Core 3 Trigonometry Answers

1(a)	$\frac{dy}{dx} = 3\sec^2 3x$		M1 A1	2	for sec $3x$ SC/$3\sec^2 x$ B1
	Alternative Use of product/Quotient rule $\frac{3\cos^2 3x + 3\sin^2 3x}{\cos^2 3x}$	(M1) (A1)			Good attempt Correct

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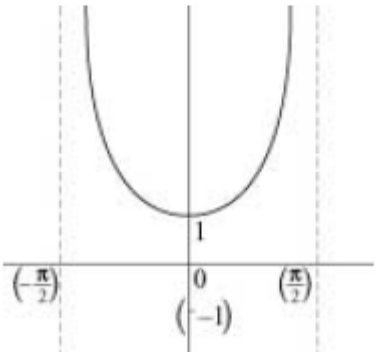
4(a)	$2\operatorname{cosec}^2 x = 5(1-\cot x)$			
	$2+2\cot^2 x = 5-5\cot x$	M1		use of $\operatorname{cosec}^2 x = 1+\cot^2 x$
	$2\cot^2 x + 5\cot x - 3 = 0$	A1	2	AG
(b)	$(2\cot x - 1)(\cot x + 3) = 0$	M1		or $2+5t-3t^2=0$ Or in $\tan x$ $(2-t)(1+3t)=0$
	$\cot x = \frac{1}{2}, -3$			
	$\tan x = 2, -\frac{1}{3}$	A1	2	AG
(c)	$x = 1.1, -2.0$ $x = -0.3, 2.8$ } AWRT	B1 B1 B1	3	Any 2 correct] In degrees: B0 Any 3 correct B1 4 correct] B2

3(a)	$\sec x = 5$ $\cos x = 0.2$	M1		
	$x = 1.37,\ 4.91$ AWRT	A1A1	3	
(b)	$\tan^2 x = 3\sec x + 9$			
	$\sec^2 x - 1 = 3\sec x + 9$	M1		for using $\sec^2 x = 1+\tan^2 x$ OE
	$\sec^2 x - 3\sec x - 10 = 0$	A1	2	AG
(c)	$(\sec x - 5)(\sec x + 2) = 0$	M1		or use of formula (attempt)
	$\sec x = 5, -2$ $\cos x = 0.2, -0.5$	A1		
	$x = 1.37,\ 4.91$	B1F		any 2 correct or ft their 2 answers in (a)
	$2.09,\ 4.19$	A1	4	all 4 correct, no extras

7(a)	$z=\dfrac{\sin x}{\cos x}$			
	$\dfrac{dz}{dx}=\dfrac{\cos x\cos x-\sin x(-\sin x)}{\cos^2 x}$	M1 A1		use of quotient rule $\left(\dfrac{\pm\cos^2 x\pm\sin^2 x}{\cos^2 x}\right)$
	$=\dfrac{1}{\cos^2 x}$			
	$=\sec^2 x$	A1	3	AG (be convinced)
(b)		M1		correct shape including asymptotic behaviour and symmetrical about $x=0$ and $y>0$
		A1	2	use of 1

(b)(i)	$y=\sin^{-1}2x$ $\sin y=2x$ and $\dfrac{1}{2}\sin y=x$	B1	1	AG (be convinced)
(ii)	$\dfrac{dx}{dy}=\dfrac{1}{2}\cos y$	B1	1	
(c)	$\dfrac{dy}{dx}=\dfrac{2}{\cos y}$	M1A1		M1 for $\dfrac{k}{\cos y}$
	$\sin y=2x$ and $\sin^2+\cos^2=1$	M1		use of to get $\cos y$ or $\cos^2 y$
	$\cos y=\sqrt{1-4x^2}$ $\dfrac{dy}{dx}=\dfrac{2}{\sqrt{1-4x^2}}$	A1	4	AG; condone omission of proof of sign

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2	Stretch (I) SF $\dfrac{1}{3}$ (II)	M1		For I + (II or III)
	Parallel to $x-$axis (III)	A1		All correct
	Translate	E1		Allow translation
	$\begin{pmatrix}0\\1\end{pmatrix}$	B1	4	Correct vector or description

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5(a)(i)	$2(\text{cosec}^2 x - 1) + 5\,\text{cosec}\, x = 10$	M1		
	$2\,\text{cosec}^2 x - 2 + 5\,\text{cosec}\, x - 10 = 0$			
	$2\,\text{cosec}^2 x + 5\,\text{cosec}\, x - 12 = 0$	A1	2	**AG**
(ii)	$(2\,\text{cosec}\, x - 3)(\text{cosec}\, x + 4) = 0$	M1		Attempt to solve
	$\text{cosec}\, x = \frac{3}{2}$ or -4	A1		Condone answers with no method shown
	$\sin x = \frac{2}{3}$ or $-\frac{1}{4}$	A1	3	**AG**
(b)	$(\theta - 0.1) = 0.73, 2.41, -0.25, -2.89$ AWRT	B1		2 correct values, may be implied later $(41.8, 138.2, -165.5, -14.5)$
	$\theta = 0.83, 2.51, -0.15, -2.79$ AWRT	B1		2 correct answers
		B1	3	+ 2 correct answers and no extra within range

(ii)	$y = x^2 \tan x$	M1		Product rule
	$\frac{dy}{dx} = x^2 \sec^2 x + 2x \tan x$	A1	2	

8(a)	$\tan x \ (+c)$	B1	1	
(b)	$f(x) = \frac{\cos x}{\sin x}$			
	$f'(x) = \frac{-\sin^2 x - \cos^2 x}{\sin^2 x}$	M1		quotient rule $\frac{\pm \sin^2 x \pm \cos^2 x}{\sin^2 x}$
		A1		
	$= \frac{-1}{\sin^2 x}$	A1		use of $\sin^2 x + \cos^2 x = 1$
	$= -\text{cosec}^2 x$	A1	4	AG CSO
				Special cases
				$f(x) = \frac{\cot x}{1}$
				$f'(x) = \frac{1 \times -\text{cosec}^2 x - \cot x \times 0}{1^2}$ M1
				$= -\text{cosec}^2 x$ A1 (max 2/4)
				Or
				$f(x) = \frac{1}{\tan x}$
				$f'(x) = \frac{\tan x \times 0 - 1 \times \sec^2 x}{\tan^2 x}$ M1 A1
				$= \frac{-\sec^2 x}{\tan^2 x}$
				$= \frac{-1}{\sin^2 x} = -\text{cosec}^2$ A1 (max 3/4)

(c)	LHS $= \tan^2 x + \cot^2 x + 2\tan x \cot x$	M1		expanding
	$= \tan^2 x + 1 + \cot^2 x + 1$	M1		correct use of trig identities
	$= \sec^2 x + \operatorname{cosec}^2 x$	A1	3	CSO
	$=$ RHS			
(d)	$\int (\tan x + \cot x)^2 \, dx = \int \sec^2 x + \operatorname{cosec}^2 x \, dx$	M1		use of identity
	$= \left[\tan x - \cot x\right]_{0.5}^{1}$	M1 A1		$\pm \tan x \pm \cot x$ OE
	$= 0.9153 - -1.2842$			
	$= 2.2$	A1	4	AWRT