	3	
	Total		**7**	

(b)	$V=4(\pi)\int_2^4(x-1)^3\,dx$	M1		$(\pi)\int y^2\,dx$
	$=4\pi\left[\frac{(x-1)^4}{4}\right]_2^4$	M1 m1		$k(x-1)^4\ (\pi)$ or in expanded form correct substitution of limits into $k(x-1)^4$
	$=\pi(81-1)=80\pi$	A1	4	CAO
(c)	Translate	E1		
	$\begin{pmatrix}1\\0\end{pmatrix}$	B1		OE
	Stretch (I) SF 2 (II) // y axis (III)	M1 A1	4	for I and (II or III) for I and II and III

4(a)	x $\quad$ y x_0 1 3 x_1 1.25 3.948(2) x_2 1.5 5.196(2) x_3 1.75 6.838(5) x_4 2 9	B1 B1		x values PI (4 +) y values correct
	$A=\frac{1}{3}\times\frac{1}{4}(3+4\times3.9482+2\times5.1962+4\times6.8385+9)$	M1		Simpson's rule
	$=5.46$	A1	4	CAO
(b)(i)	$f(x)=3^x-x-3$ $f(0.5)=-1.77$ $f(1.5)=0.696$ change of sign $\therefore$ root	M1A1	2	

(ii)	$3^x = x+3$ $\ln 3^x = \ln(x+3)$	M1		correct use of logs
	$x\ln 3 = \ln(x+3)$ $x = \dfrac{\ln(x+3)}{\ln 3}$	A1	2	correct with no mistakes; AG
(iii)	$x_1 = 0.5$ $(x_2 = 1.14)$ $x_3 = 1.29 = 1.3$	M1 A1	2	CAO
(iv)		M1 A1	2	staircase x_2, x_3 correct and labelled on x-axis
	Total		**12**	